Brief introduction to vehicle dynamics

The following is a very brief introduction to vehicle dynamics, with emphasis on a solarcar-like vehicle.

Assume all numbers in this document are approximate at best, and wrong at worst- read it only for qualitative information!

All angles (especially front tire angles) are exaggerated for clarity.

If you are interested in learning more I highly recommend taking Prof. Gerdes excellent vehicle dynamics class.

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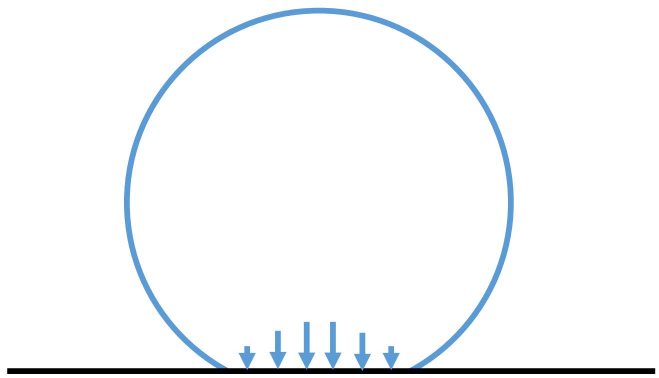
The Contact Patch

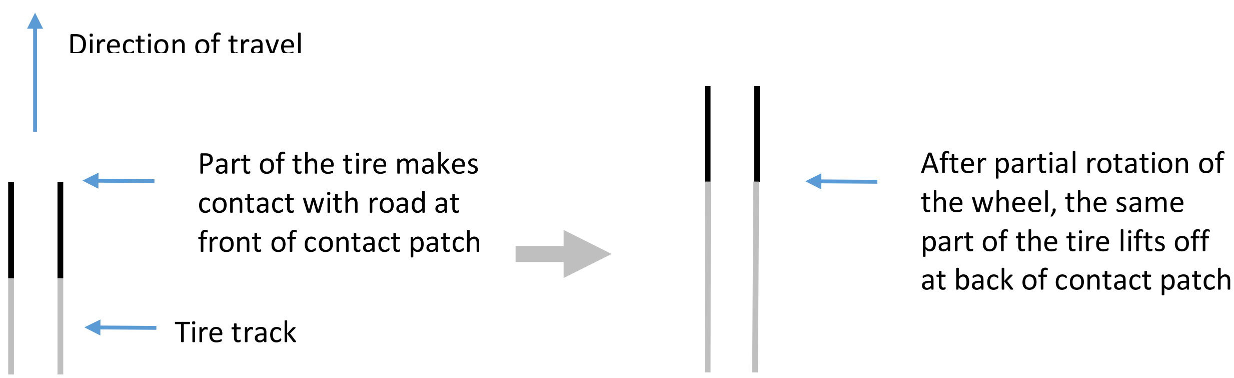
There is a finite region of the tire, called the contact patch, that is in contact with the ground. The weight of the car is distributed across this region. The distribution looks something like this:



The tire is more compressed in the middle so pressure is correspondingly higher.

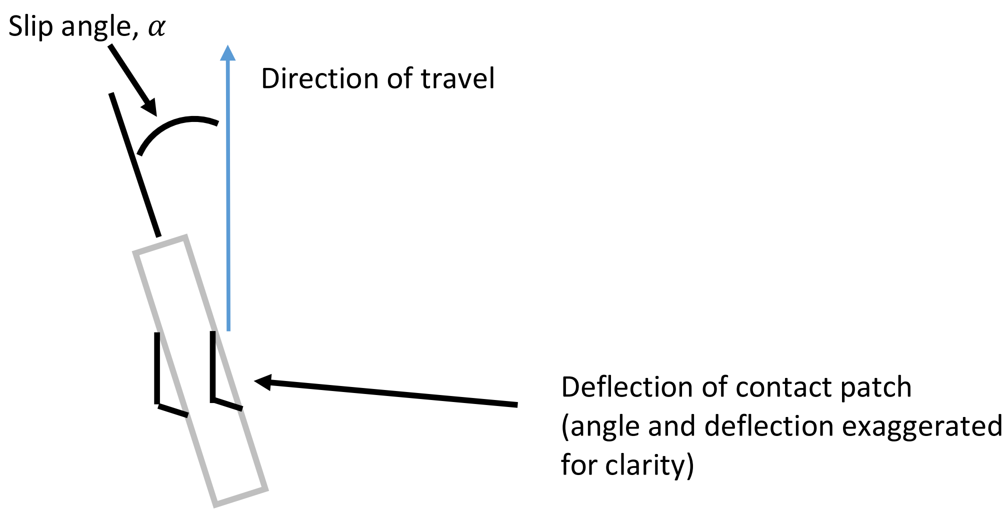
Slip Angle

Under normal driving conditions, the contact patch is stationary relative to the road that it is in contact with.

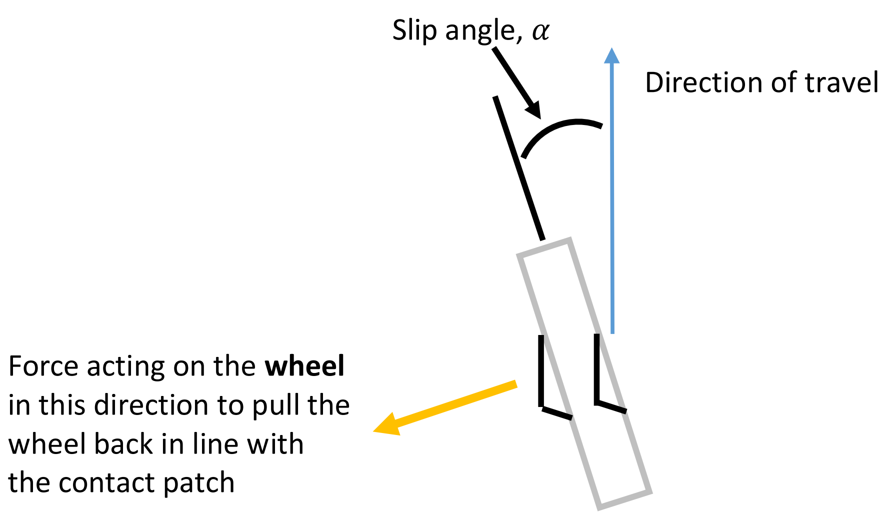


In this case the wheel is aligned with the direction of travel, so the tire imparts no lateral force.

