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HANDOUT 14

Borealis II Solar Vehicle Structural and Safety Report

UNIVERSITY OF MINNESOTA

Solar Vehicle Project

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Introduction

Borealis II, the University of Minnesota Solar Vehicle Project's American Solar Challenge 2003 entry, incorporates many design features from its predecessors, the Aurora generation of vehicles, as well as the ASC 2001 entrant, Borealis. The main design efforts for Borealis II were to improve on the reliability and safety features from the previous vehicles. The team began addressing these issues after racing the Borealis in the 2002 Formula Sun event; the beginning of the design conceptualization stage.

The following report highlights the safety and design considerations that went into the Borealis II design. The intent was to address specific design concerns as outlined by the American Solar Challenge regulations, which includes the following topics: front suspension, rear suspension, brakes, steering, wheels and tires, battery enclosure, and vehicle impact analysis. Other regulation controlled design issues concerning driver cockpit penetration and roll cage, chassis frame materials and construction techniques, battery attachment, and driver harness mounting points are also addressed. There is also narration on the logical progression of the most important crash loading situations, followed by a structural analysis performed to verify the vehicle integrity in the loading situations required by the American Solar Challenge rules.

Jason Halpern

Jason Halpern
Mechanical Design Team Co-Leader
UMNSVP - Borealis II

Vehicle Concept

Borealis II is at the cutting edge of solar car design and manufacturing. Based off of the Borealis I car, Borealis II has made improvements on every component in the car. The vehicle's design is focused around driver's safety, mechanical reliability, and overall efficiency.

To best achieve this design the car is built with a lightweight monocoque frame. The predicted weight of the vehicle including driver and batteries is 540 - 550 pounds. The composite frame accounts for merely 20 pounds of the total vehicle weight. By placing the batteries in the front of the vehicle the driver is allowed more crush space, while maintaining a center of gravity behind the front axle line by a distance of 1/3 the wheel base.

The vehicle's chassis includes a composite frame integrated with the bottom of the vehicle's shell, suspension components, and roll cage. Borealis II's chassis is designed to transfer loads from the driver and other components to the suspension mounts. The vehicle's top shell is completely detachable from the chassis to allow redirection of the array while the car is stationary. The shell also has a removable canopy for driver egress when necessary.

Each of the vehicle's 3 wheels is suspended independently. The front two wheels ride on double A-arm suspension whose axle line is at the front of the driver's compartment. The driving wheel is suspended by a swing arm suspension mounted directly behind the driver's compartment. The combination of these three wheels gives the vehicle a wheelbase of 91.5 inches and a front track of 51.5 inches.

The driver seated in the cockpit is fully enclosed and isolated from the road and is clear of all moving parts. The cockpit contains a six-point harness, headrest, and a driver ventilation system. The cockpit is designed to allow driver maximum visibility, vehicle impact protection, and unstressed driving of the vehicle.

The batteries are held in a detachable box, which is fully enclosed, attaches ahead of the cockpit, and is electrically isolated from the rest of the vehicle and driver. All high voltage equipment is out of reach or protected from the driver.

Mechanical Analysis

Introduction

Loading Conditions for All Analyses

Appendix A, Figure 7a shows the forces on the bottom of the tires for the following conditions. The total vehicle weight is W and the center of gravity is at one third of the wheelbase behind the front axle. We expect to achieve this value within one inch, so it is assumed for the analysis.

- The static load on each wheel is 1 G or 1/3 W.
- The bump load is 4G or 4/3 W.
- The brake load is 1G and assumed at the front wheels only so each front wheel braking load is 1/2 W.
- The cornering load is shown for a right hand turn and assumes a 1G lateral acceleration and incipient tipping, with the "outside" front wheel providing lateral force 2/3 W, the "inside" front wheel providing no lateral force, and the rear wheel providing 1/3 W.
- The drive and braking forces at the rear wheel are relatively small and produce small chassis loads compared to the bump and cornering, and so are ignored.

These load cases are similar to those used in the GM Sunraycer Case History, Lecture 6-1: Structure (Chassis) by Ray Morgan and Herman Drees, and have proven successful on all the University of Minnesota solar vehicles since 1993.

Mounting to Composite Paneling

Two part aluminum and Torlon inserts are used at all locations that require fastening to the composite panels. The aluminum inserts are tested by the manufacturer to 1500 lbs for in-plane shear, and are used for all attached components under load. The design of the insert is intended to handle bolt clamping load while distributing axial and shear loads to the sandwich panels. These inserts were used successfully on all previous U of M solar vehicles.

Front Suspension Analysis

Material Specifications

The front suspension system is a double A-arm design incorporating a 60° even split between lower a-arms and a 22°/27°, (total 51°), uneven split between upper A-arms. The lower A-arm is made from 0.75" OD x 0.058" wall, 4130 chrome-moly steel tubing. The upper A-arm is machined into 0.5" x 0.5" solid square section A-arms from 7075-T6 aluminum. The wheel hubs and uprights are CNC machined from 7075-T6 aluminum. The axle is made from 17 mm OD heat-treated and ground steel.

As for bearings and mountings, the lower A-arm uses a 7/16" bore spherical bearing at the lower upright joint, and is mounted to the chassis with 1/4" high angle rod ends with a 3/8" shank. The lower A-arm brackets are machined from 7075-T6 aluminum and each attaches to two chassis panels, spreading the load and reinforcing the panel joint. The top A-arm uses a 3/8" bore spherical bearing at the top upright joint, and is mounted with 5/16" rod ends. They are attached to angled chassis panels via plates of 0.125" 6061-T6 aluminum, which sandwich both the panel and the rod ends. The panels are aligned with the a-arm legs, so the loads are in the plane of each panel. The a-arms are designed so that NO rod ends are in bending under any load condition, allowing each arm to be treated as a two-force member. The assembled layout of the front suspension is shown in Appendix A, Figure 10.

All fastening hardware used in the front suspension are AN-aircraft or MS-military spec grade bolts and nuts. All structural brackets are fastened to chassis panels with a minimum of three bolts.

System Analysis

The first step in design was to free body diagram the a-arms and upright to trace the input force from the tire contact patch all the way to the chassis mountings. The highest forces in the system were found to be approximately 1990 lbf in the front lower a-arm, and 1060 lbf in the top rear a-arm.

Loading Conditions w/ Associated Forces *(G's)*

Fb	0	0	1	1	0	1	1	0	*(G's)*
W	1	4	4	4	4	1	1	1	
Fc	0	0	0	1	1	0	1	1	
Fr	-220	-879	-384	403	-92	275	1062	567	(lbf)
Ff	-220	-879	**-1983**	-1196	-92	-1324	-537	567	
FR	34	136	-508	-951	-307	-610	-1053	-409	
FF	28	112	736	371	-254	652	287	-338	

Table 1 - Front Suspension Loading Conditions / Forces

Fb= braking force, W= static weight or bump force, Fc= cornering force
Fr= lower rear arm, Ff= lower front arm, FR= upper rear arm, FF= upper front arm
(See Appendix A, Figures 7b and 8 for complete force spreadsheet and free-body diagrams.)

Bending moments in the front upright structures were next calculated and used to design upright cross-sections with suitable moments of inertia. This entailed yielding safety factors greater than 1 with respect to aluminum 7075-T6 fatigue strength, and greater than 4 with respect to yield strength under all loading cases. Both of criteria were satisfied. (See Appendix A, Figure 9b for sectional bending stresses at the upright's lower joint, top joint, and axle.)

Last, buckling calculations were done to prove that our current A-arm designs would not fail under load. For both the A-arms, the Johnson mode of buckling was found to be applicable. Using the maximum loads from the force analysis, safety factors of 3.3 for the lower a-arm and 9.4 for the upper a-arm were found. See Appendix A, Figure 11 for buckling calculations for the lower and upper a-arms. Note tubing wall thickness for lower a-arm used in calculation, 0.049", is smaller than actual 0.058" wall of the part, therefore safety factor is actually greater than listed calculated value.

Carbon Fiber Tubing

This tubing is to be used for the steering tie-rods, the rear suspension shock link, and potentially as upper A-arms when used with a to-be-designed aluminum piece, which will serve as the outer end of the upper a-arm and house the upright top joint bearing.
The carbon fiber tubing is a unidirectional vinyl ester tube utilized as two force members when aluminum inserts are glued into the tube ends and rod ends are installed into these inserts. The only size tubing used on the car is 0.75" OD x .095" nominal wall made of 33m.s.i. carbon fibers. The internally threaded inserts are all machined from 0.75", 2024-T6 or 6061-T6 aluminum hex stock, and are then mated to the carbon tubes with 3M brand DP-460 NS or Vantico Epibond 420 A/B structural adhesives. The tubing manufacturer lists for a similar 0.75" OD x 0.085" walled tube a tensile strength of 200 ksi. The DP-460 NS is the same structural adhesive used to glue the chassis composite panels together.

Rear Suspension Analysis

Material Specifications

The swing arm assembly is constructed out of 4130 steel tube and plate. The shock links are water-jet cut out of 7075-T6 aluminum and one piece of the carbon fiber tubing, All chassis mounting brackets for rear suspension pieces are made out of 7075-T6 aluminum.

System Analysis

Borealis II is lower than our 2001 vehicle, Borealis I. We estimated that the anti-squat effect, which raises the rear of the vehicle under acceleration, would be small with a conventional swing arm, rather than the more complicated double A-arm design of Borealis I, which exhibited no squat effect. A Fox spring/shock unit acts directly on the arm, with its line of action aimed at the intersection of the arm centerline and the vertical line through the axle. This produces little or no bending moment in the arm due to bump loads. The main load in the arm is torsion with large values produced in bump and cornering. Estimated stresses, using simplified models neglecting additional supports, at critical points were all below yield by safety factors of at least 1.86 for the 1 g turn and 4 g bump. The upper end of the spring/shock unit is supported by a linkage, consisting of two legs of a triangle which connect to the outer ends of the swing arm cross tube with rod ends, and a horizontal member with rod ends which attaches to the upper chassis panel, transferring loads directly into the panel. With this arrangement, bump loads at the rear wheel are transferred to the swing arm mounting points, but not through the arm itself. The horizontal member is adjustable for length, which can change rear ride height without using up shock travel. The chassis mounted pivots for the swing arm are spherical bearings in custom aluminum housings, one on each end of the cross tube. Each housing attaches to two chassis panels with five ferrules or inserts. Toe and camber adjustment is accomplished by changing one housing.

Brakes

Borealis II uses two independent hydraulic braking systems acting at the front wheels, assisted by regenerative braking through the motor at the rear wheel.

