Tire Modeling

Lateral and Longitudinal Tire Forces

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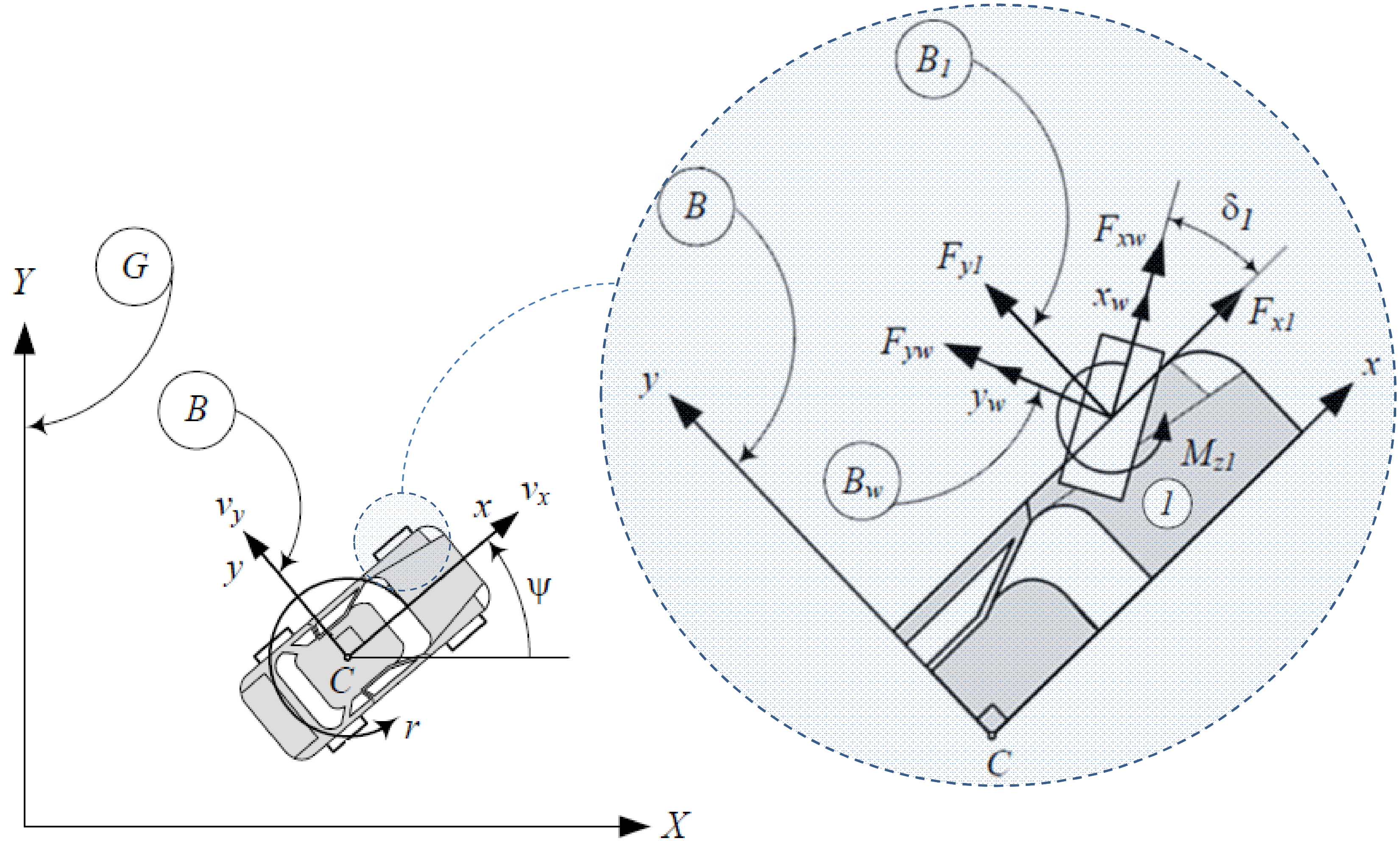
Why Tires are important for Vehicle Control Systems?

- Tires generate the forces that drive and maneuver the vehicle.
- The knowledge of **magnitude**, **direction** and **limit** of the tire forces are essential and valuable for vehicle control systems.
- However, the estimation of these variables in **all driving conditions** and in **real-time** is a very challenging task.

Horizontal Tire Forces

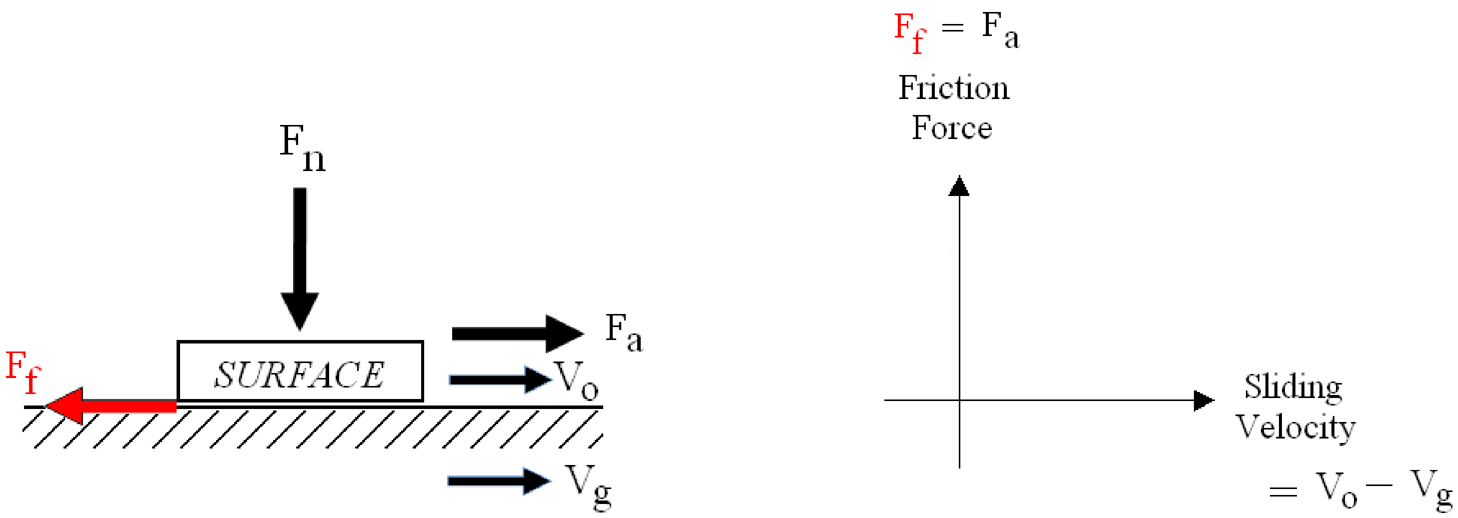


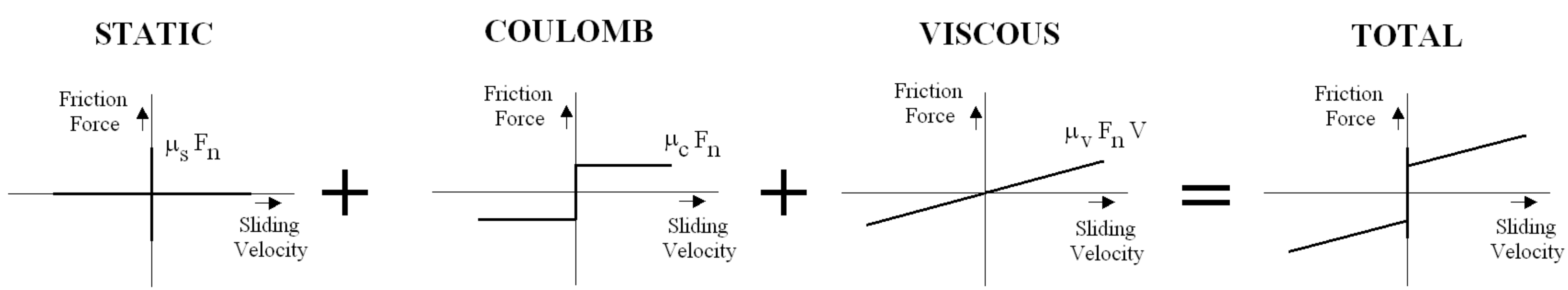
How tire force is generated?

- Tire forces are generated inside the **contact patch**, in other words between the tire and the ground.