If the wheel is not aligned with the direction of travel, we call this angle the slip angle. When the rubber of the tire makes contact with the road it is held to the road by friction. This causes the tire to deflect.

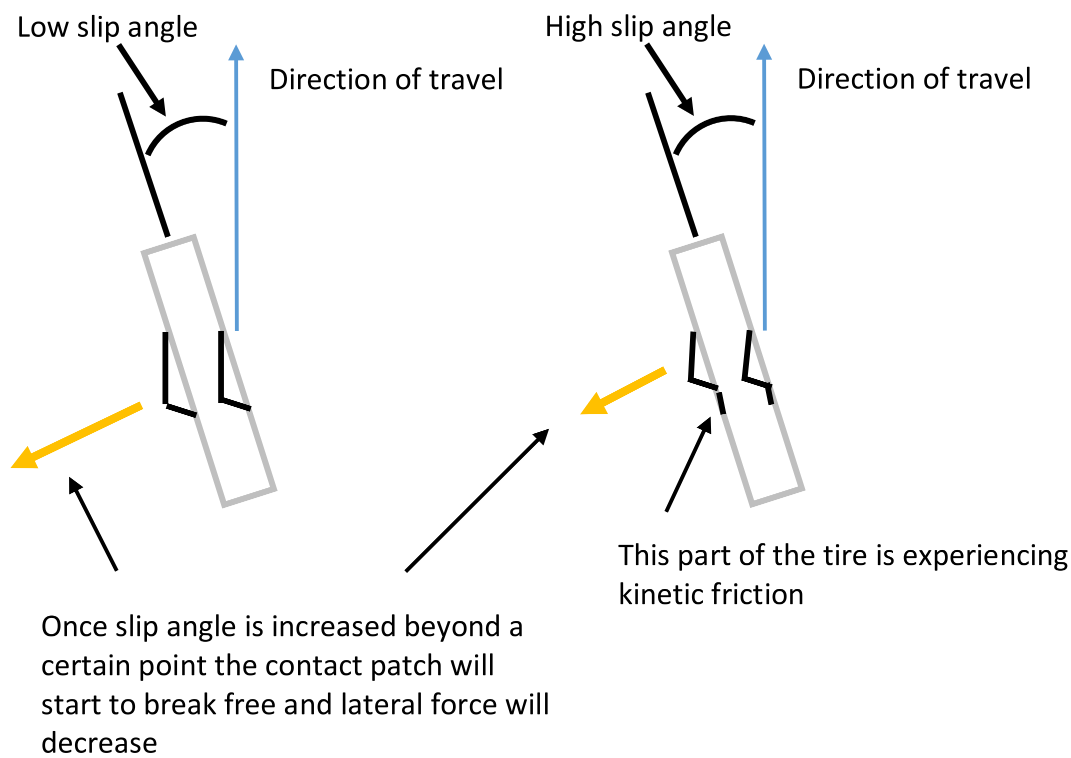


The tire is a type of spring; deflection of the tire results in a lateral force acting on the wheel at the contact patch which is acting to restore the form of the tire.



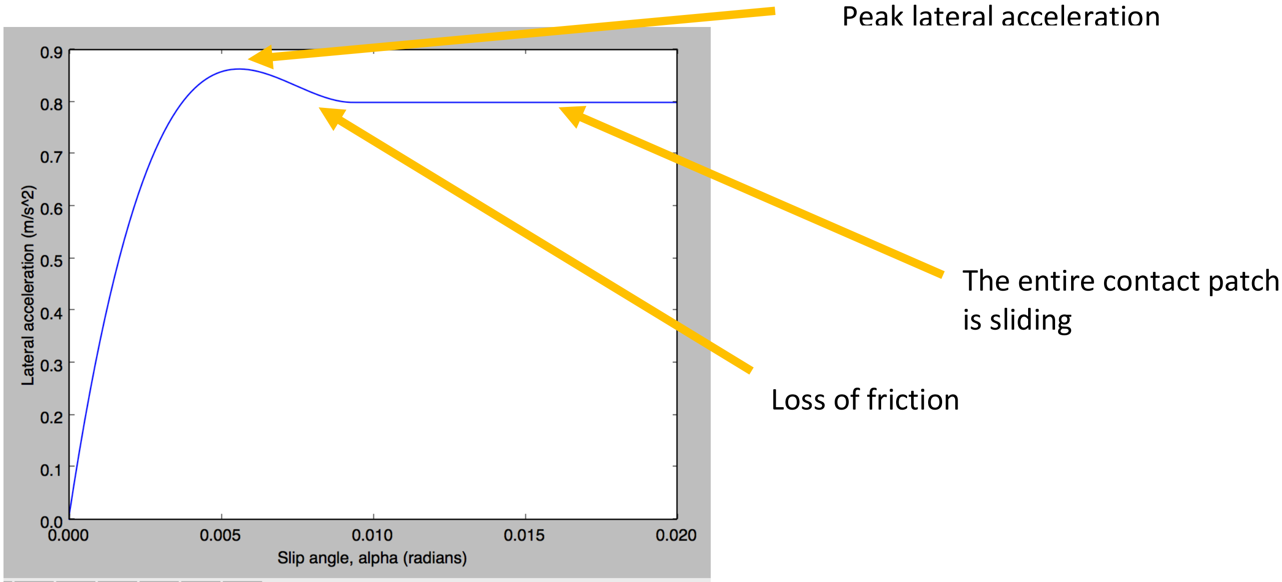
The tire can deflect a certain amount (the exact amount depending on the friction of the tire, road surface, weight on the wheel and stiffness of the tire) before the spring force will overcome friction between the tire and the road surface and the tire will snap back.