Material Specifications

Each hydraulic system has a master cylinder, with a 0.875 inch diameter piston, which actuates two cylinders, one at each front wheel. The master cylinders are from Tolomatic, Inc. and the calipers are from Martin Custom Products and have 1.0 inch diameter pistons. There is a custom 8.0 inch diameter brake disc at each wheel. The discs are machined from 7075-T6 aluminum and hard coated with "Alpha Coating" from Surface Solutions, Fridley, MN, which provides a surface that is twice the hardness of titanium nitride. A custom pedal assembly actuates both hydraulic systems simultaneously and is adjustable to balance wear on the pads. The regenerative brake is controlled with a lever on the steering wheel. Components were sized based upon an optimistic tire-road coefficient of friction of 1.0, which with front wheel braking only, corresponds to a deceleration of 0.85 g using the previous values for wheelbase and CG location. (The value of 1 G deceleration shown previously was used for sizing suspension components, but here we need a more realistic value to check pedal force and stresses, and line pressures. A pedal force of 108 lbf will supply line pressure of 565 psi and produce deceleration of 0.85 G. The master cylinders are rated for 1200 psi. If one hydraulic system fails, the pressure in the remaining one at 0.85 G would double to 1130 psi, still below the rated value. The pedal is constructed from 0.75 inch O.D. x 0.065 inch wall 4130 steel tubing and its maximum bending stress under this load produces a safety factor of 2.44. It should be pointed out that the rules require a braking deceleration of 0.5 G, and at this level, the above pedal force and line pressure will reduce and the pedal safety factor will increase.

Steering

Material Specifications

A conventional rack and pinion steering layout is used, consisting of a composite steering wheel, which is removable, a tubular aluminum steering column, a custom rack and pinion unit, aluminum rack extensions in bushings, connecting to carbon fiber tie rods and finally to steering arms which bolt on the suspension uprights. The rack and pinion are steel items from Stockdrive Products, and have been used successfully on our last 3 solar vehicles. High performance Aurora rod ends are used on the steering arms. A top view of the steering system is included in Appendix C.

System Analysis

Special attention was given to maintaining Ackerman effects and zero bump steer. There is 5 degrees of caster, approximately 1 inch of kingpin offset and 3.5 degrees of kingpin inclination. The caster produces about 1 inch of trail, which was used to determine the steering loads on the outside wheel during 1 G cornering. The resulting tie rod load is about 60 lbf, which is negligible for the rod ends, but causes bending on the steering arms, and rack extensions where they pass through the bushings. The corresponding bending stresses were computed and gave safety factors of 4.4 and above based on a yield of 40,000 psi for the 6061 T6 aluminum materials.

Wheels and Tires

Borealis II will be using the NGM wheels with Bridgestone Ecopia tires in all races.

Battery Enclosure

The Battery box, although not structurally part of the chassis it is made of the same carbon fiber paneling and uses the same non-conductive 3M™ Scotch-Weld™ DP-460 NS two part epoxy to join paneling. The inside of the battery box is lined with a layer of Kevlar to insure electrical isolation from the car. The top of the battery box is fastened to the bottom half by 6, 1/4" diameter fasteners, the same fasteners that hold on suspension components and will hold the batteries inside the box in the event of a vehicle roll over. The box is removable and is fastened to the chassis by 6 AN grade grommets and #10 fasteners. The batteries' weight is distributed to the bottom of the chassis by a set of carbon fiber ribs. At least two panels must be penetrated for any of the batteries to contact the driver.

Vehicle Impact Analysis

Body Construction

The body is divided into two separate parts, the bottom and top shells. The bottom shell is mostly structural and is integrated into the chassis. The top shell serves to support the array and act as an aerodynamic shell. The top shell is not designed to support highly localized loads.

The bottom shell was made at Northwestern Airlines Advanced Composite Laboratory. The wet lay-up of the bottom shell was done using the EPON Resin 826/EPI-CURE Curing Agent with the EPI-CURE Curing Agent Accelerator 537. This resin system was used in conjunction with 3k x 3k, 2x2 Twill Weave, 5.7oz Graphite Fabric. Two layers of the graphite fabric where used on each side of the core.

Two types of core were used in the construction of the nonstructural areas of the bottom shell depending on the severity of the curves in the mold. HRH – 78 Nomex Commercial Grade Honeycomb (hexagonal), 1/4'' thickness, 1/8'' cell diameter was used on gentle curves and HRH – 78 Nomex Commercial Grade Honeycomb (flexcore)1/4'' thickness, 1/8'' cell diameter was used on severely curved portions. HRH – 78 Nomex Commercial Grade Honeycomb (hexagonal), 1/2" thickness, 1/8" cell diameter core was used to form the structural portion of the bottom shell to increase rigidity.

Composite grade balsa was used to reinforce all edges and seam areas in the bottom shell. Microballons mixed with the resin system were used to join all core pieces together into a unified sheet and was also used to fill any gaps in the core joints. By reinforcing the edging and filling gaps in the core we greatly reduce the chances of delamination. The bottom shell can be seen in Figure 1 below.

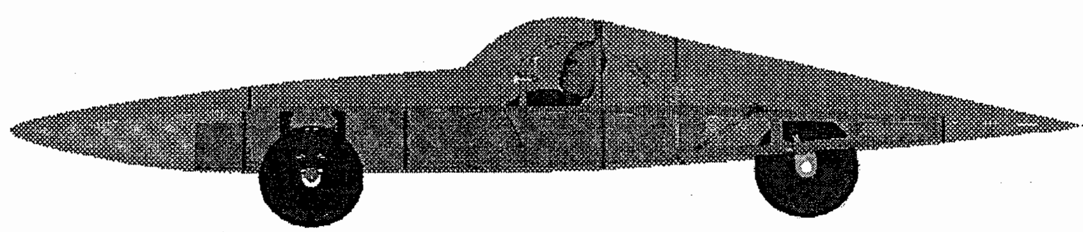


Figure 1 - Borealis II Shell

Chassis Construction

The Monocoque chassis satisfies the needs of our vehicle far better than other design and construction methods. Monocoque refers to the construction method that directs the forces in the chassis through the skins of the composite material. The carbon fiber paneling we have chosen offers us several advantages over a space-frame chassis. Carbon fiber paneling offers the same lightweight and high strength advantages as an aluminum space-frame design while satisfying several other needs that the chassis must meet. It provides a fully enclosed space for the driver, attachment points for electrical and suspension components, and significantly simplifies construction and design over a space-frame design.

Carbon fiber paneling owes its great strength to the interlacing fibers in the skins of the paneling. The facing acts like thousands of tiny cables all strung in the same direction held in place by the matrix of resin combined with the facing fabric in it's production. Many fabrics are available, examples being Kevlar, glass, carbon, and Carbon/Kevlar combination. Each has a specific property that is advantageous. Carbon has the highest strength in tension and compression of the materials listed, which is why it was chosen for our highly stressed chassis.

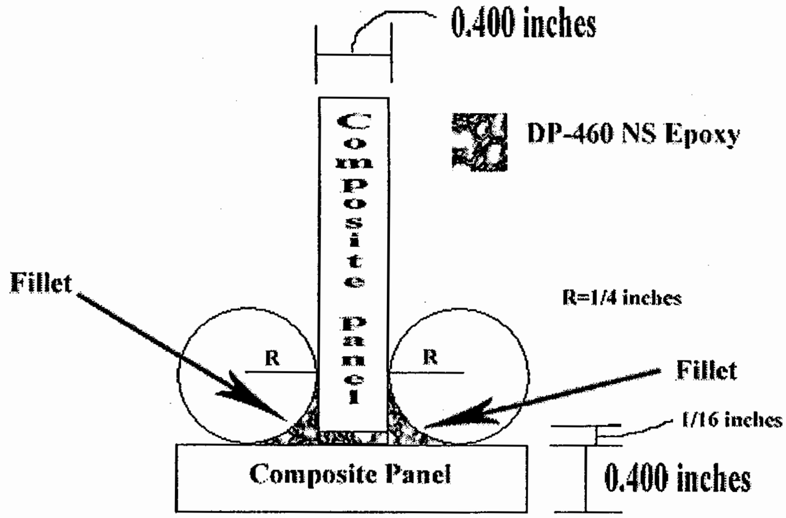
The paneling is constructed in a box beam geometry with panels assembled at 90-degrees to each other which simplifies construction and ensures consistent high quality joints. The panels were designed in such a way that all loads are transferred into the chassis in a direction parallel to at least one prefabricated panel. The composite panel chassis' were successfully tested with the last four solar cars built, Aurora II, Aurora III, Aurora IV, and Borealis I.

A mock-up full-scale plywood chassis was constructed before the chassis design was finalized to have model of what the driver cockpit would be like. It helped to integrate the roll bar, pedals, steering, and driver visibility. A complete vehicle assembly in CAD was also used to enhance the integration between all vehicle parts. The combination of these two tools enabled optimal driver placement, layout of components, and integration of vehicle systems.

Material Specifications

Hexcel Composites Fiberlam 2000 prefabricated paneling was used to construct the majority of the chassis. Theses composite panels are typically used for aircraft flooring. The panels have a thickness of .400 inches consisting of Nomex honeycomb core, cell diameter 1/8 inch, and a nominal thickness of .380 inches. Each side of the core is covered with 2 layers of unidirectional carbon fiber fabric laid perpendicular to each other. Each layer of unidirectional carbon fiber has a thickness of .0045 inches. There is also a .001 inch thick layer of fiberglass on each side of the paneling to prevent marring and reduce electrical conductivity. The paneling has an average weight of .37 lbs/ft^2. Data from the published four point bending test was used to show a maximum allowable stress in the face sheet of 57,000 psi in Appendix D.

3M™ Scotch-Weld™ DP-460 NS was used to join the paneling at all joints. This glue is very similar to glue used on previous cars but has a much higher viscosity which assures us that it will not flow away from the joints and will maintain correctly shaped fillets. The epoxy is rated at 4900 psi under the surface preparation conditions used and room temperature curing. The typical joints on the car have approximately .4 square inches of shear area per joint inch. See Figure 2 below. The joint can then be assumed to carry a maximum shear load of (.4inches)(4900psi)=1960 lbs per inch of joint.



Cross section of a typical chassis joint bonded with DP-460 NS epoxy

Figure 2 - Chassis Joint Bond

Construction
The process used to machine the 4'x12' Hexcel panels was a water jet cutting machine. This process uses a high-pressure stream of water combined with a garnet aggregate to erode away the material it cuts. The mixture of water and garnet is pressurized to 40,000 psi and focused through a carbide tip into a beam that is 40/1000" in diameter. The water jet tool is the best suited tool for cutting this type of carbon fiber paneling because there is no clamping needed, no carbon fibers released into the air, no forces exerted on the material by the tooling, and it cuts to very high tolerances.

The paneling was placed on a bed of plastic/cardboard energy absorbing material, which is above a steel water tank. The paneling was held in place by its own weight, no clamping was necessary. After the paneling was cut it was quickly cleaned off and dried to prevent delamination. The paneling was also laid out in a warm dry environment for two days to ensure all the moisture was removed from the paneling before it was glued together.

The paneling was prepped by abrading all glue surfaces followed by extensive cleaning of the glue joints with isopropyl alcohol. Using 3M™ Scotch-Weld™ DP-460 NS two part epoxy the composite paneling was assembled. The epoxy was applied using the .4 liter Pneumatic Thunder Epoxy dispenser adapted with 10mm deluxe mixing tips to ensure complete mixing between the resin and the hardener. The epoxy was filleted with a 1/4" radius (See Figure 2) on the joints, which adequately transfers the forces from one panel to the next as if the chassis was a unified piece. Many of the chassis joints are also reinforced by attachment of the Roll bar, suspension, and other components on the car.