- Tire forces are a combination of two factors:
  - **Friction/sliding** in the contact patch, and
  - **Elastic deformations/slipping** of the tire.

Background – Friction Forces



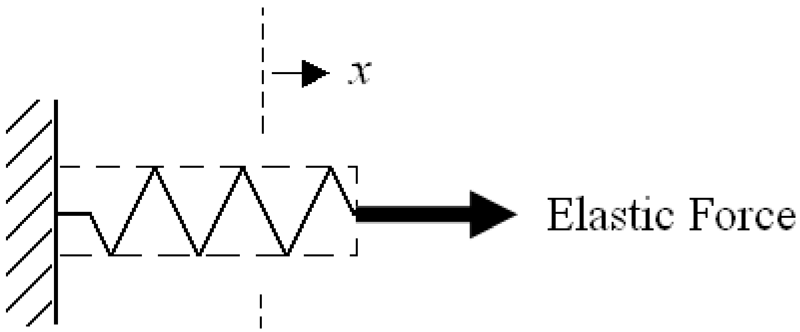


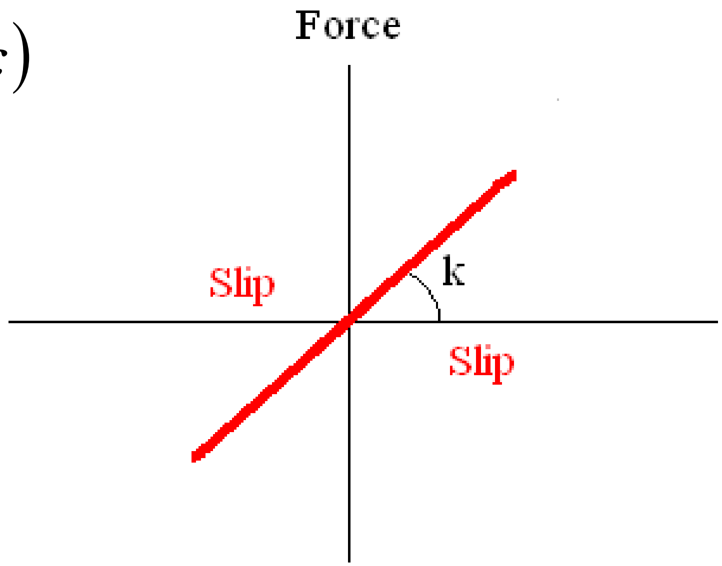
Background – Friction Forces

- Stribeck Effect
  - Stribeck (1902) observed that the friction force is decreasing continuously with increasing velocities ***for low velocities***.

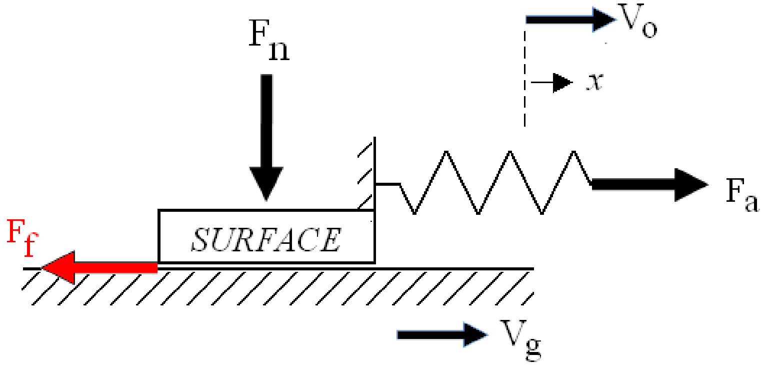
Background – Elastic Force

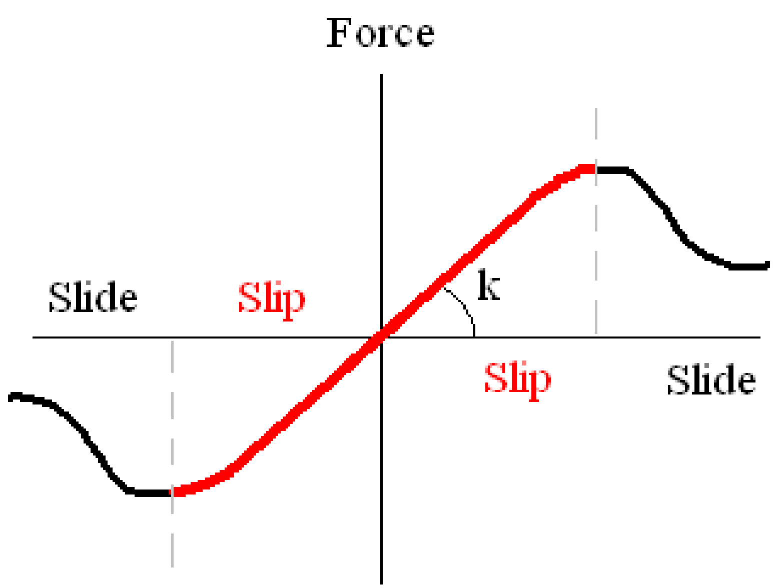
$$F = k \times x$$
$$(\sigma = E \times \varepsilon)$$





Background – Friction/Elastic Forces





Classification of Tire Mathematical Models

- Based on how you attack the problem…

← empirical   theoretical →

from experimental data only	using similarity methods	***through simple physical model***	through complex physical model

- Based on which time behavior you can capture…

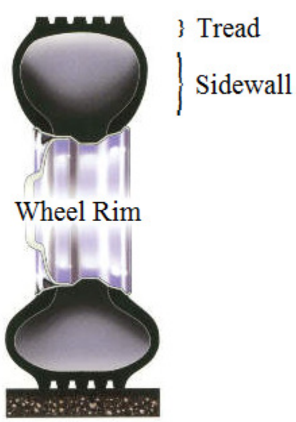
Dynamic (Transient) Models	***Steady State Models***

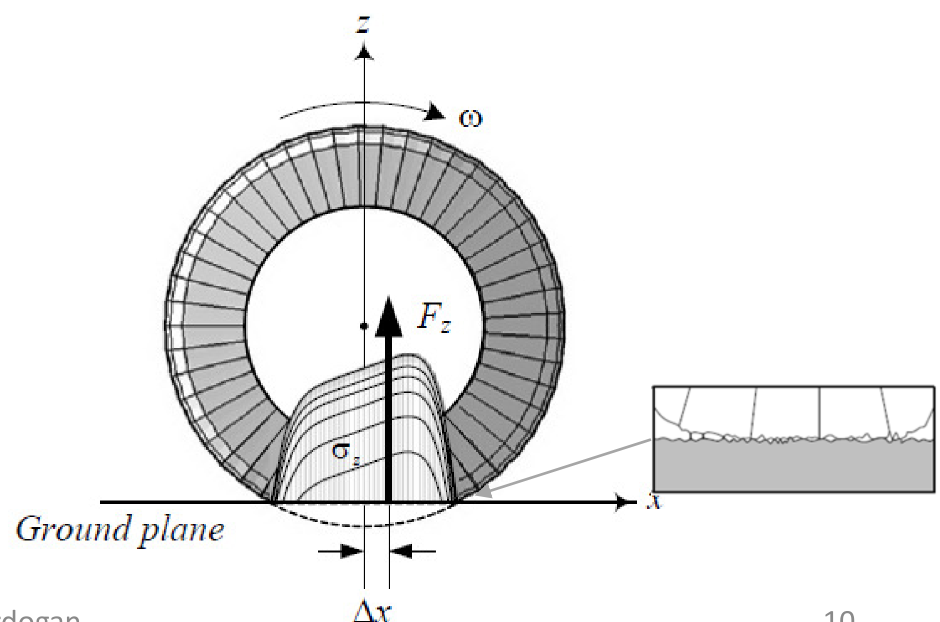
Tire Models

through simple physical models

- There are four main players in tire modeling through simple physical models

1. Tread deflection	(with and w/o)
2. Carcass/belt deflection	(with and w/o)
3. Distribution of contact pressure	(symmetric and asymmetric dist.)
4. Tire-road friction properties	(variable friction)