As the slip angle is increased, the tire will reach maximum deflection sooner and more of the contact patch will break free of the road surface. As the contact patch starts to break free from the surface of the road, the lateral force on the wheel is reduced.



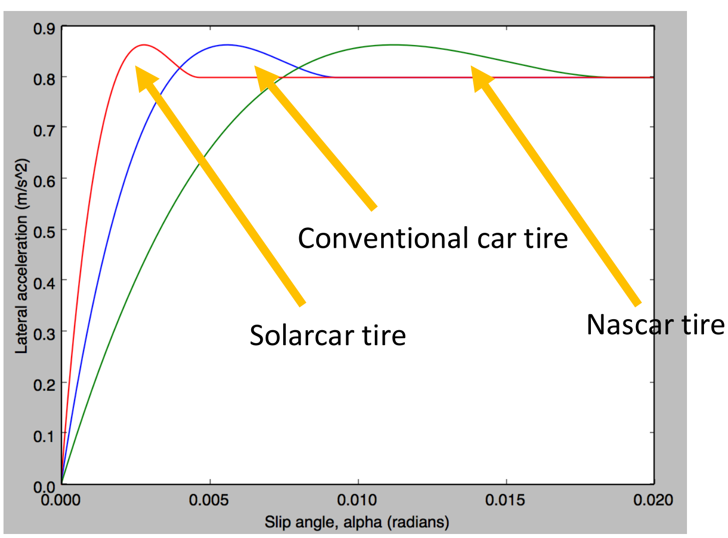
Tire Curves

The relationship between slip angle and lateral force (assuming negligible torque on the wheel due to braking or acceleration) looks something like this:



Note that I have normalized lateral force by dividing by mass on the wheel to give lateral acceleration.

What would the graph look like with different tires?

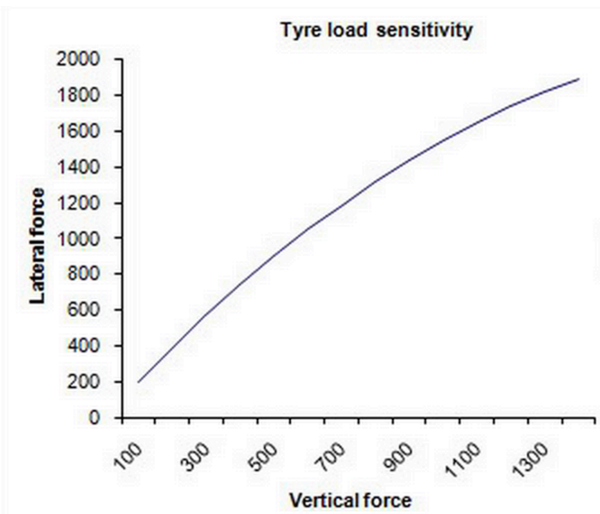


A solarcar tire is very narrow and inflated to a very high pressure to reduce rolling resistance. This makes it very stiff, so the curve becomes very steep and it will reach peak friction and saturate at a low slip angle. A race car tire, particularly one used in Nascar, will be very soft and have a very wide, predictable curve.

Changing the tire compound may also increase or decrease the peak lateral acceleration you can get out of the tire, but this is a less significant effect. Most tires can achieve around 0.8 to 1.1g peak lateral acceleration.

Effect of weight on tire curve

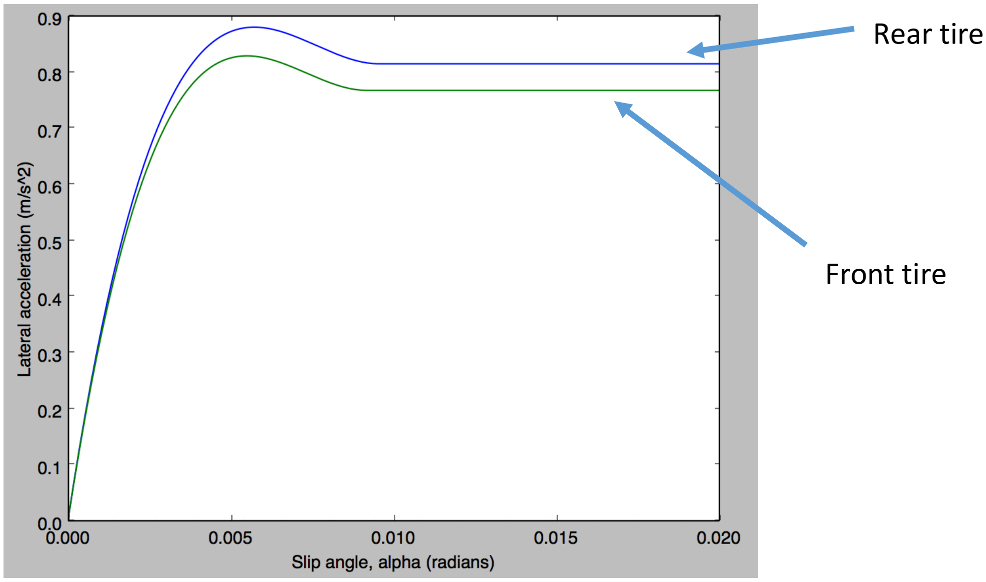
High school teaches us that frictional (lateral) force is proportional to load (vertical) force and a friction coefficient. This is true for most metals, but it isn't true for rubber. When a rubber tire is in contact with tarmac it deforms at the microscopic level. The net result is that rubber has disproportionally high friction at lower loadings, and doubling the load on a tire results in less than double the lateral frictional force.