The finished chassis edges have been filled with lightweight edge filler that ensures moisture and dirt cannot compromise the composite's integrity. The edge filler also reduces delamination that might otherwise occur.

Crush Space

In the event of a collision, a system of progressive safety features will prevent the driver from being injured. The Borealis II design places the driver within a safety capsule, with no part of his or her body extending beyond the structural chassis. The driver's shoulders are positioned below the upper plane of the chassis, such that, the torso, containing most vital organs, is located in the center of the structural chassis. The driver's head will be encompassed by a roll cage structure designed to protect the vulnerable driver's head which protrudes through the car's body.

ASC rules require 5.9 inches (15 cm) horizontal distance between the diver's shoulders, hips and feet and the car's outer body surface. (Section 5.3.5) Borealis II's minimum crush space of greater than 2 feet is over four times that required by ASC rules. This large crush space around the driver was made possible by locating the cockpit in a central location in the car. (See Figure 3) In a rear collision, the 8 feet of solar car behind the driver will act to absorb much of the impact energy. Likewise, in side collisions the driver resides in the center 20 inches of the 5.9 foot wide car, allowing over 2 feet of crush zone on either side. The shell material will crush and the driver is then protected by the driver's compartment as shown in the various crash analysis sections.

Front crush space was maximized beyond race rule requirements with 42.5 inches of crush space. The 42.5 inch nose of the car will crush easily, allowing the crash to be stopped by the structural chassis. The front batteries will help block penetrating objects and decelerate the impacting body due to their mass.

Roll Cage Description

The roll cage is comprised of three elements:

1. A main hoop behind the driver, with legs that extend down to the bottom of the chassis. It is attached to the chassis in the corners of the driver's seat bulkhead and the side panels via inserts, along with flanges on the top of the chassis with inserts.
2. Two rearward facing supports welded near the top of the main hoop, and attaching to the top of the chassis via inserts.
3. Two forward facing tubes are welded near the top of the hoop, and extend horizontally forward, and then bend downward and outward to meet the edges of the driver's compartment, where they attach to the top and side chassis panels via welded brackets and inserts. These tubes provide "anti-decapitation" protection from rearward or lateral movements of the body relative to the chassis.

The entire structure is constructed out of 1.25 inch O.D. x 0.049 inch wall 4130 steel tubing, and 0.065 sheet for bracketry. The tubing is cold drawn normalized with yield strength reported as 75,000 psi. This tubing has an EI stiffness that is 32% larger than that of the 1.0 inch O.D. x 0.083 wall as specified in the ASC rules. All the welding was performed by a professional using the TIG process.

Center of Gravity

Borealis II is a 3 wheel layout with a 91.5 inch wheelbase, a 51.5 inch front track. The estimated location of the CG based upon measurements on the 2001 Borealis vehicle, and the design changes for the 2003, yield a CG height of 20 inches (which will be reduced for Formula SUN to 19.5 inches through suspension adjustments) and a distance of 32.3 inches behind the front axle. This corresponds to 35% of the weight on the rear wheel, which is very close to the desired 33.3%. There is a relationship between these distance parameters and the level of steady state lateral g's to initiate tipping: that is, to pick up the inside wheel (see "Designing Stable Three Wheeled Vehicles with Applications to Solar Powered Racing Cars", P.J. Starr, working paper sent to Sunrayce organizers, revised May 1996). For the Borealis II estimated parameters, set up for Formula SUN, the level of lateral g's to lift a wheel is 0.85, presuming the tire-road coefficient of friction can accommodate this.

Driver Restraint Description

Drivers Compartment

The drivers safety capsule is designed to remain un-violated in the event of a collision and constrain the driver inside. The 20.0 inch driver's compartment width holds most drivers snug from left to right which would be beneficial in a side impact. Also, the drivers lower extremities are confined within the drivers compartment from all sides and cannot "flail" in an accident. The side panels are 10.7" high at the driver's shoulders, and when belted in, only the driver's head is above the upper plane of the chassis. The driver head is constrained from moving rearward during a rear impact by a padded headrest. This reduces the risk of a whiplash injury.

Safety Harness

A six point safety harness will be utilized in Borealis II. The harness attachment points are located at the intersection of two or more panels, providing strength in multiple directions. The driver is reclined at approximately 27 degrees from horizontal. This configuration is similar to that addressed in SAE's Baja Buggy Competition Rules. These rules provided a guideline for harness mounting point location. The shoulder belts are secured below the top of the shoulder, constraining the driver in the event of a roll over. (See Figure 3) The lap belts are positioned three inches ahead of the intersection of the belly pan and the seat back. This ensures that the belts cross the hips and not the lower abdominal region. The submarine belts secure to the same location holding the driver in a frontal collision. The mounting brackets are bolted through he composite using composite inserts to distribute loads and transfer them to the panels.

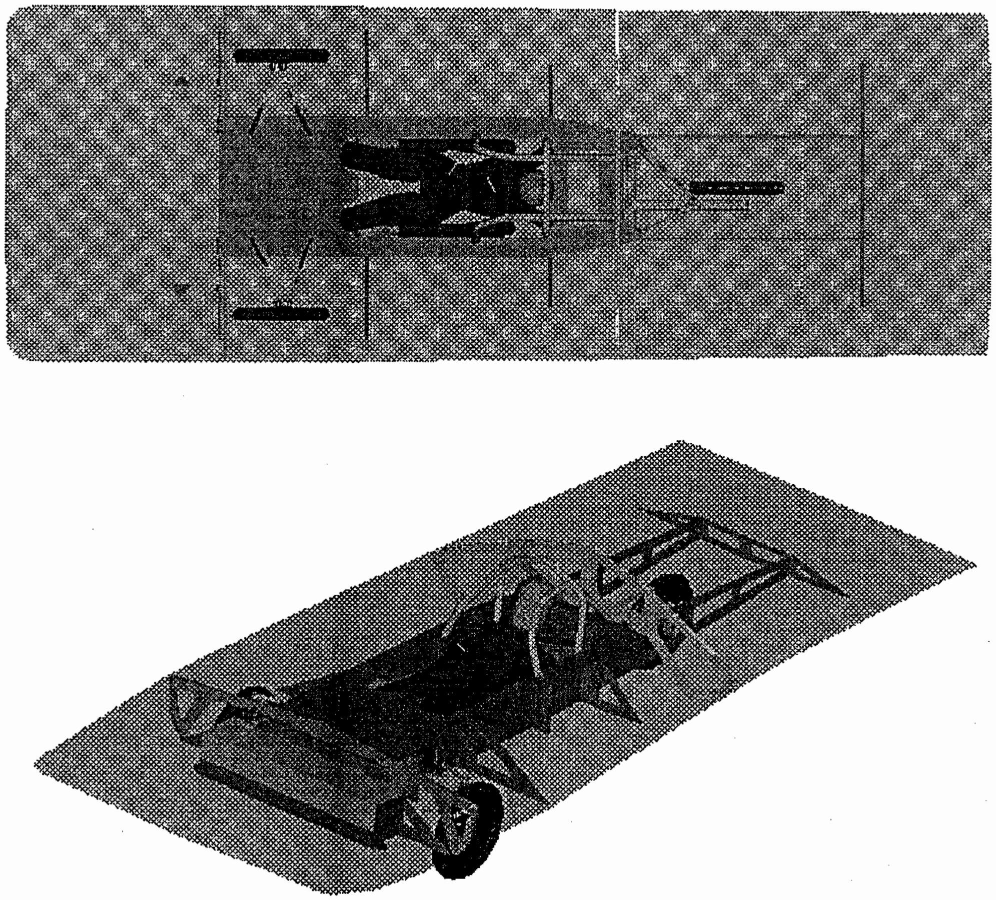


Figure 3 - Safety Harness Illustration

Crash Loading Analysis

Front and Rear 5 G Collisions

The loading on various panels will be traced and specific joints and panels will be analyzed for strength using accepted procedures based upon the properties of the panels and the bonding agent.

The bumper height ranges from 13.8 inches (35 cm) to 17.7 inches (45 cm) and the front chassis lateral bulkhead ranges from 10.5 inches to 21.5 inches. Thus the bumper will hit the chassis bulkhead and battery box, once the nose collapses. The body may or may not move rearward as the nose collapses. If the latch and guides for the body fail and the body moves rearward, the drivers canopy will hit the forward facing bends of the anti-decapitation bars and deflect up and rearward, coming off of the car. The canopy opening extends forward on the body approximately 16 inches ahead of the driver's head. If the body moves rearward, the edges of the canopy opening will hit the downward sloping portions of the anti-decapitation bars which will either deflect the upper body above the driver, or start tearing the body apart. Eventually, the bumper will contact the battery box and the front chassis bulkhead.

Figure 4 shows a top view of the chassis at this stage, with the inertia forces of the major components:

- The front batteries (66 lb)
- The driver (176 lb)
- The body (110 lb)
- The chassis, electrical components and suspension (198 lb)

- Total Weight = 550 lb

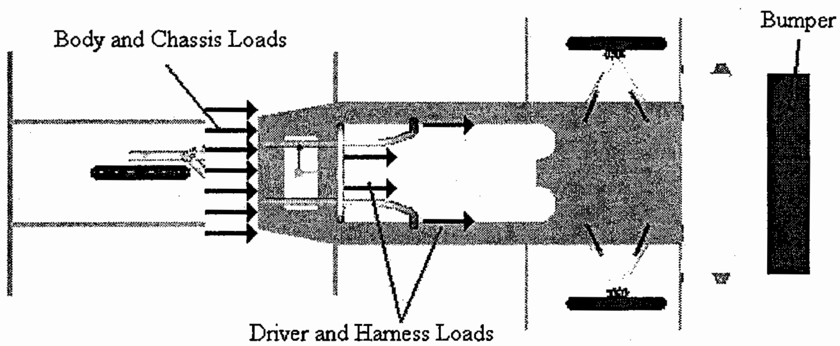


Figure 4 - Assumed Loads in Chassis

Each of these component weights is multiplied by five to estimate the 5G loading. The center of mass of the front battery pack is within the bumper height, and the battery pack is in front of the front bulkhead, so the inertia force due to the front battery pack acts directly upon the bumper. It does not load the chassis in front end collision. Thus the load on the chassis can be reduced by the 5G force on the front battery pack i.e. Force from bumper upon chassis = 5•(550-66)=5•(484)=2420 lbs (We will use 2500 lb). This load will be initially felt by the two vertical panels, one on each side of the driver. The front chassis bulkhead is glued to these panels, and so will distribute the bumper load across the front vertical face of each panel as shown in Figure 5.