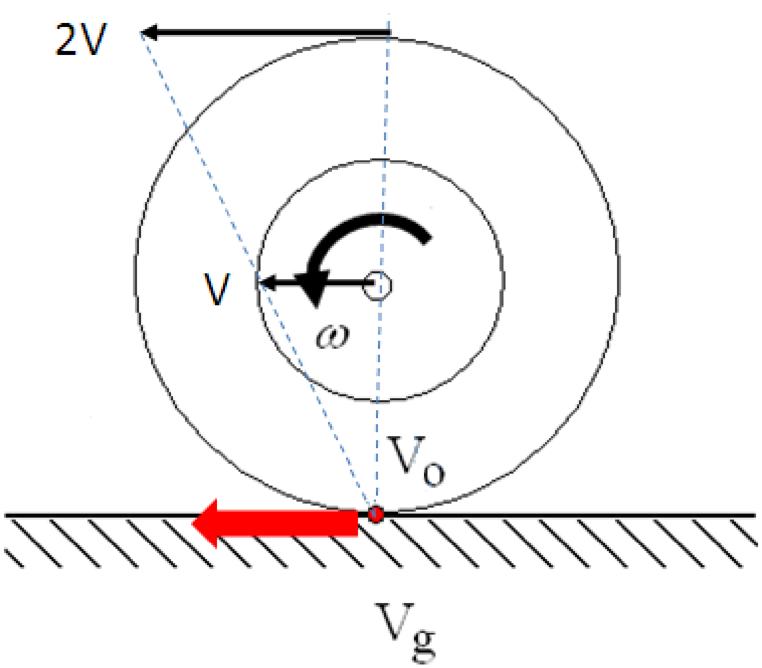
Tire Models

through simple physical models

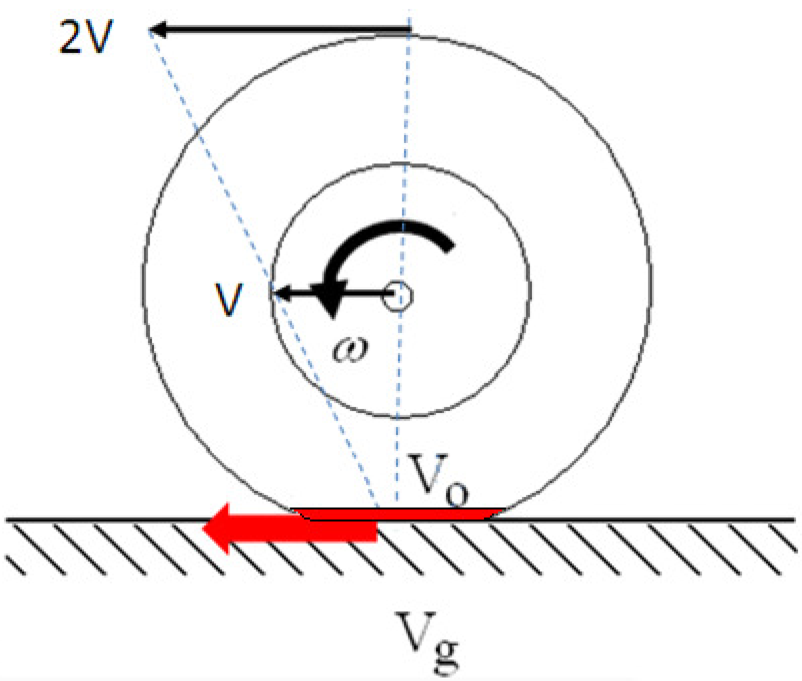
- **Brush Model**
  - **BM (Rigid Carcass)**
  - BM + Linear Carcass Defection
  - BM + Parabolic Carcass Defection
  - BM + Asymmetric Carcass Deflection
- String Model
  - Stretched String (No Tread Element)
  - BM + Stretched String
- Beam Model

Some Concepts - Instantaneous Center of Rotation

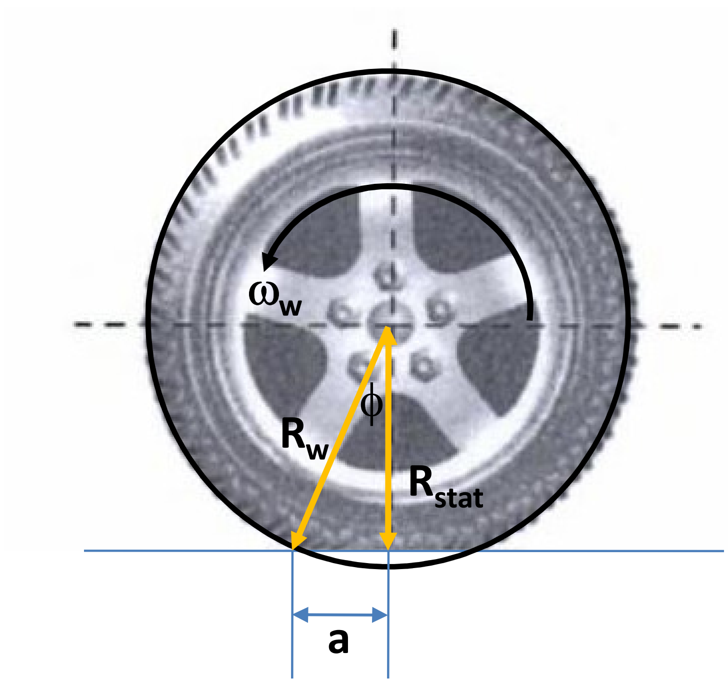
Ideally:



Reality:



Some Concepts – Effective Radius



$$V_{eff} = r_{eff}\,\omega_w = \frac{a}{t}$$

$$= r_{eff}\,\frac{\phi}{t}$$

$$r_{eff} = \frac{a}{\phi}$$

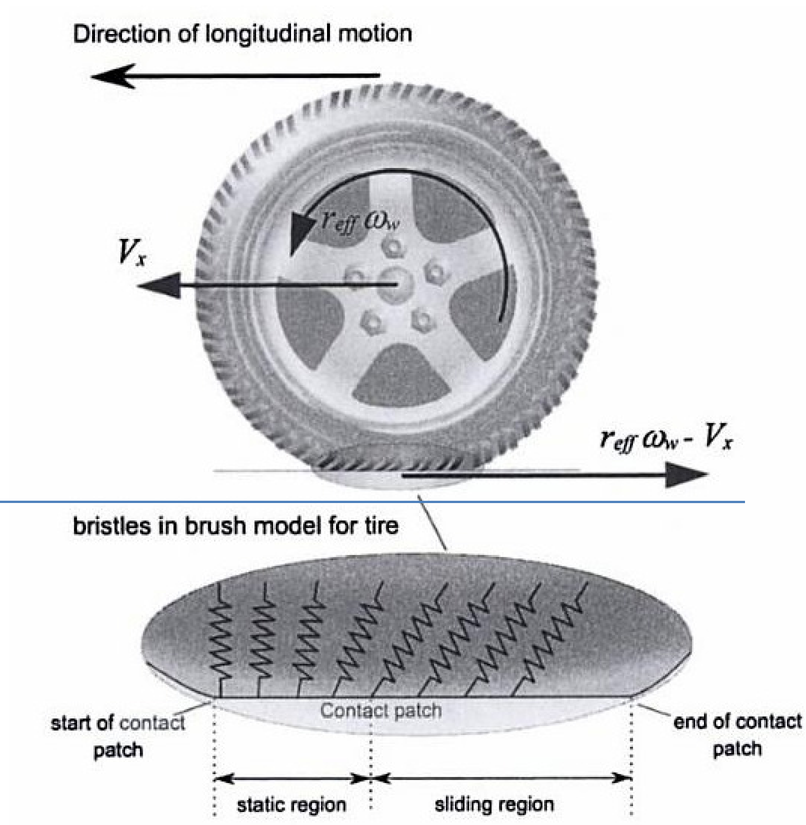
$$R_{stat} < r_{eff} < R_w$$

Brush Model – Pure Longitudinal Slip

through simple physical models

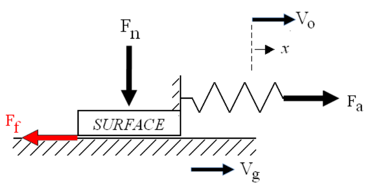
Longitudinal Tire Deformation

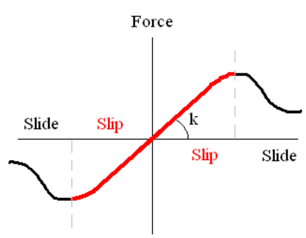
Slip Ratio: $$\sigma_x = \frac{r_{eff}\omega_w - V_x}{\max\left(V_x, r_{eff}\omega_w\right)}$$