http://smg.photobucket.com/user/Quicksilver4000/media/Dynamics/Loadsensitivity.jpg.html

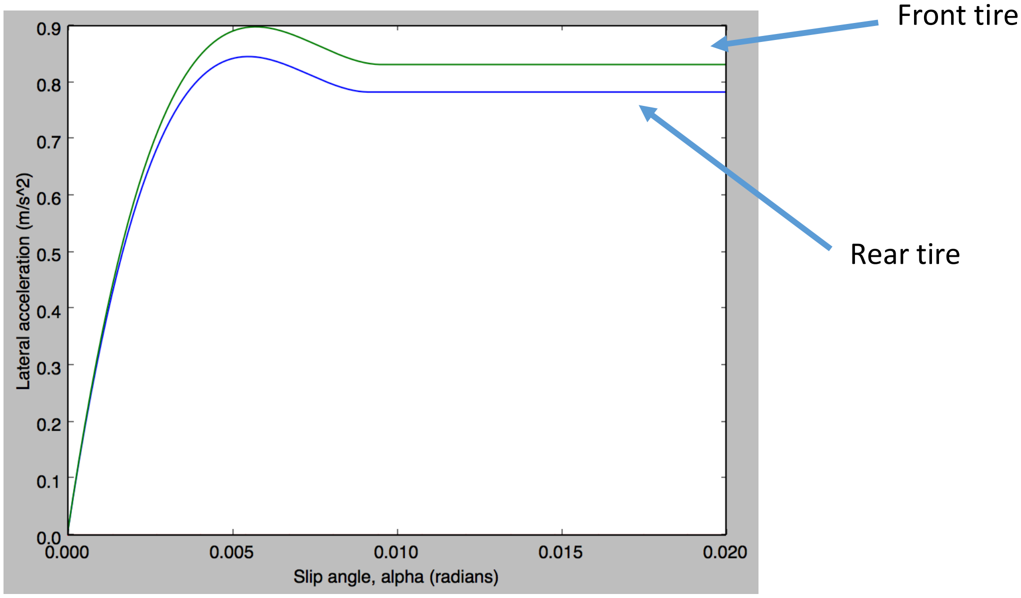
Assume that all four tires on a vehicle are identical. If weight is evenly distributed in the vehicle, the tire curve will look the same for all four wheels.

Suppose now that twice as much mass is in the front of the vehicle as is in the back. Lateral friction available to the front wheels will be slightly less than double, due to the characteristics of the rubber tire described above. Mass is double, so when we calculate $a = \frac{F}{m}$ we find that the peak lateral acceleration must be less on the front tires than it is on the rear tires.



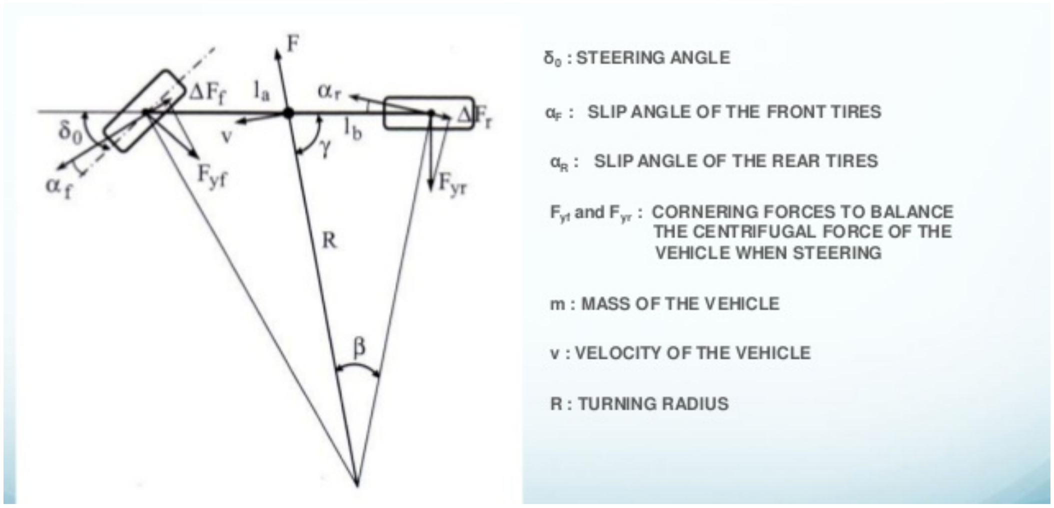
Tire curves with more than 50% vehicle mass at the front of the vehicle

Similarly, if more than 50% of the vehicle mass is towards the rear of the vehicle, the rear tire will have a lower peak lateral acceleration than the front tire.