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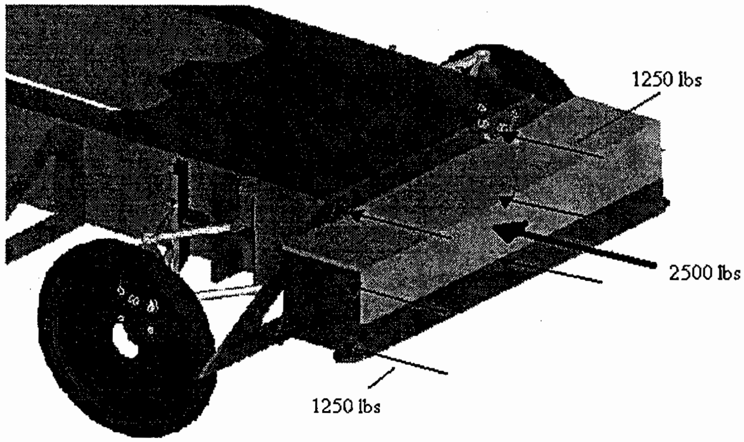

Figure 5 - Net Bumper Loads on Vertical Panels

The following will argue that the forces are taken by these vertical panels and have sufficient strength to withstand the front bumper loads under very conservative assumptions. The upper and lower panels are used to proved edge stability to the vertical panels and are not figured into the crash analysis. The inertia forces on the driver, body and chassis will all be assumed to be located at the rear of the driver's compartment, and will only be resisted longitudinally by the two vertical panels. These panels are a minimum of 10 inches high and 68 inches long. This is a conservative estimate because some of the impact force would be transmitted to the upper and lower panels as well as the vertical panels.

Each panel of Figure 5 has similar loading, so only one will be examined. The presumed mode of failure is buckling, and the critical stress level can be found using the methods in Successful Composite Techniques, by K. Noakes, Osprey Publishing, 1992, p 133-141.

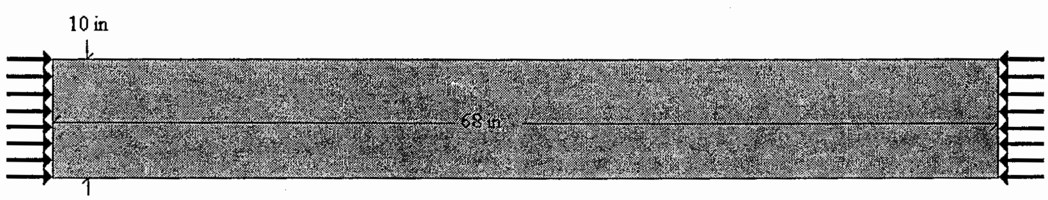

Figure 6 - Assumed Load on Vertical Panels

When conservatively treated with simply supported edges, the buckling stress in the face sheets was calculated to be 199,859 psi. This figure greatly exceeds the maximum skin stress of 57,000 psi as determined from the panel specifications, so in the following, we will use the lower 57,000 psi stress level. The actual compressive stress in the skin is simply the 1250 lb load divided by the skin area which gives a compressive stress of 6,250 psi. This is far below the 57,000 psi maximum value, giving a safety factor of 9. Thus the driver's compartment can easily withstand the 5 G frontal impact.

Rear Impact

The trailing edge of the vehicle at nominal ride height is 15.7 inches, while the bumper ranges from 13.8 inches (35 cm) to 17.7 inches (45 cm). Upon impact, the bumper will strike the tail section of the body which will start to collapse and be driven forward. The roll bar has two rearward sloping supports and these will serve to tear apart the rear body shell before it could impact the driver's headrest, or to drive the body up and over the driver's head. Then the bumper will hit the rear wheel, which has a radius of 9.5 inches, while the rear of the chassis is about 12 inches above the ground. The bumper will hit the rear wheel above its axle line, thereby pushing it forward and downward. The spring-shock unit will reach its full extension and sever, while the suspension, wheel and motor will be forced below the vehicle chassis as the suspension rod ends fail, being rotated beyond their design limits without the constraint of the shock unit. Finally the bumper will contact the rear chassis bulkhead the same way as for the front impact. The bulkhead will distribute the load to the two longitudinal vertical panels. It has already been shown that these vertical panels can easily handle this 5 G impact. The only difference here is the added front battery pack inertial loading. This creates an inertial loading of 2,750 lbs and a safety factor of about 7. The front battery pack is positioned in front of the front bulkhead and is 36 inches wide. This is wider than the two vertical panels are apart which means these panels will stop the battery pack. Thus we can conclude the chassis will withstand both front and rear 5 G impacts with the specified bumper.

Bending Rigidity

To assess the side impact and rollover strength of the chassis, it is necessary to estimate the bending moments caused by inertial loads of the components, and the locations of the chassis supports. Appendix D shows the estimated location and weights of components, which can be considered a 1 G load. The supports are the rear structural chassis plane and the front axle line. The maximum bending moment due to the loads and supports is seen to be 7071 inch-lb, occurring in the driver's compartment. The Appendix also shows computation of the chassis moments of inertia about a horizontal axis and vertical axis through the driver's compartment. This data will be used in the side impact and rollover analysis.

Side Impact at 5 G

This will be treated similarly to the front/rear end collisions. The driver is within the center 20 inches of the 28 inch wide structural chassis, with the legs of the roll hoop on each side of her head and the anti-decapitation bars running forward and to each side. The side impact will be considered a plane which contacts the vehicle, crushing the over 20 inches of body and array before impacting the structural chassis. The canopy would contact the side bars and fly off, and the body, which angles upward toward the driver from the side, would slide upward along the roll bar hoop and/or tear itself apart on the roll bar and the fore and aft supports. Eventually the side of the chassis would be contacted.

The load analysis of Appendix D can apply here, by presuming that the front and roll bar bulkheads are the supports. This would move the supports closer together than in Appendix D reducing the maximum bending moment, so by presuming the maximum 1

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G bending moment is still about 7000 in-lb to be conservative, a 5 G load would give a 35,000 in-lb moment. Appendix D shows the maximum C/I value about the vertical axis in the driver's compartment of 18/165.6 = 0.1087, so the maximum bending stress is 3,804 psi, well below the allowable 57,000 psi.

Rollover Scenario

Roll Cage Analysis

When the vehicle is in a rollover condition, the roll bar and the front of the chassis will support it. The following will show that the roll bar structure is able to support the entire 3 G load at various angles. This result will be used to argue that such loads may be placed upon the chassis to determine its strength in bending due to rollover. In the following, it will be shown that the main hoop alone is able to withstand the vertical and angular side loads in the plane of the hoop. Then it will be shown that the rearward supports and the main hoop, without the forward tubes, are sufficient to support the front-to-rear loads. Then any combination of load between these extremes will reduce to these individual loads and hence be accommodated.

Main Hoop Loads

The main hoop of the roll bar was analyzed in three loading conditions. These were analyzed using Roark's Formulas for Stress and Strain, Sixth Edition, 1989, curved beam formula 5c, Table 18, p 293-294. These formulas identify the ground loading and moments, which in turn can be used to find the maximum stress (Appendix E). Using the 3 G load of f=1620 (540 lb car) the maximum stress occurs in the attachment area where the hoop crosses the upper chassis plane. The hoop is attached to the chassis with brackets holding 4 inserts on each leg, 2 going into the seat back bulkhead and 2 into the side vertical bulkhead (reinforcing the bonded joint). Also a flange is welded where each leg crosses the upper chassis plane, and two more inserts are placed into the side and rear surfaces of the upper chassis. The stresses due to bending are much larger than due to shear or vertical loading and are shown below.

1620 lb vertical load on top of hoop: max stress =31,067 psi
1620 lb load at 30° on hoop: max stress = 48,764 psi
1620 lb load at 60° on hoop: max stress = 65,543 psi

these are all below the yield stress of 75,000 psi for the 4130 CDN steel tube. It must be emphasized that the forward and rearward supports play no role in this analysis, so it is conservative. Since these loads do not have fatigue considerations, the small safety factors are not a concern.

Fore-Aft Loading of the Roll Bar

Appendix E shows 3 loading cases, in which the rear supports play a role. Each support is an intermediate column with a low L/r ratio and is in the Johnson theory range. It is shown that the safety factors for the fore-aft load cases exceed 13, based upon the yield strength.

Thus it can be concluded that loads placed upon the roll bar structure are transmitted to the chassis structure without causing failure of the roll bar. These loads can be used to determine chassis strength in rollover.

Typical Roll Over Situation

The typical roll over situation would see the vehicle transferring inertial load to the ground at two points: the roll bar peak and the front, upper corner of the frame. The load analysis of Appendix E can apply with both supports moved forward about 12 inches, representing the roll bar hoop at the rear, and the top of the structural chassis at the front. This would change the maximum 1 G moment a bit, but it would still be in the 7000 in-lb range. For the 3 G impact, the moment would be about 21,000 in-lb. The maximum C/I value about the horizontal axis is computed in Appendix E as C/I = 8.216/22.6, so the corresponding maximum face stress of 7634 psi, well below the allowable 57,000 psi.

Crash Load Increments between Horizontal and Vertical

Viewed from Front

These loading configurations can be inferred to be satisfactory from the calculations performed in previous sections. A planar load applied to the chassis from any angle between the vertical and horizontal, as viewed from the front, can never violate the drivers compartment. The plane will be forced to load the roll bar hoop and the top plane of the chassis at its widest point. The resulting localized loads would be much lower than those shown above. Therefore, since the chassis has very large safety factors in the vertical and horizontal load directions these lower loads could only produce higher safety factors resulting in a safe condition.

Viewed from Side

Again, any planar load between the vertical and horizontal loads would not violate the driver's cockpit. Instead, they would load the top of the roll hoop and the foremost portion of the top plane. The resulting localized loads would be much lower than those already found to be safe in preceding sections. Therefore the chassis is deemed safe in all incremental loading situations as viewed from the side.