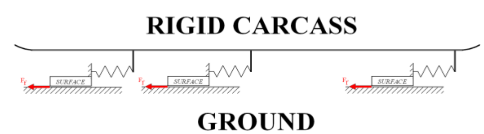


BM – Pure Longitudinal Slip

Remember…





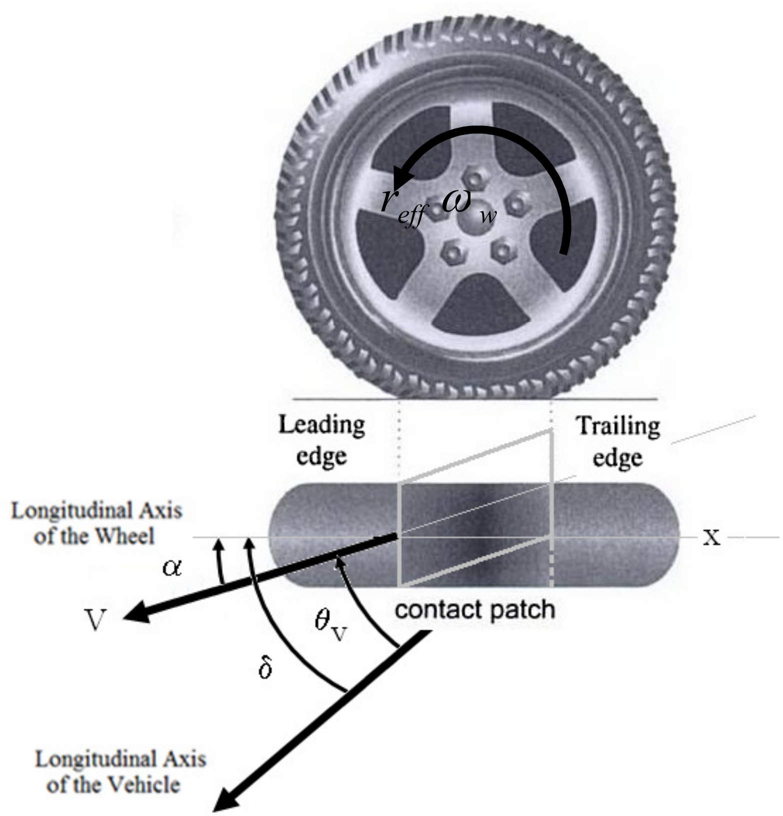


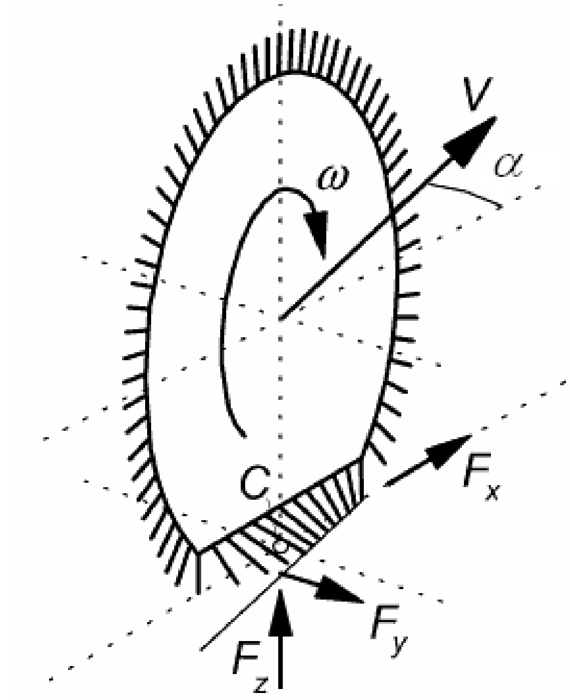
$V_o = r_{eff}\omega_w - V_x$

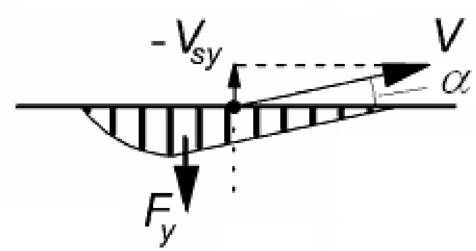
$V_g = 0$

Brush Model – Pure Side Slip

through simple physical models





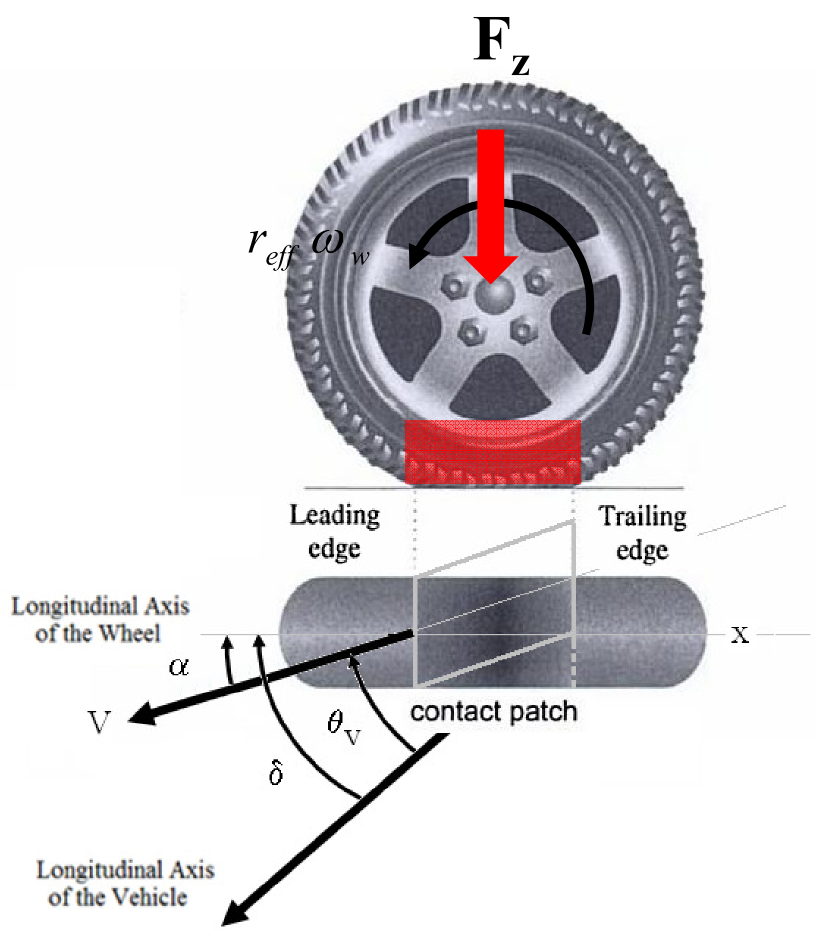


Slip Angle: $\alpha = \delta - \theta_V$

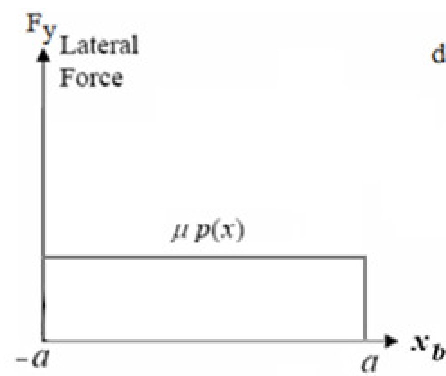
Brush Model – Pure Side Slip

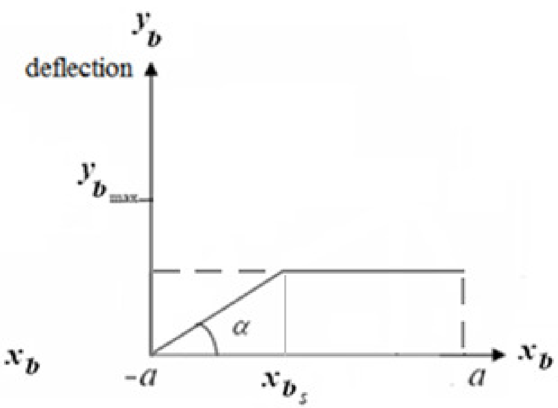
through simple physical models

- **Uniform** Normal Force Distribution



$$p(x) = \frac{F_z}{2a} \qquad F_z = \int_{-a}^{a} q_z dx$$





Brush Model – Pure Side Slip

through simple physical models

- **Uniform** Normal Force Distribution

Lateral Tire Force :