Tire curves with more than 50% vehicle mass at the rear of the vehicle

Cornering



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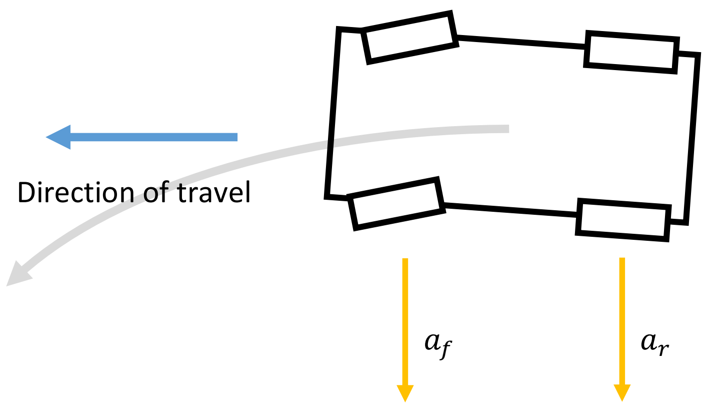
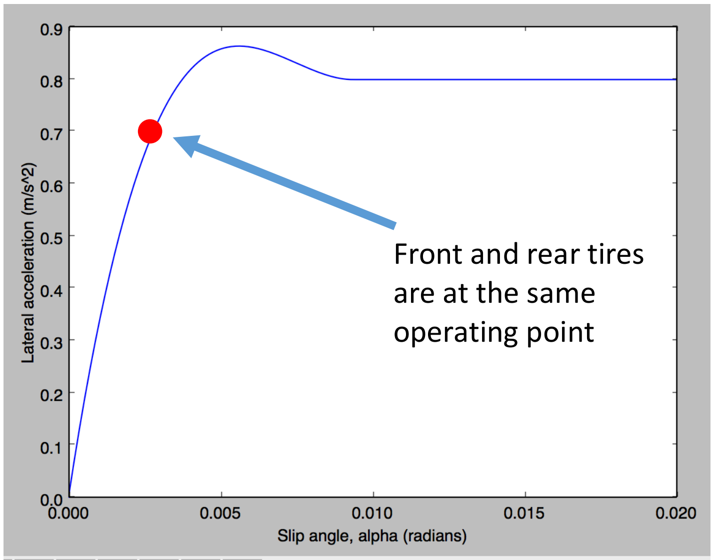
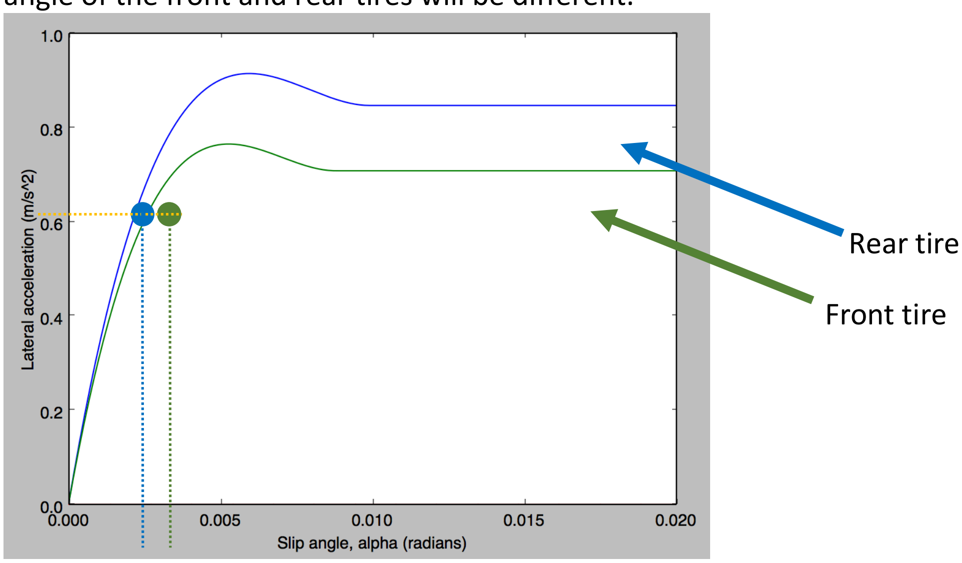


When a vehicle is cornering, assuming negligible angular rate and a stiff vehicle body, the lateral acceleration on the front and rear tires (a_f and a_r) will be equal. The lateral acceleration is the centripetal acceleration that keeps the vehicle on a circle.

If the tires are the same and the weight distribution is 50-50, this implies that the slip angles α_F and α_R will be equal:



Suppose the weight distribution is biased to the front. In this case the lateral acceleration of the front and rear tires will still be the same (the car is making the same turn), however the slip angle of the front and rear tires will be different.



As the lateral acceleration increases (corresponding to a higher speed, or a tighter corner), the difference between slip angle on the front and rear tire increases. In this case, with the vehicle weight biased towards the front, as lateral acceleration increases the slip angle of the front tire becomes larger than the slip angle on the rear tire.

Understeer

This effect is known as **understeer**. The driver must turn the steering wheel a greater amount than they expect to make the same turn.

An understeering vehicle is dynamically stable, and has a positive understeer gradient (positive K value).

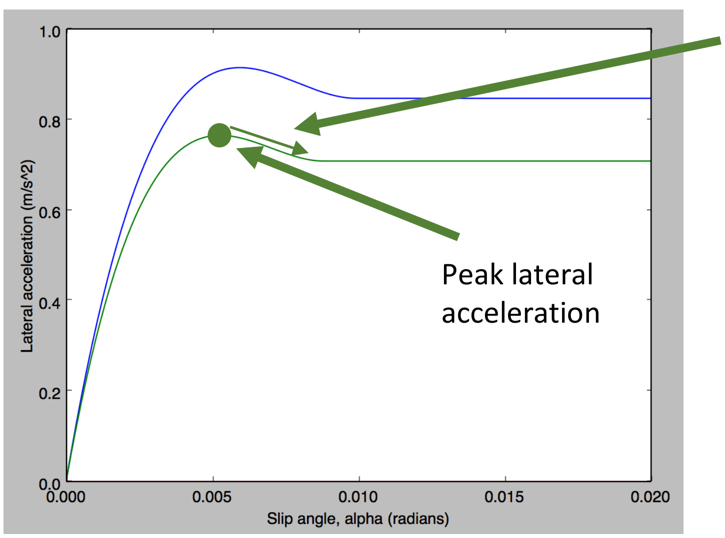
Oversteer

If the majority of the vehicle weight is distributed to the rear of the car, the opposite effect occurs. The driver must turn the wheel a lesser amount than they expect to make the same turn. This is known as **oversteer**.

An oversteering vehicle is dynamically unstable above some critical velocity, and has a negative understeer gradient (negative K value).