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Appendices

Appendix A – Front Suspension

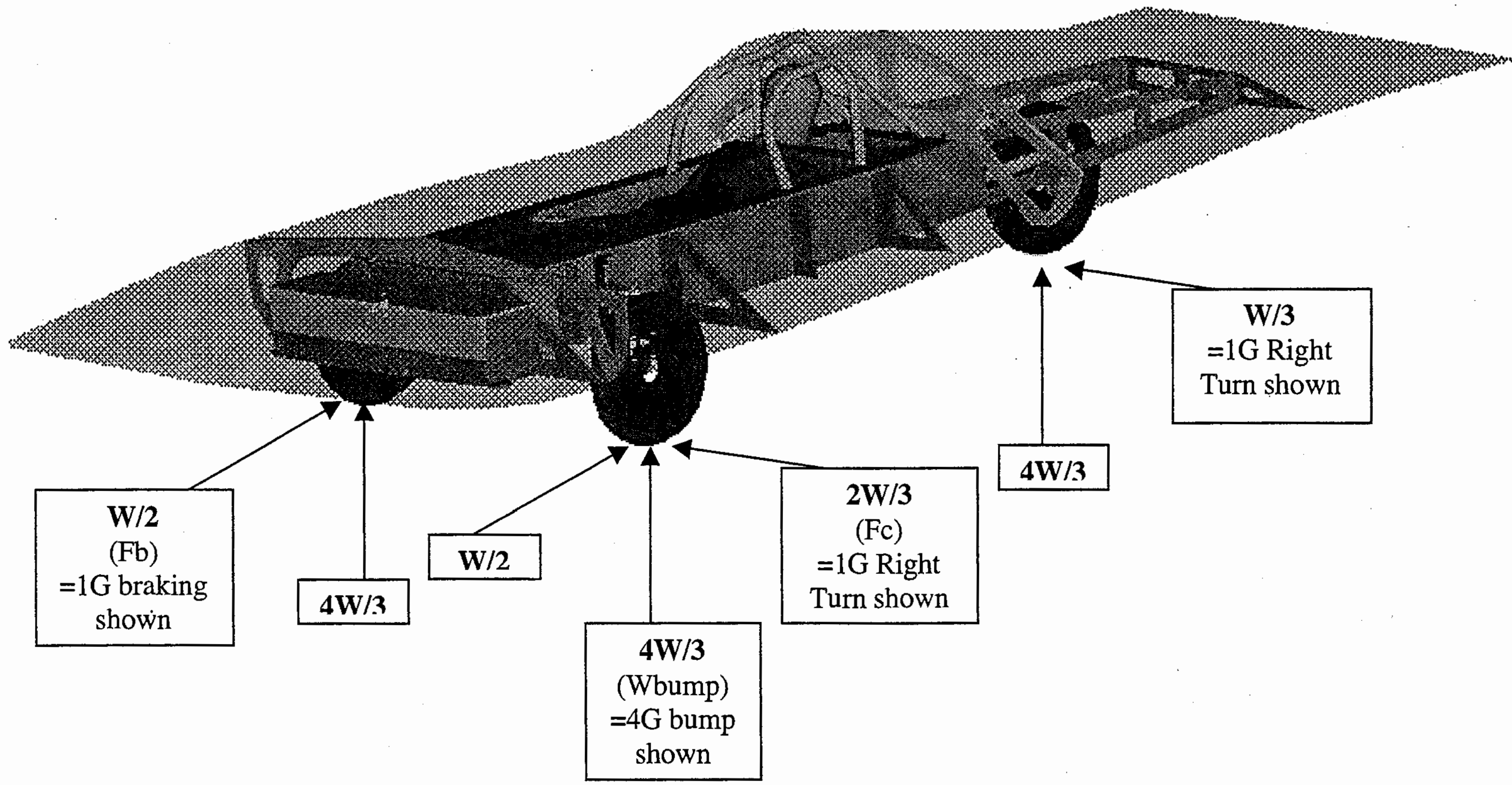

Figure 7a - Vehicle External Loading Diagram for Suspension Analysis

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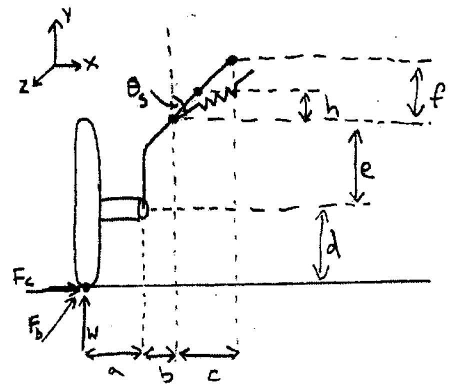
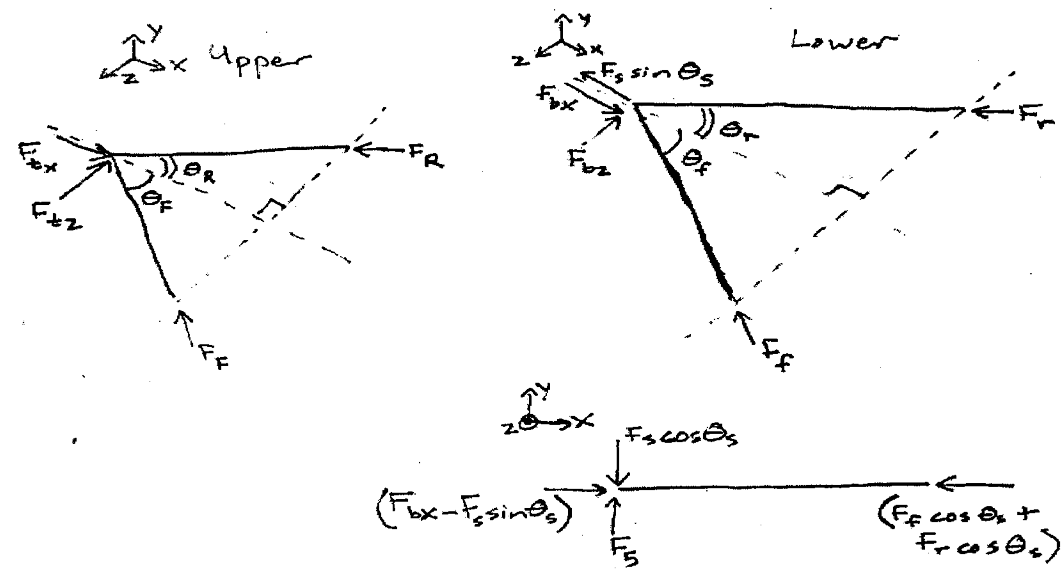
Figure 7b - Force Free-Body Diagrams

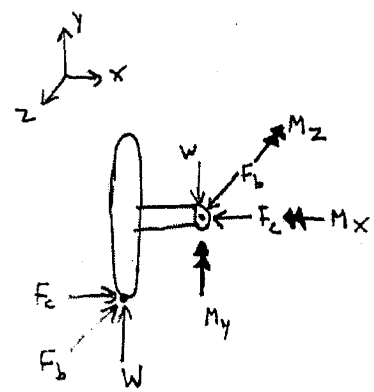
A) Front Suspension Assembly



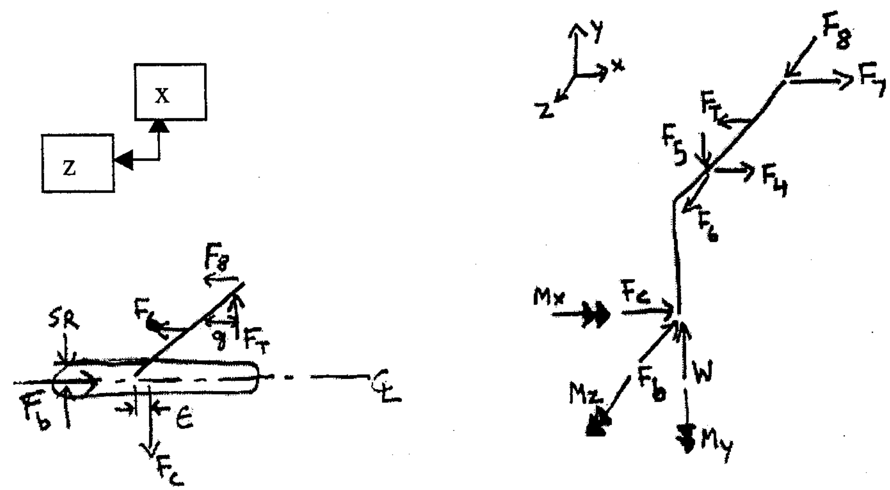
B) Upper and Lower A-arms



C) Wheel and Axle



D) Upright



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Composition of Inputs on one Wheel: (8 Possible Cases)

Loading Input (G's applied) (W = 1 --> static weight)

Fb	0	0	0.5	0.5	0	0.5	0.5	0	*(% W)*
W	0.333333	1.3333333	1.3333333	1.3333333	1.3333333	0.3333333	0.3333333	0.3333333	
Fc	0	0	0	0.6666667	0.6666667	0	0.6666667	0.6666667	

Fb	0	0	1	1	0	1	1	0	*(G's)*
W	1	4	4	4	4	1	1	1	
Fc	0	0	0	1	1	0	1	1	

Reaction Forces: *(lbf)*

									direction
F4	56	226	275	-747	-796	106	-916	-965	x
F5	183	733	733	733	733	183	183	183	y
F6	0	0	800	800	0	800	800	0	z
F7	-56	-226	-184	552	511	-15	721	680	x
F8	0	0	-525	-525	0	-525	-525	0	z
Fs	293	1173	1173	1173	1173	293	293	293	xy
Fr	-220	-879	-384	403	-92	275	1062	567	xz
Ff	-220	-879	**-1983**	-1196	-92	-1324	-537	567	xz
FR	34	136	-508	-951	-307	-610	-1053	-409	xz
FF	28	112	736	371	-254	652	287	-338	xz
FT	0	0	91	172	81	91	172	81	xz

θ_s	0.89570	radians	spring angle	51.32	(degrees)
θ_r	0.52360	radians	btm rear arm angle	30	(degrees)
θ_f	0.52360	radians	btm front arm angle	30	(degrees)
θ_R	0.38397	radians	top rear arm angle	22	(degrees)
θ_F	0.47124	radians	top front arm angle	27	(degrees)

1983 lbf Largest Force Magnitude

INPUTS:

Braking 1G (lbf)	550	Fb
Static Weight 1G (lbf)	550	W
Bump 4G (lbf)	2200	W
Cornering 1G (lbf)	550	Fc

(refer to FBD's 4th iteration)

Distances: (Figure 1)

Name	*Value*	*Units*
a	1.821	*inch*
b	0.179	*inch*
c	0.4	*inch*
total	2.4	*inch*

(SVP Mech Sys Notes, pg. 85)

(FBD 5 a,b)

g	3.75	*inch*
SR	1.237	*inch*
E	0.83	*inch*

Heights: (Figure 1)

Name	*Value*	*Units*
d	9.5	*inch*
e	2.9	*inch*
f	6.5	*inch*
total	18.9	*inch*

(SVP Mech Sys Notes, pg. 85)

(Figure 1)

h	2.956	*inch*

Figure 8 – Front Suspension Force Analysis

Figure 9a - Upright Section Moment of Inertia and Stress Calculation Formulas

Variables List:
li= length of individual section
wi= width of individual section
zbari= centroid of individual section in z-direction
xbari= centroid of individual section in x-direction
Izzi= Moment of Inerita contribution about axis z-z by individual section
Ixxi= Moment of Inertia contribution about axis x-x by individual section
Ai = individual section 2-D area
dzzi= distance from individual section centroid to the neutral axis z-z
dxxi= distance from individual section centroid to the neutral axis x-x

There are four section types in the upright cross-section:
1- horizontal center, 2- vertical sides, 3- open side flanges, 4- closed side flanges

Section Centroids:
Subscript 'i' refers to which section type is being calculated for.

$$\bar{x}_{total} = \frac{\sum (\bar{x}_i * A_i)}{\sum A_i}$$

By symmetry of the cross section, we know that the centroid in the z-direction will be exactly in the middle.

$$\bar{z}_{total} = \frac{\sum (z_i * A_i)}{\sum A_i} = 0$$

$$dzz_1 = \bar{x}_{total} - \bar{x}_1$$

dxxi and dzzi vary for each section type.