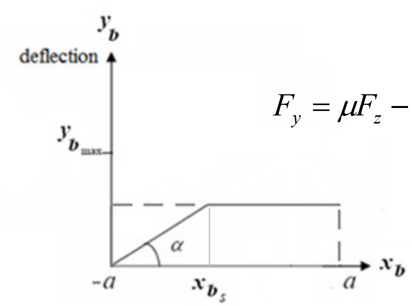
$$F_y = \mu F_z - \frac{(\mu F_z)^2}{c_{py} 8a^2 \tan(\alpha)}$$

Tire Moment:

$$M_z^{'} = \frac{(\mu F_z)^2}{8c_{py} a \tan(\alpha)} - \frac{(\mu F_z)^3}{48c_{py} a^3 \tan^2(\alpha)}$$

Friction Coefficient :

$$\mu = \frac{2c_{py} a \tan(\alpha)}{F_z} x_{bs}$$

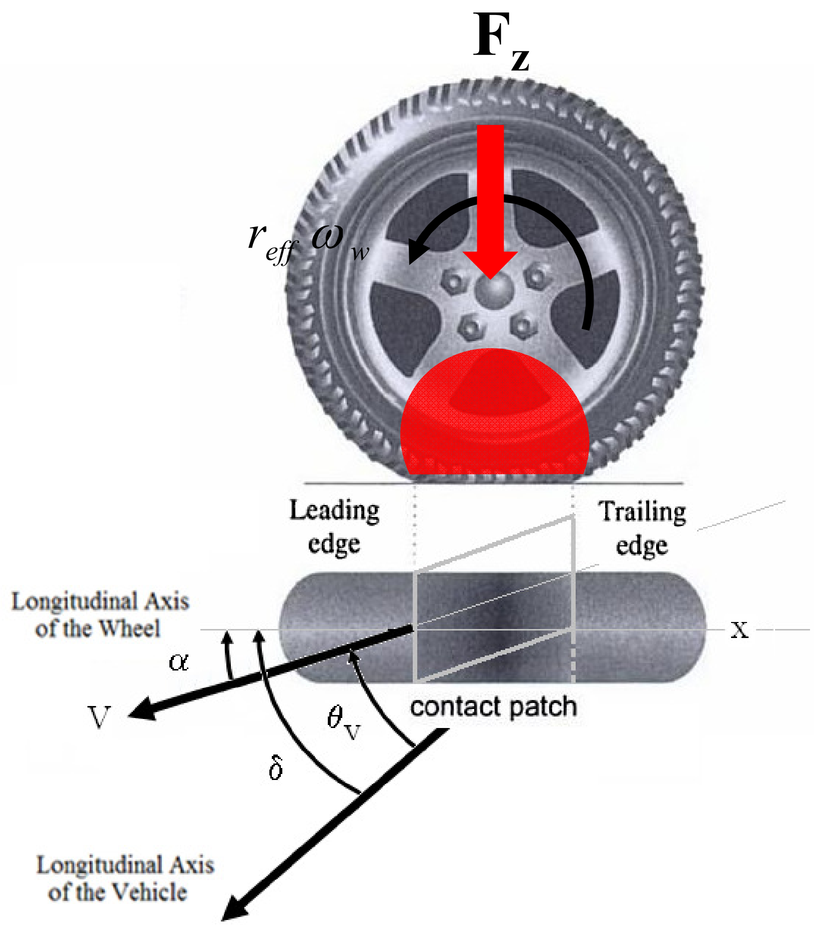


$$F_y = \mu F_z - \frac{y_{b\max} x_{bs}}{2} = \mu F_z - \left(\frac{\mu F_z}{2a}\right)\left(\frac{\mu F_z}{c_{py} 2a} \frac{1}{\tan(\alpha)}\right)\frac{1}{2}$$

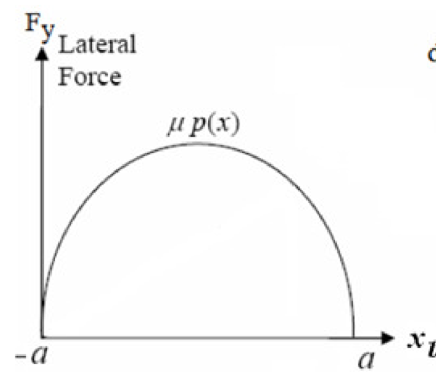
Brush Model – Pure Side Slip

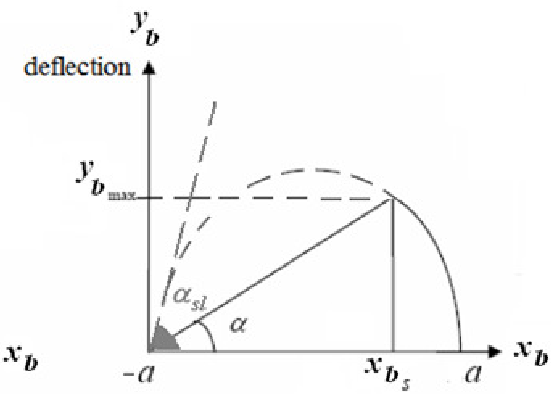
through simple physical models

- **Parabolic** Normal Force Distribution



$$p(x) = \frac{3F_z}{4a}\left(\frac{a^2 - x^2}{a^2}\right) \qquad F_z = \int_{-a}^{a} p(x)dx$$