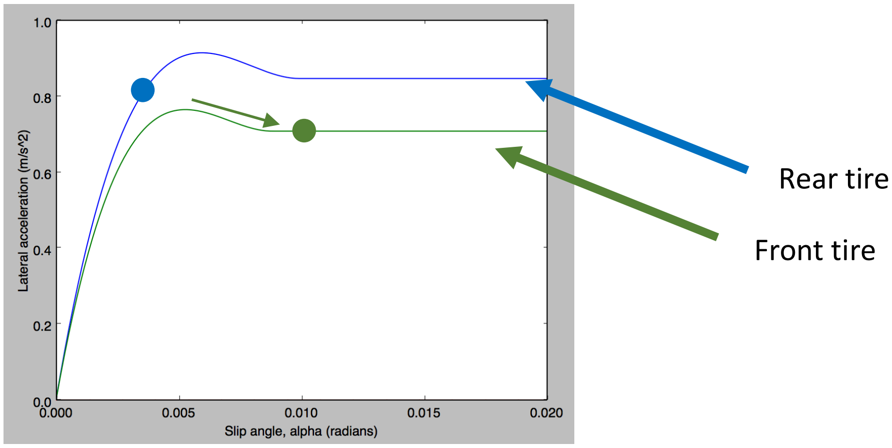
The Traction Limit

If the slip angle is increased (by increasing the speed of the turn, or by turning the steering wheel to a tighter steering angle), the tires will at some point exceed the traction limit.

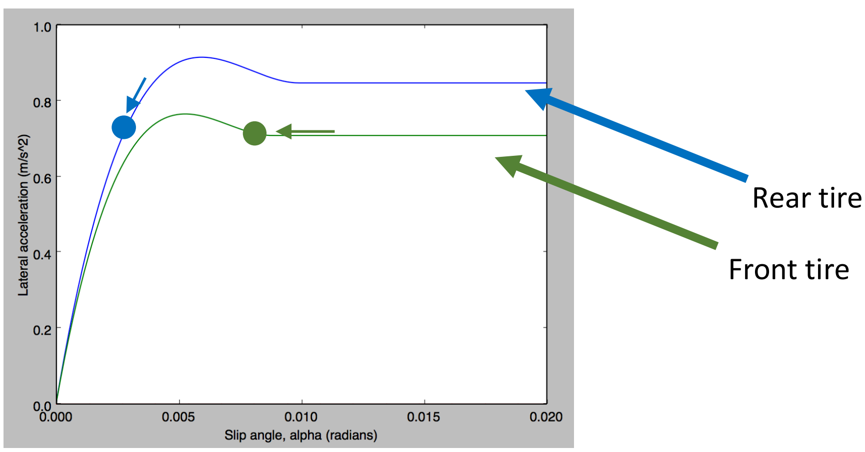


Limit Understeer

A vehicle is said to have **Limit Understeer** when the front tire loses friction first.

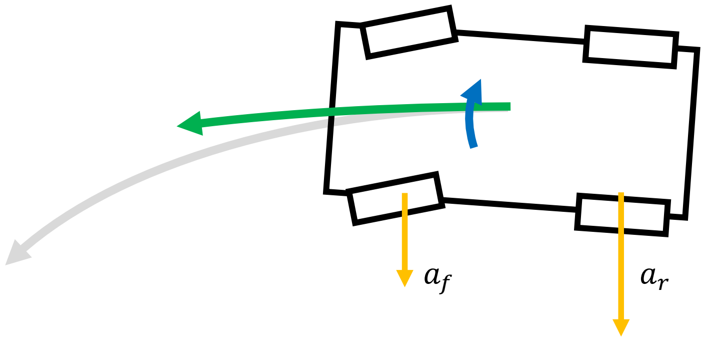


When this occurs, lateral acceleration on the front tire is reduced. This causes the vehicle to straighten out of the turn. The direction of motion of the vehicle becomes more aligned with the orientation of the rear tires.



As the vehicle becomes more aligned with the orientation of the rear tires, the slip angle on the rear tires is reduced, so the lateral acceleration is reduced and the operating point of the rear tires moves down the tire curve to a less-saturated position.

Taken to the limit, if the front tires were to lose friction completely, the vehicle would continue in a straight line tangent to the circle it was originally traversing. The rear tires would operate at the origin of the tire curve.

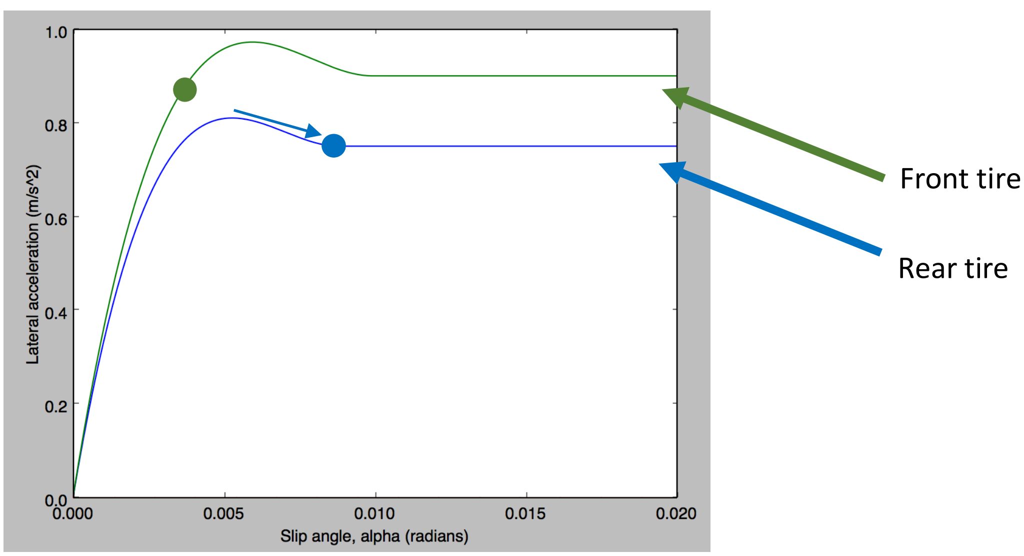


Recovery from limit understeer is achieved by straightening the front wheels (thereby reducing the slip angle) until traction is regained.