Parallel Axis Theorem: ex: $Izz_1 = \frac{(l_1 \bullet w_1^3)}{12} + A_1 \bullet dzz_1^2$

Total Moments of Inertia and Bending Stress Distance:

$$I_{zztotal} = \sum I_{zzi}$$

$$\mathrm{I}_{xxtotal} = \sum I_{xxi}$$

$$z_{\max} = \frac{l_1}{2} + l_2 + \max[l_3, l_4]$$

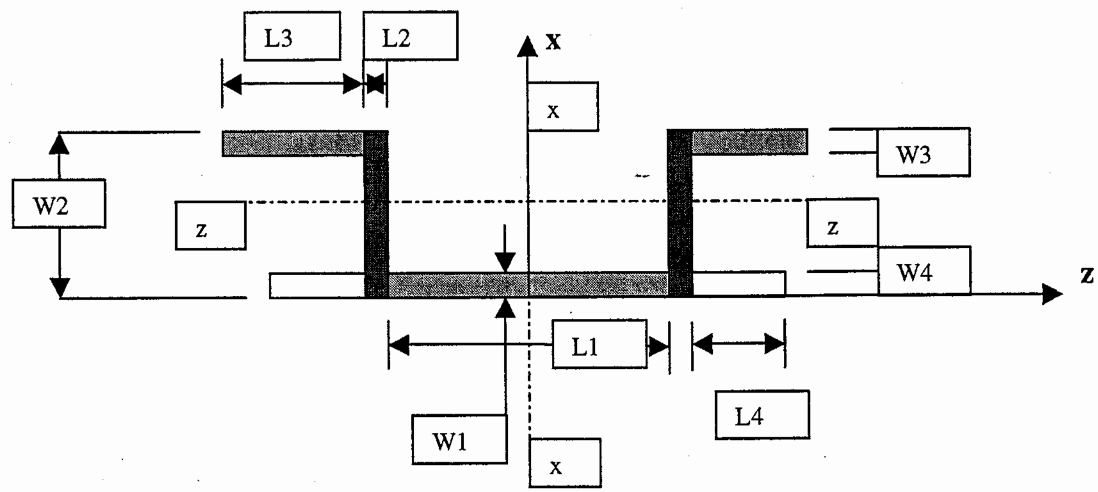
$$x_{\max} = \max[\bar{x}_{total}, (w_2 - \bar{x}_{total})]$$

Bending Stresses:

$$\sigma_x = \frac{(z_{\max} * M_{bending})}{I_{xxtotal}}$$

$$\sigma_z = \frac{(x_{\max} * M_{bending})}{I_{zztotal}}$$

Figure 9b - Upright Cross Sections



Moment Axis	X braking	Y	Z cornering		Max σ: X	Z	axis
Height1- axle	0	0	0	[FBD letter]			
inch value	0	0	0	[inch from axleline]			
Mmax1	2613	805	3149	[inch-lbf]	4735	14106	[psi]
Height2- lower joint/str tie rod	e	e+h	e	[FBD letter]			
inch value	2.9	5.856	2.9	[inch from axleline]			
Mmax2	3410	805	4213	[inch-lbf]	3860	12477	[psi]
Height3- top joint	e+f	e+f	e+f	[FBD letter]			
inch value	9.4	9.4	9.4	[inch from axleline]			
Mmax3	2000	161	2000	[inch-lbf]	5844	11823	[psi]

(Max σ using max tension and compression values, not Max σ using Xmax and Zmax values.)

7075-T6 Aluminum Properties:

Fatigue Stress=23000 psi @ 500,000,000 cycles (www.matweb.com)

Yield Stress=70000psi

(Fundamentals of Machine Component Design 3rd, by Juvinall and Marshek)

Figure 9c - Moment of Inertia Calculations for Front Suspension Upright Cross Sections at Areas of Direct Load Application

Borealis 2 Inputs:
Concept 2

At Lower Joint

Upright Height Location e — 2.9 **[inches] from axleline**

Section	1	2	0.35	4	
li	2.30	0.150	0.40	0.40	[in] }free choices
wi	0.10	1.50	0.20	0.20	[in] }free choices
zbari	0.0	1.225	1.5	1.5	[in]
xbari	0.05	0.75	1.4	0.1	[in]
Izzi	0.06701	0.04802	0.05288	0.01940	[in^4] I about zz axis
Ixxi	0.10139	0.33806	0.18107	0.18107	[in^4] I about xx axis
Ai	0.23	0.225	0.08	0.08	[in^2]
dzzi	0.539	-0.161	-0.811	0.489	[in] dist to zz
dxxi	0.0	1.225	1.5	1.5	[in] dist to xx
			0.0800	0.0800	

Cross-section Mass assuming depth of 1 unit
0.1020 lbm = Atotal*density of Aluminum

[in^4] [full section] IzzTotal	IxxTotal	[in] [stress distances] ZbarTotal	XbarTotal
0.3076	1.5018	0.000	0.589

	Design C/I [in^-3]	Axis
	Zmax	***Xmax***
	1.7	1.5
	σ_{zz}	σ_{xx}
tension	8067	3860
compression	12477	
Safety Factors:		
s yield	5.6	18.1
s fatigue	1.8	6.0
Max σ:		
	X	**Z**
	16628	4769

Just Below Top Joint

Upright Height Location e+f — 9.4 **[inches] from axleline**

Section	1	2	3	4	
li	1.40	0.100	0.30	0.30	[in] }free choices
wi	0.10	1.20	0.20	0.20	[in] }free choices
zbari	0	0.75	0.95	0.95	[in]
xbari	0.05	0.6	1.1	0.1	[in]
Izzi	0.02550	0.01625	0.02358	0.00867	[in^4] I about zz axis
Ixxi	0.02287	0.06760	0.05460	0.05460	[in^4] I about xx axis
Ai	0.14	0.12	0.06	0.06	[in^2]
dzzi	0.426	-0.124	-0.624	0.376	[in] dist to zz
dxxi	0	0.75	0.95	0.95	[in] dist to xx

Cross-section Mass assuming depth of 1 unit
0.0632 lbm = Atotal*density of Aluminum

[in^4] [full section] IzzTotal	IxxTotal	[in] [stress distances] ZbarTotal	XbarTotal
0.1225	0.3765	0.000	0.476

	Design C/I [in^-3]	Axis
	Zmax	***Xmax***
	1.1	1.2
	σ_{zz}	σ_{xx}
tension	7768	5844
compression	11823	
Safety Factors:	should not be any bendin	
s yield	5.9	12.0
s fatigue	1.9	3.9
Max σ:		
	X	**Z**
	0	5844

At the Axle Height

Upright Height Location 0 — 0 **[inches] from axleline**

Section	1	2	3	4	
li	2.00	0.125	0.20	0.20	[in] }free choices
wi	0.10	1.50	0.20	0.20	[in] }free choices
zbari	0	1.0625	1.225	1.225	[in]
xbari	0.05	0.75	1.4	0.1	[in]
Izzi	0.05209	0.04196	0.02839	0.00858	[in^4] I about zz axis
Ixxi	0.06667	0.21191	0.06016	0.06016	[in^4] I about xx axis
Ai	0.2	0.1875	0.04	0.04	[in^2]
dzzi	0.510	-0.190	-0.840	0.460	[in] dist to zz
dxxi	0	1.0625	1.225	1.225	[in] dist to xx

Cross-section Mass assuming depth of 1 unit
0.0750 lbm = Atotal*density of Aluminum

[in^4] [full section] IzzTotal	IxxTotal	[in] [stress distances] ZbarTotal	XbarTotal
0.2099	0.7311	0.000	0.560

	Design C/I [in^-3]	Axis
	Zmax	**Xmax**
	1.325	1.5
	σ_{zz}	σ_{xx}
tension	8392	4735
compression	14106	
Safety Factors:		
s yield	5.0	14.8
s fatigue	1.6	4.9
Max σ:		
	X	**Z**
	0	0

Figure 10 - Pro-Engineer Front Suspension System Assembly

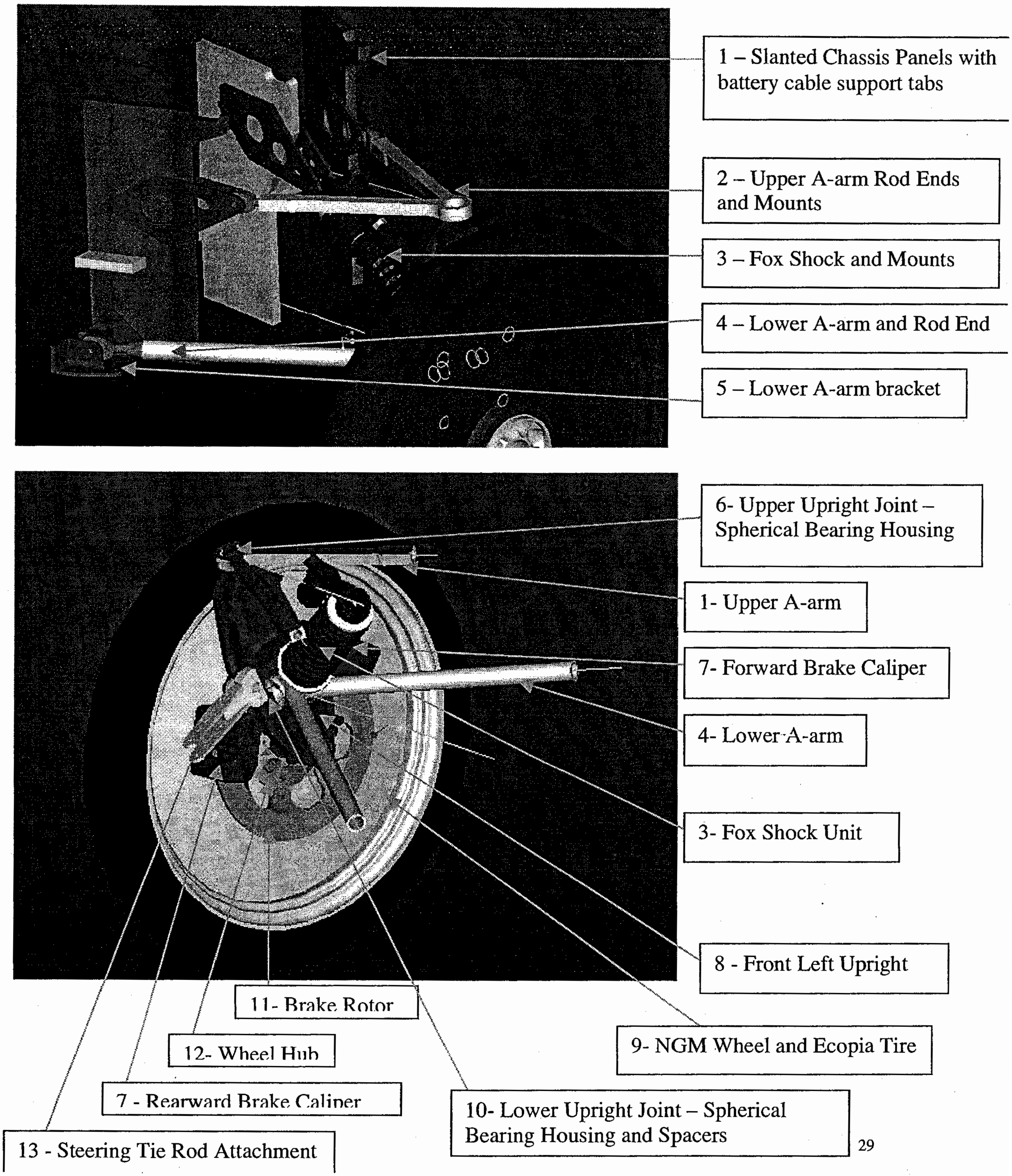


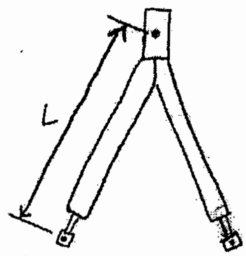
Figure 11 - Buckling Calculations

Jason Halpern
3/23/2003 UMNSVP
<u>B2 A-arm Buckling Analysis</u>

<u>Lower A-arm:</u>

Material: 4130 Chromoly steel Hollow Tubing.
3/4" nom OD × .049" wall
E = 29,700 ksi (www.matweb.com) S_y = 70.0 ksi (www.emjmetals.com)
Model as a fixed-pinned column.