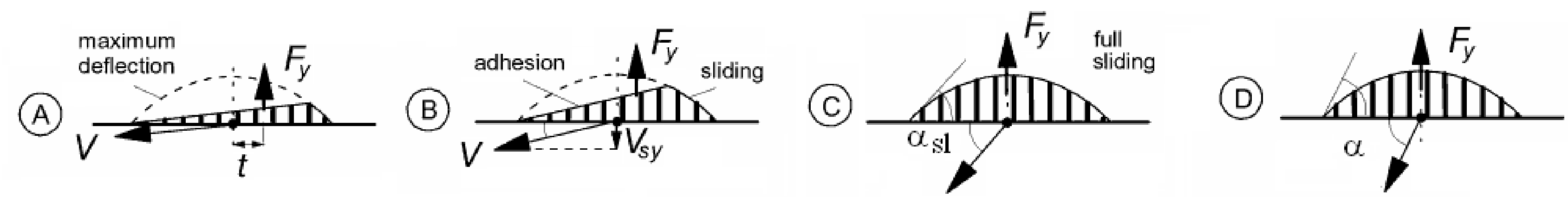


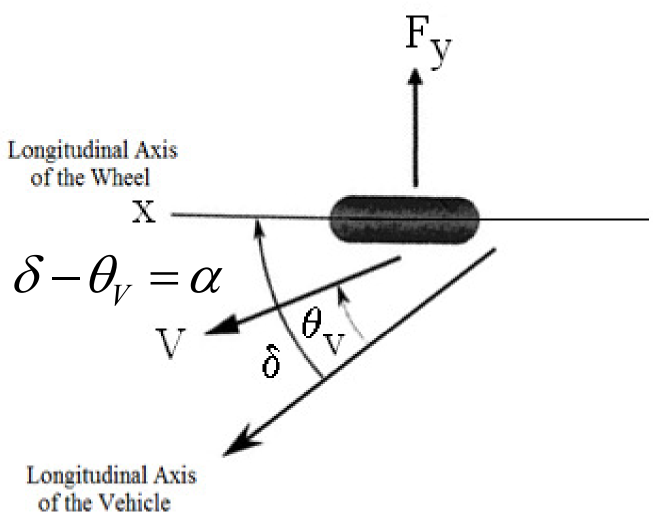


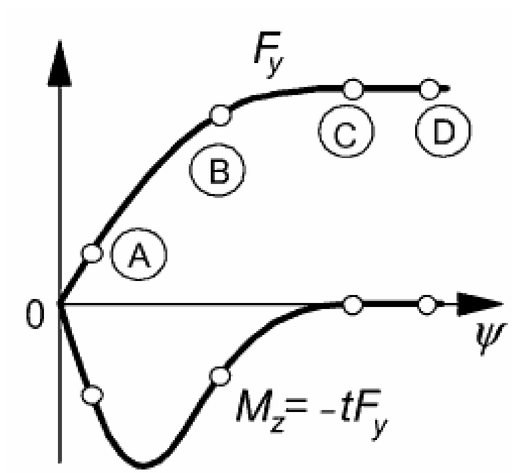
Brush Model – Pure Side Slip

through simple physical models

- Parabolic Normal Force Distribution



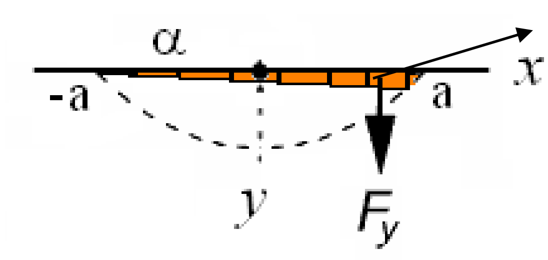




Brush Model – Pure Side Slip

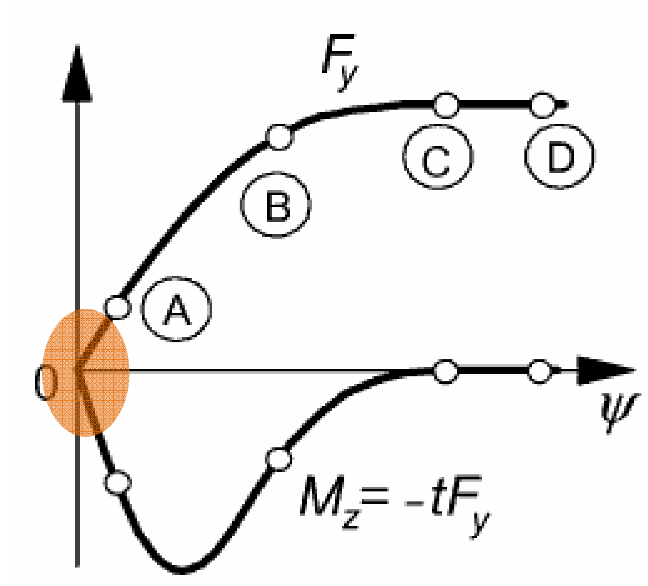
through simple physical models

- Parabolic Normal Force Distribution
  - **Small Slip Angle (as α→0, tan(α)→ α)**



$$q_y = c_{py} y$$

$$y = (a - x)\tan(\alpha) = (a - x)\alpha$$



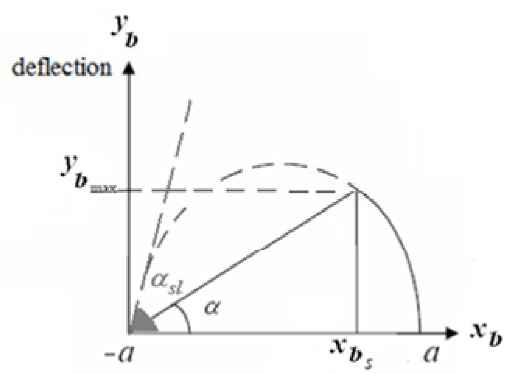
$$F_y = \int_{-a}^{a} q_y \, dx = c_{py}\alpha \int_{-a}^{a} (a - x)\, dx = c_{py}\, 2a^2\, \alpha = C_{F\alpha}\alpha$$

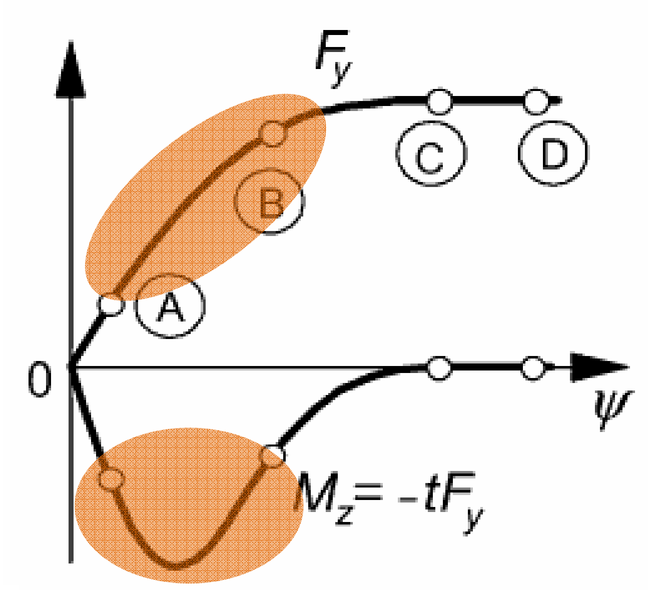
$$M_z = \int_{-a}^{a} q_y \, x \, dx = c_{py}\alpha \int_{-a}^{a} (a - x)\, x \, dx = -c_{py}\, 2a^2 \frac{a}{3}\alpha = \frac{a}{3} F_y$$

Brush Model – Pure Side Slip

through simple physical models

- Parabolic Normal Force Distribution
  - **Large Slip Angles**





Lateral Tire Force :
$$F_y = 3\mu F_z \frac{\tan(\alpha)}{\tan(\alpha_{sl})}\left(1-\left|\frac{\tan(\alpha)}{\tan(\alpha_{sl})}\right|+\frac{1}{3}\frac{\tan^2(\alpha)}{\tan^2(\alpha_{sl})}\right)$$

Tire Moment (Lateral):
$$M_z^{'} = -\mu F_z a \frac{\tan(\alpha)}{\tan(\alpha_{sl})}\{1-3\left|\frac{\tan(\alpha)}{\tan(\alpha_{sl})}\right|+3\left(\frac{\tan(\alpha)}{\tan(\alpha_{sl})}\right)^2-\left|\frac{\tan(\alpha)}{\tan(\alpha_{sl})}\right|^3\}$$

Friction Coefficient :
$$\mu = \frac{2c_{py}a^2}{3F_z}\tan(\alpha_{sl})$$

Brush Model – Combined Slip

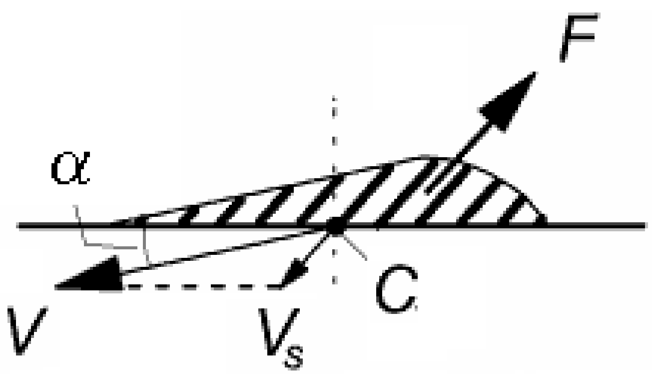
through simple physical models

- Theoretical Slips

$$\sigma_x = \begin{cases} \kappa = \dfrac{V_r - V_x}{V_x} = -\dfrac{V_{sx}}{V_x} & brake \\[2ex] \dfrac{\kappa}{1+\kappa} = \dfrac{V_r - V_x}{V_r} = -\dfrac{V_{sx}}{V_r} & drive \end{cases}$$

$$\sigma_y = \begin{cases} \tan(\alpha) = -\dfrac{V_{sy}}{V_x} & brake \\[2ex] \dfrac{\tan(\alpha)}{1+\kappa} = -\dfrac{V_{sy}}{V_r} & drive \end{cases}$$

$$V_r = r_{eff}\,\omega_w$$



$$u = (a - x)\sigma_x$$

$$v = (a - x)\sigma_y$$

$$\sigma = \sqrt{\sigma_x^2 + \sigma_y^2}$$

Brush Model – Combined Slip

through simple physical models

Tire Force :

$$F = 3\mu F_z \frac{\sigma}{\sigma_m}\left(1 - \frac{\sigma}{\sigma_m} + \frac{1}{3}\frac{\sigma^2}{\sigma_m^2}\right)$$

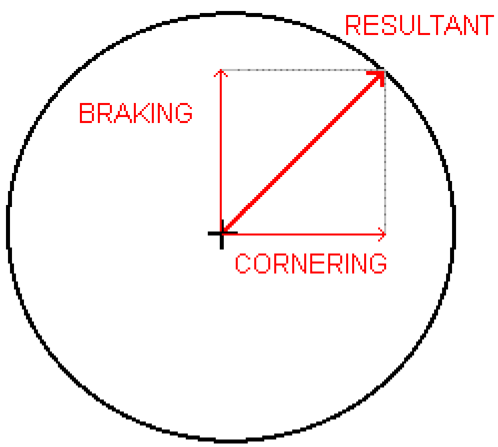
Tire Moment:

$$M_z' = -\mu F_z a \frac{\sigma}{\sigma_m}\{1 - 3\frac{\sigma}{\sigma_m} + 3\frac{\sigma^2}{\sigma_m^2} - \frac{\sigma^3}{\sigma_m^3}\}$$