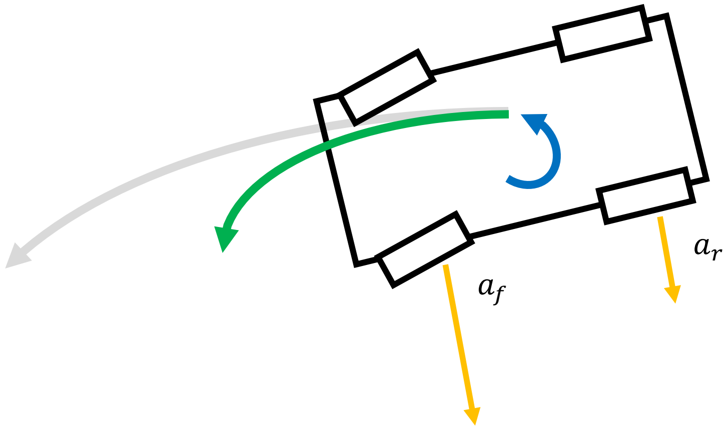
When a vehicle is in limit understeer, applying the brakes will further reduce lateral friction available to the front wheels. This will cause the vehicle to travel in an even straighter line. However, it will not cause the vehicle to become unstable (unless the brakes are applied fully and both front and rear wheels are locked).

Limit Oversteer

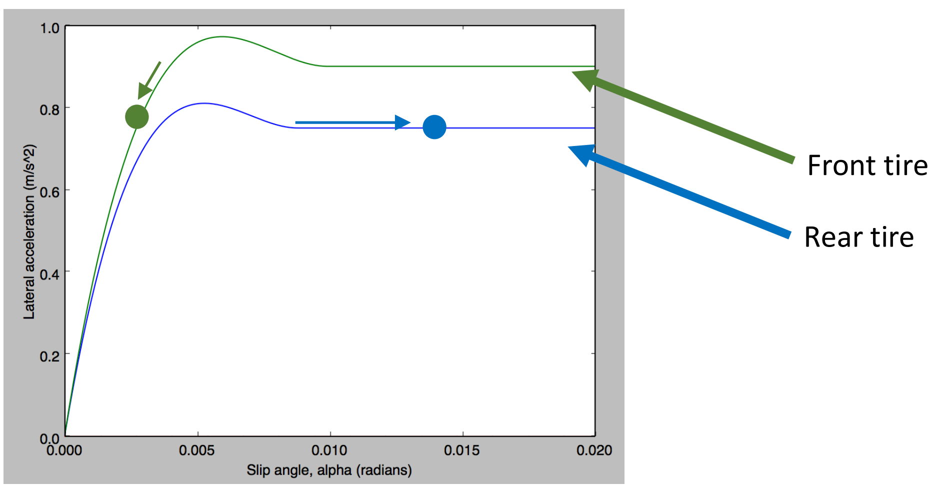
A vehicle is said to have **Limit Oversteer** when the rear wheels lose traction first.



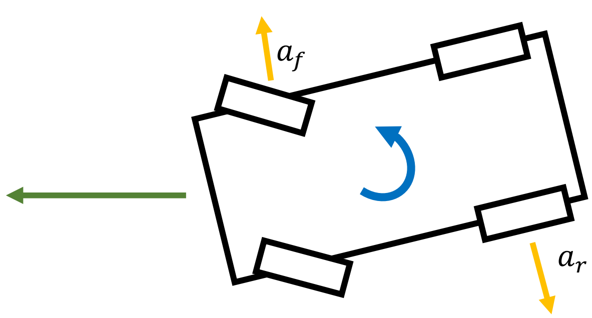
When this happens the lateral acceleration available at the rear of the car is reduced. This causes the vehicle to turn deeper into the corner.



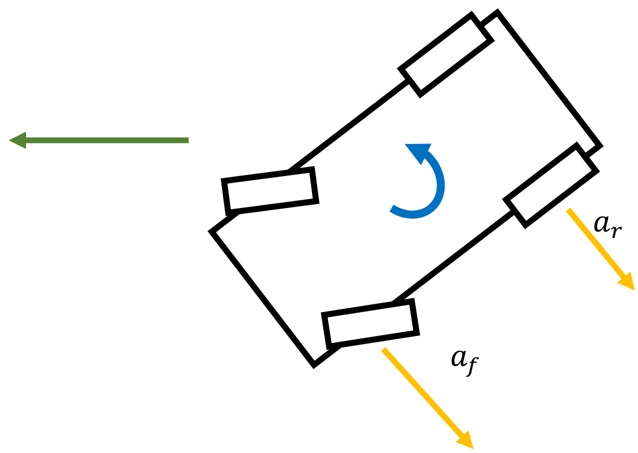
As the car rotates into the turn the slip angle on the rear wheels continues to increase. Note that this is the opposite behavior to the slip angle on the front wheels in the understeering case.



In order to regain control of the vehicle the driver must reduce the slip angle on the rear tires such that the operating point is moved back into the region of static friction. This is done by applying opposite steering lock, also known as steering into the slide:



The driver only has a very small amount of time to affect this steering input. If the driver waits too long, the maximum steering angle of the front wheels is less than the direction of travel of the vehicle plus the maximum slip angle of the front wheels. At this point the front wheels are saturated so even a maximum steering input will do nothing to correct the motion of the vehicle.



All four wheels are saturated and sliding. In an oversteering vehicle the front tire has more available kinetic friction (see tire curves above) so the angular rate of the vehicle increases.