$L_{eq} = .8 * L$
$L_{eq} = 11.54''$



L = 14.43"

slenderness Ratio Dividing Point:

$$(Sr)_D = \sqrt{\frac{2\pi^2 E}{S_y}} = \sqrt{\frac{2\pi^2 (29{,}700\text{ksi})}{(70.0\text{ ksi})}} = 91.5$$

$Sr = \frac{L_{eq}}{\rho}$, $\rho = \sqrt{\frac{I}{A}} = .2484$ (Shelby Seamless Round Tubing – SVP Mech Notes page 232)

$I = .0067 \text{ in}^4$
$A = .1079 \text{ in}^2$

$Sr = 46.46$

→ $Sr < (Sr)_D$ ∴ use Johnson mode of buckling. ←

$$P_{cr} = A_{xsec}\left[S_y - \frac{S_y^2}{4\pi^2 E}\left(\frac{L_{eq}}{\rho}\right)^2\right] = (.1079)\left[70 - \frac{(70^2)}{4\pi^2(29{,}700)}\left(\frac{11.54}{.2484}\right)^2\right]$$

$P_{cr} = 6.580$ kips $= 6580\ Lb_f$

$P_{max\ actual} \approx 1990\ Lb_f < P_{cr}$ ∴ $\left[\text{Safety Factor} = \frac{6580}{1990} = 3.3\right]$

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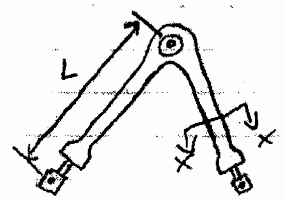
Jason Halpern
3/23/2003 UMNSVP
B2 A-arm Buckling
Analysis

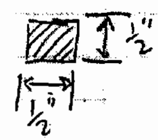
Upper A-arm:

Material: 7075-T6 Aluminum
$\frac{1}{2}'' \times \frac{1}{2}''$ Square Solid Section
$E = 10400$ ksi $\qquad S_y = 73.2$ ksi
Model as a fixed-pinned column.

$L_{eq} = .8 * L$
$L = 8.80''$
$L_{eq} = 7.04''$



section X-X:



$A = .25 \text{ in}^2$

$I = \frac{(.5)(.5)^3}{12}$

$I = 0.0052 \text{ in}^4$

Slenderness Ratio Dividing Point:

$$(S_r)_D = \sqrt{\frac{2\pi^2 (10{,}400 \text{ ksi})}{(73.2 \text{ ksi})}} = 52.9$$

$$\rho = \sqrt{\frac{.0052}{.25}} = .14$$

$$S_r = \frac{L_{eq}}{\rho} = \frac{7.04''}{.14} = 50.29$$

→ $S_r < (S_r)_D$ ∴ use Johnson mode of buckling ←

$$P_{cr} = (.25)\left[73.2 - \frac{(73.2^2)}{4\pi^2 (10400)}\left(\frac{7.04}{.14}\right)^2\right] = 10.05 \text{ kips}$$

$P_{cr} = 10{,}050$ Lb_f

$P_{max\ actual} = 1060$ Lb_f $< P_{cr}$ ∴ $\left[\text{Safety Factor} = \frac{10{,}050}{1{,}060} = 9.4\right]$

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Appendix B – Rear Suspension

3-D PERSPECTIVE

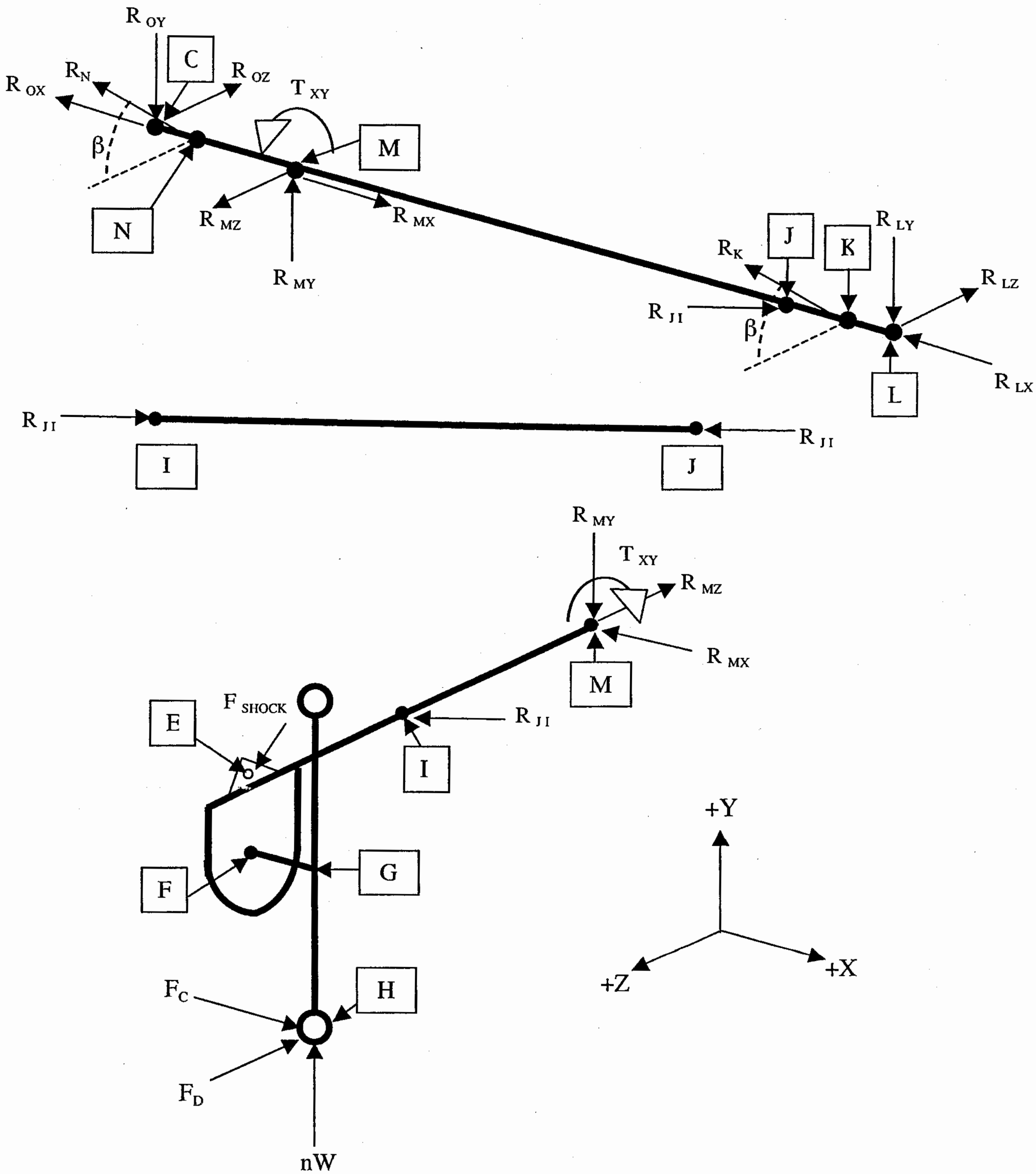

FIGURE 2A-THIS FIGURE SHOWS THE FORCES ACTING ON THE REAR SUSPENSION FROM A 3-D PERSPECTIVE.

YZ-PLANE

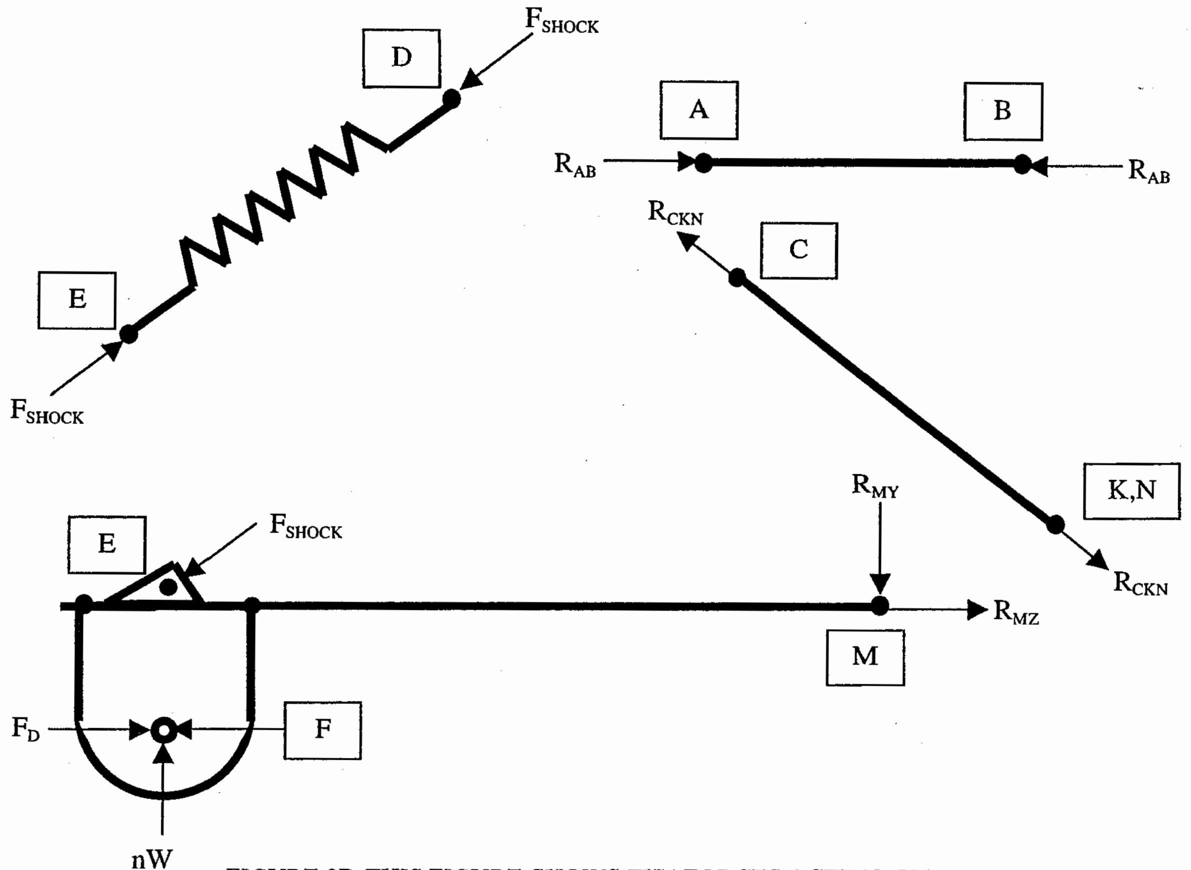

FIGURE 2B-THIS FIGURE SHOWS THE FORCES ACTING ON THE REAR SUSPENSION IN THE YZ-PLANE.