Friction Coefficient :

$$\mu = \frac{2c_p a^2}{3F_z}\sigma_m$$

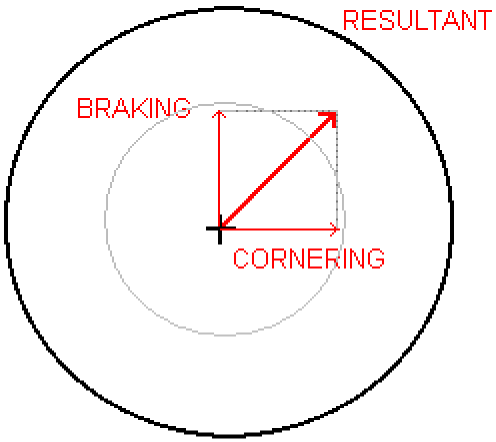
Friction Circle:

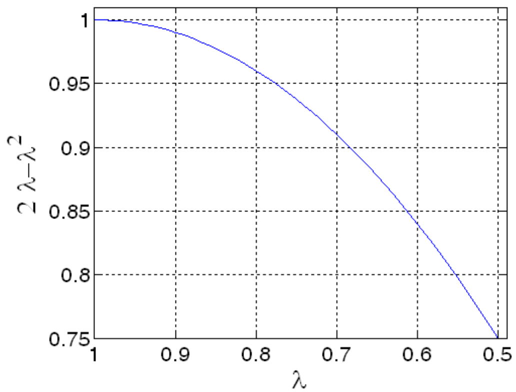


Brush Model – Combined Slip

through simple physical models

- Dugoff's Model



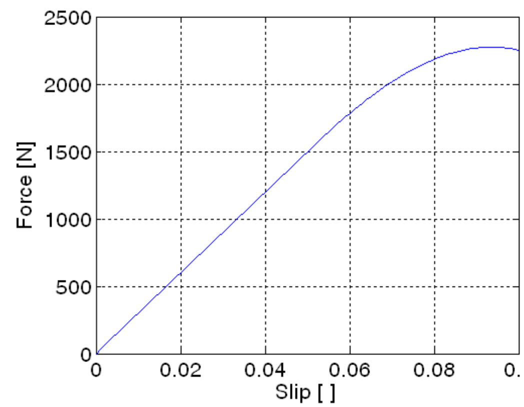


$$F_{x\_lin} = C_\kappa \sigma_x$$
$$F_{y\_lin} = C_\alpha \sigma_y$$

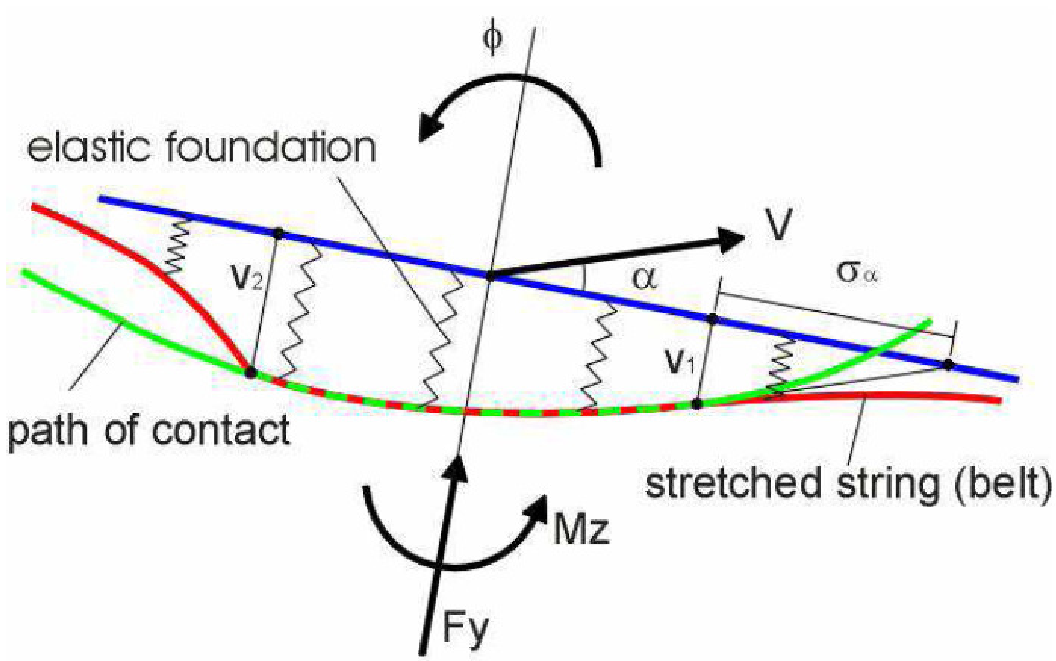
$$if \qquad \sqrt{F_{x\_lin}^2 + F_{y\_lin}^2} \le \frac{\mu F_z}{2}, \qquad \begin{aligned} F_x &= F_{x\_lin} \times 1 \\ F_y &= F_{y\_lin} \times 1 \end{aligned}$$

$$if \qquad \sqrt{F_{x\_lin}^2 + F_{y\_lin}^2} > \frac{\mu F_z}{2}, \qquad \begin{aligned} F_x &= F_{x\_lin} \times \left(2\lambda - \lambda^2\right) \\ F_y &= F_{y\_lin} \times \left(2\lambda - \lambda^2\right) \end{aligned}$$