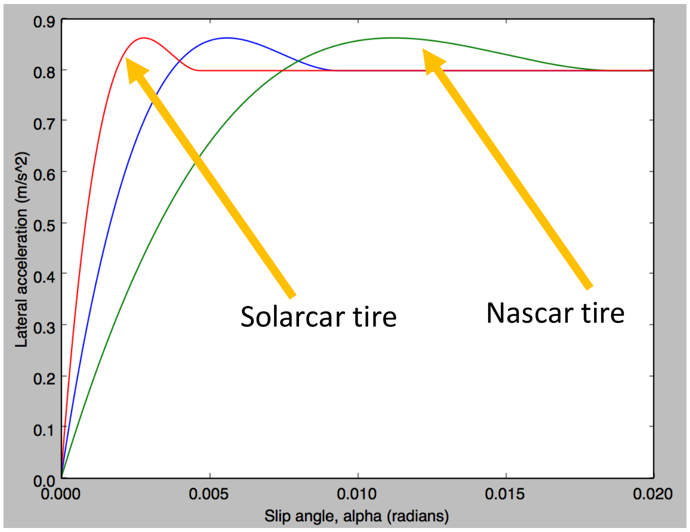
When a vehicle exhibits limit oversteer, applying the brakes will have little or no impact on the trajectory of the vehicle. If the brakes are applied before the front tires saturate, it may cause the front tires to saturate early, which may reduce angular acceleration, but the car already has some angular velocity so it will continue to spin. If the brakes are applied after the front tires saturate they will have negligible impact.

Sports car handling

Sports cars have all of the same fundamental characteristics.

They tend to be fairly evenly balanced, so front and rear tire curves are similar, which increases the lateral acceleration because the tire curves are more evenly matched.

Recall the tire curve showing cars with different tire stiffnesses:



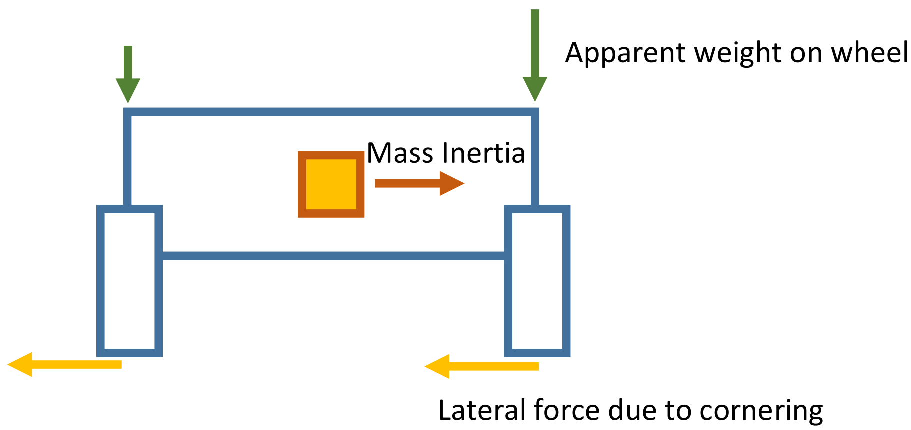
A Nascar tire is designed to be very soft. This gives it a long rolling tire curve. When a driver gets close to the friction limit they can feel the car start to slide, and when they pass the friction limit they have more time to take corrective action and bring the car back under control. This allows race drivers to stay much closer to the peak friction point on the tire curve.

A solarcar tire is designed to be very efficient, which means that it should lose as little energy as possible to deflection (which becomes heat). It should therefore be very stiff and inflated to a high pressure. This means that a solarcar tire exhibits a very steep tire curve. This is a very unforgiving curve- once the point of peak friction is exceeded, the tire will quickly saturate.

Once the friction limit is exceeded, however, in the absence of driver input, a solarcar will do the same fundamental thing as a Nascar.

Weight Transfer

As a vehicle corners it will experience some amount of weight transfer.

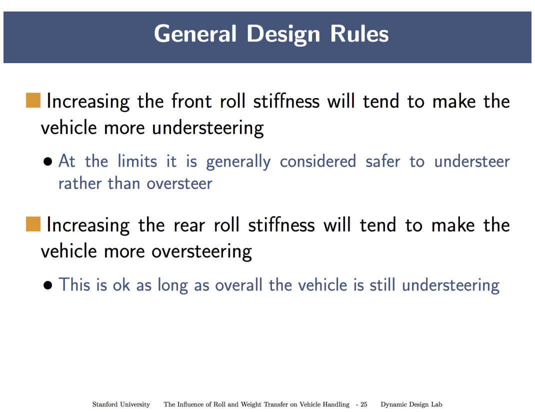


Direction of motion is into the page

In this case, the right hand wheels experience a δm increase in apparent mass and the left hand wheels experience a δm reduction in apparent mass.

Each suspension element on the vehicle has some corresponding spring stiffness. This is a combination of spring stiffness in the suspension elements and tire pressure. As additional mass is shifted to the side of the vehicle during cornering, that mass will be shifted to the front or the rear of the vehicle depending on spring stiffness.

If the front spring elements are stiffer, mass will shift to the front and the vehicle will become more understeering. If the rear spring elements are stiffer, mass will shift to the rear and the vehicle will become more oversteering.



Additional reading material

https://en.wikipedia.org/wiki/Slip_angle
This has a good explanation of tire deflection, slip angle and the effect of slip angle on the front and rear on the dynamics of the vehicle.

http://www.autozine.org/technical_school/handling/tech_handling_6.htm
Front/rear weight distribution and how you might want to set up a race car.

https://en.wikipedia.org/wiki/Understeer_and_oversteer
Understeer gradient, understeer and oversteer, limit understeer and limit oversteer.

http://www.eng-tips.com/viewthread.cfm?qid=190555
Vehicle engineers talk about how cars are designed and how much understeer is appropriate: "In fact, most cars made in the past 10 years have 2 to 3 degrees per g of understeer. (N = many hundreds) Sort of bi-modal with sportier types low 2 and others 3-ish. Trucks are more towards 3 and higher. Virtually no vehicles below 1 deg/g (well maybe 1 or 2)."

https://en.wikipedia.org/wiki/Weight_transfer
Load transfer during cornering.

http://www.crcarlson.com/Academia/Research/VehicleRoll.pdf
DDL lecture on vehicle roll (see slide 25 onwards for conclusions).