$$\lambda = \frac{\frac{\mu F_z}{2}}{\sqrt{F_{x\_lin}^2 + F_{y\_lin}^2}}$$



String Model
through simple physical models



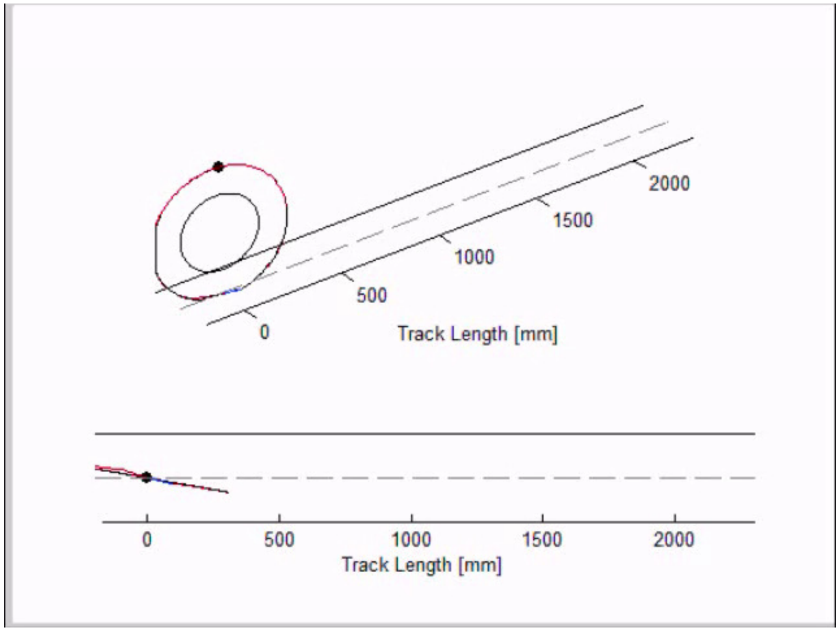
Dynamic (Transient) Tire Models

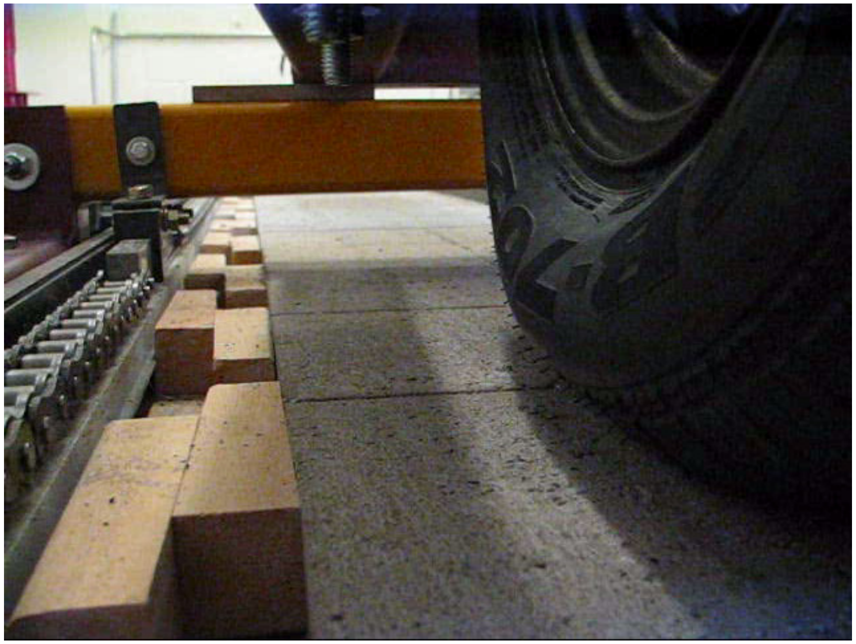
through simple physical models

- The steady state tire models are handy when we have constant linear and angular velocities.
- Experimental data used to validate the slip/force curves are obtained using specialized equipment that allows independent linear and angular velocity modulation so as to cover the whole slip range.
- This steady-state point of view is rarely true in reality, especially when the vehicle goes through continuous successive phases between acceleration and braking.
- Dynamic models capture the transient behavior of the tire-road contact forces **under time-varying** velocity conditions.
  - Bliman
  - Kinematic TM
  - **Dahl TM**
  - **LuGre TM**

Dynamic Models – Relaxation Length

- **Relaxation length**
  - is related to the distance needed by the tire to reach a the steady state situation after a step change in slip.
  - is the distance needed to build up the steady state tire forces.





Dynamic Models – Dahl Models

through simple physical models

The Dahl model is essentially **Coulomb friction with a lag.**

$$F_{deformation} = F_{friction}$$

$$\sigma_o z = F_{coulomb}\left(1 - e^{\frac{-\sigma_o |x|}{F_{coulomb}}}\right)\operatorname{sgn}(v)$$

$$\dot{z} = v - \frac{\sigma_o |v|}{F_{coulomb}} z$$

Derivation

$$\frac{dF_{deformation}}{dt} = \frac{dF_{friction}}{dt}$$

$$\dot{z} = e^{\frac{-\sigma_o |x|}{F_{coulomb}}} |v| \operatorname{sgn}(v)$$

$$e^{\frac{-\sigma_o |x|}{F_{coulomb}}} = \frac{\dot{z}}{|v| \operatorname{sgn}(v)}$$

Steady State:

$$\dot{z} = 0 \quad \Rightarrow \quad z_{ss}$$

$$0 = v - \frac{\sigma_o |v|}{F_{coulomb}} z_{ss}$$

$$z_{ss} = \frac{F_{coulomb}}{\sigma_o} \operatorname{sgn}(v)$$

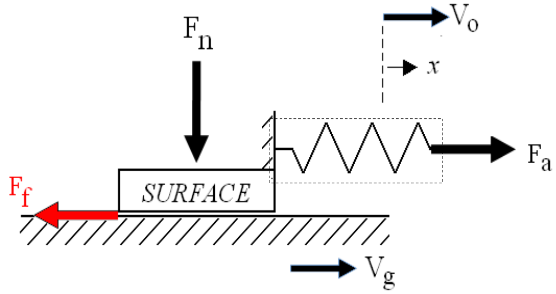
Dynamic Models –LuGre Models

through simple physical models

$$F_{deformation} = F_{friction}$$

$$\sigma_o z = F_{coulomb}\left(1 - e^{\frac{-\sigma_o|x|}{g(v)}}\right)\operatorname{sgn}(v), \quad or$$

$$\sigma_o z + \sigma_1 \dot{z} + f(v) = F_{coulomb}\left(1 - e^{\frac{-\sigma_o|x|}{g(v)}}\right)\operatorname{sgn}(v)$$



$$g(v) = F_{coulomb} + \left(F_{static} - F_{coulomb}\right)e^{-|v/v_{stribeck}|^{\alpha}}$$

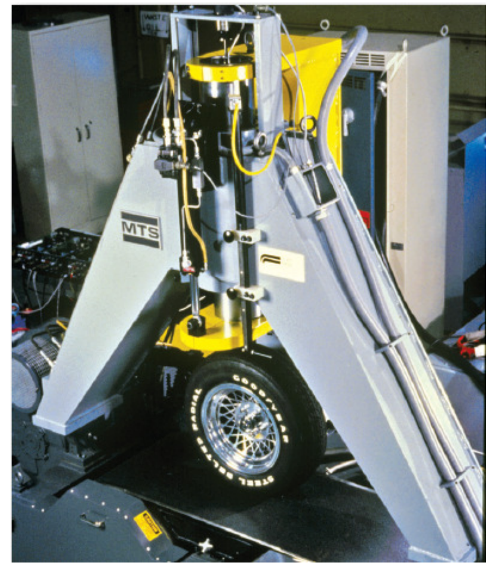
$$f(v) = \sigma_2 v$$

$$\dot{z} = v - \frac{\sigma_o|v|}{g(v)}z$$

Tire Models

from experimental data only

- Pacejka TM
- Burckhardt TM
- Kiencke and Daiss TM



Create TIRE DEFORMATION

TIRE

BLACK BOX

$$Force = f\left(Slip\right)$$

Measure TIRE FORCE

Pacejka Tire Model (Magic Formula)

from experimental data only

$a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7\ a_8 \quad F_z$

$$D = a_1 F_z^{\,2} + a_2 F_z$$

$$BCD = a_3 \sin(a_4 \arctan(a_5 F_z)) \qquad \text{(lateral force)}$$

$$BCD = \frac{a_3 F_z^{\,2} + a_4 F_z}{e^{a_5 F_z}} \qquad \text{(longitudinal force)}$$

$$E = a_6 F_z^{\,2} + a_7 F_z + a_8$$

$$C = \frac{2}{\pi}\sin^{-1}\left(\frac{y_s}{D}\right)$$

$$x = X - S_h$$

$$y = D\sin[C\arctan\{Bx - E(Bx - \arctan Bx)\}]$$

$$Y(X) = y(x) + S_v$$

B stiffness factor

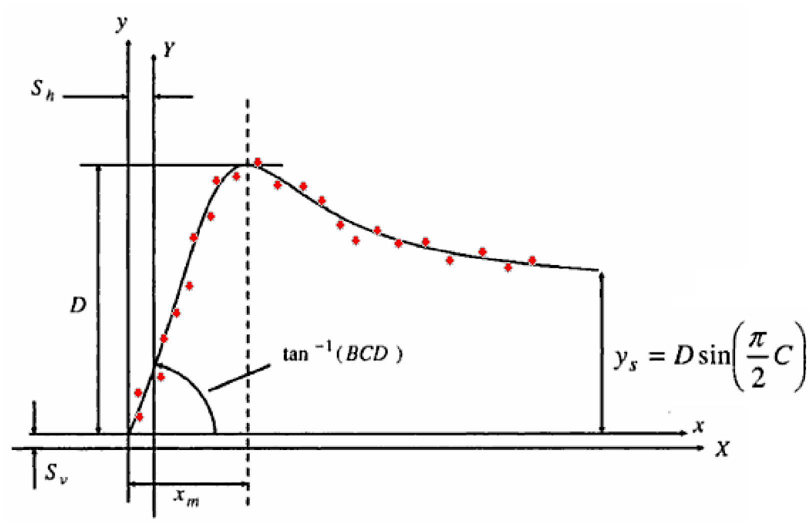
C shape factor

D peak value

E curvature factor

S_h horizontal shift

S_v vertical shift



References

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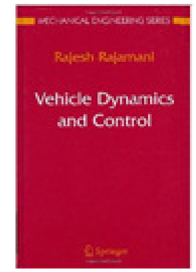

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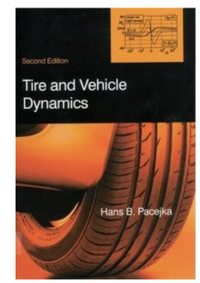

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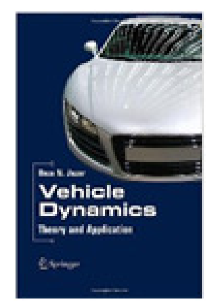

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