PLANE CKLNO

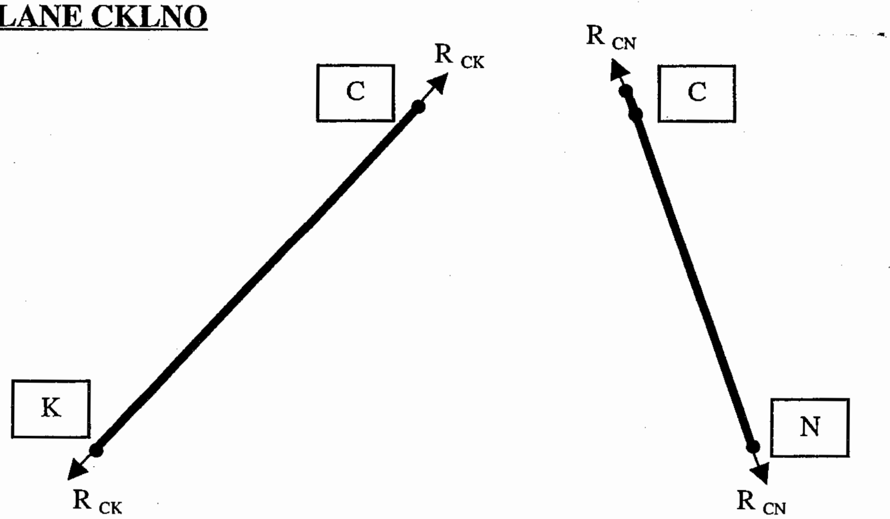

FIGURE 2C-THIS FIGURE SHOWS THE FORCES ACTING ON THE REAR SUSPENSION IN PLANE CKLNO.

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REACTION FORCE AND TORQUE EQUATIONS

$$F_{SHOCK} = nW * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right)$$

$$R_{MX} = F_C \quad R_{MY} = nW$$

$$R_{MZ} = nW * \left(\frac{b1}{b1-c3}\right) * \left(\frac{c5-c3}{h4-h2}\right) - F_D$$

$$R_{AB} = nW * \left(\frac{b1}{b1-c3}\right) * \left(\frac{c5-c3}{h4-h2}\right)$$

$$R_{CKN} = nW * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta)$$

$$R_{CK} = nW * \left(\frac{e9}{e7}\right) * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta) * \left(\frac{\frac{1}{2}d3 - r - f6 + d4}{d3 - 2*(f6-d4)}\right)$$

$$R_{CN} = nW * \left(\frac{e8}{e7}\right) * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta) * \left(\frac{\frac{1}{2}d3 + r - f6 + d4}{d3 - 2*(f6-d4)}\right)$$

$$R_{JI} = F_C * \left(\frac{b1}{b3}\right) * \left(\frac{d1}{\frac{1}{2}d3 + r - d2}\right)$$

$$T_{XY} = F_C * (R + a2) + nWr$$

$$R_N = nW * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta) * \left(\frac{\frac{1}{2}d3 + r - f6 + d4}{d3 - 2*(f6-d4)}\right)$$

$$R_{NY} = nW * \left(\frac{f4}{f3}\right) * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta) * \left(\frac{\frac{1}{2}d3 + r - f6 + d4}{d3 - 2*(f6-d4)}\right)$$

$$R_{NZ} = nW * \left(\frac{f5}{f3}\right) * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta) * \left(\frac{\frac{1}{2}d3 + r - f6 + d4}{d3 - 2*(f6-d4)}\right)$$

$$R_K = nW * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta) * \left(\frac{\frac{1}{2}d3 - r - f6 + d4}{d3 - 2*(f6-d4)}\right)$$

$$R_{KY} = nW * \left(\frac{f4}{f3}\right) * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta) * \left(\frac{\frac{1}{2}d3 - r - f6 + d4}{d3 - 2*(f6-d4)}\right)$$

$$R_{KZ} = nW * \left(\frac{f5}{f3}\right) * \left(\frac{b1}{b1-c3}\right) * \left(\frac{ls}{h4-h2}\right) * \cos(180° - \theta - \beta) * \left(\frac{\frac{1}{2}d3 - r - f6 + d4}{d3 - 2*(f6-d4)}\right)$$

$$R_{LX} = F_C$$

$$R_{LY} = R_{NY} * \frac{f6-d4}{d3} + nW * \frac{\frac{1}{2}d3 - r}{d3} + R_{KY} * \frac{d3 - f6 + d4}{d3} - \frac{\frac{1}{2}F_C * (R + a2) - \frac{1}{2}nWr}{\frac{1}{2}d3 + r}$$

$$R_{LZ} = R_{NZ} * \frac{f6-d4}{d3} + R_{KZ} * \frac{d3 - f6 + d4}{d3} + R_{MZ} * \frac{\frac{1}{2}d3 - r}{d3}$$

$$R_{OX} = F_C$$

$$R_{OY} = R_{NY} * \frac{d3 - f6 + d4}{d3} + nW * \frac{\frac{1}{2}d3 + r}{d3} + R_{KY} * \frac{f6-d4}{d3} - \frac{\frac{1}{2}F_C * (R + a2) + \frac{1}{2}nWr}{\frac{1}{2}d3 - r}$$

$$R_{OZ} = R_{NZ} * \frac{d3 - f6 + d4}{d3} + R_{KZ} * \frac{f6-d4}{d3} + R_{MZ} * \frac{\frac{1}{2}d3 + r}{d3}$$

BOREALIS II REAR SUSPENSION FORCE ANALYSIS
COMPARISON OF FOUR DIFFERENT LOADING SITUATIONS

Suspension Dimensions:		
a1 =	3.50	inches
a2 =	3.75	inches
b1 =	21.25	inches
b2 =	0.50	inches
b3 =	14.19	inches
c1 =	1.84	inches
c2 =	0.57	inches
c3 =	1.75	inches
c4 =	0.74	inches
c5 =	7.18	inches
ls =	7.68	inches
d1 =	18.18	inches
d2 =	1.00	inches
d3 =	17.06	inches
d4 =	0.57	inches
e1 =	14.68	inches
e2 =	18.22	inches
e3 =	13.53	inches
e4 =	0.85	inches
e5 =	5.72	inches
e6 =	0.50	inches
e7 =	14.75	inches
e8 =	15.37	inches
e9 =	18.45	inches
f1 =	1.38	inches
f2 =	0.83	inches
f3 =	1.63	inches
f4 =	0.83	inches
f5 =	1.40	inches
f6 =	1.40	inches
h1 =	9.50	inches
h2 =	15.04	inches
h3 =	12.75	inches
h4 =	20.35	inches
w =	18.20	inches
t =	8.35	inches
R =	9.50	inches
r =	3.38	inches
theta =	44.360	degrees
beta =	30.662	degrees

Loading Conditions:	When steering to the left:		When steering to the right:	
Number of g's, n =	1	4	1	4
Weight at Rear Wheel, W (lbf) =	183.33	183.33	183.33	183.33
Input Forces:				
Drive Force, F_D (lbf) =	15	15	15	15
Bump Force, nW (lbf) =	183.33	733.32	183.33	733.32
Cornering Force, F_C (lbf) =	-183.33	-183.33	183.33	183.33
Reaction Torques and Forces:				
F_{SHOCK} (lbf) =	288.95	1155.80	288.95	1155.80
R_{MX} (lbf) =	-183.33	-183.33	183.33	183.33
R_{MY} (lbf) =	183.33	733.32	183.33	733.32
R_{MZ} (lbf) =	189.30	802.19	189.30	802.19
R_{AB} (lbf) =	204.30	817.19	204.30	817.19
R_{CKN} (lbf) =	-74.68	-298.72	-74.68	-298.72
R_{CK} (lbf) =	-26.20	-104.82	-26.20	-104.82
R_{CN} (lbf) =	-55.99	-223.96	-55.99	-223.96
R_{JI} (lbf) =	-457.49	-457.49	457.49	457.49
T_{XY} (in-lbf) =	-1809.47	49.50	3048.78	4907.74
R_N (lbf) =	-53.73	-214.92	-53.73	-214.92
R_{NY} (lbf) =	-27.40	-109.61	-27.40	-109.61
R_{NZ} (lbf) =	-46.22	-184.88	-46.22	-184.88
R_K (lbf) =	-20.95	-83.80	-20.95	-83.80
R_{KY} (lbf) =	-10.68	-42.74	-10.68	-42.74
R_{KZ} (lbf) =	-18.02	-72.08	-18.02	-72.08
R_{LX} (lbf) =	183.33	183.33	-183.33	-183.33
R_{LY} (lbf) =	119.81	173.30	-84.15	-30.65
R_{LZ} (lbf) =	37.75	164.59	37.75	164.59
R_{OX} (lbf) =	183.33	183.33	-183.33	-183.33
R_{OY} (lbf) =	277.07	400.79	-194.60	-70.89
R_{OZ} (lbf) =	87.30	380.64	87.30	380.64

***Note that R_{NY}, R_{NZ}, R_{KY}, and R_{KZ} are the respective y-components and z-components of R_N and R_K.**

BOREALIS II REAR SUSPENSION STRESS ANALYSIS

Stress analysis will be performed for components of the rear suspension which are made of the following materials. The stress analysis will consider a worst case scenario of 4-g bump and 1-g cornering. With this type of loading, reaction forces and torques are greatest when the car is cornering to the right as shown in the force analysis.

Material Property Information:			
7075-T6 Aluminum			
	Yield Strength, S_Y =	73000	psi
	Modulus of Elasticity, E =	10400000	psi
Normalized 4130 Steel			
	Yield Strength, S_Y =	75000	psi
	Modulus of Elasticity, E =	29700000	psi

Component Stress Analysis

For the following four links, column buckling is a potential result since they experience axial compressive stress for worst case loading. The critical stress formulas that will be used for Johnson and Euler Column Buckling are as follows:

For Johnson Column Buckling:

$$S_{CR} = S_Y - \frac{1}{E} * \left(\frac{S_Y S_R}{2\pi}\right)^2$$

For Euler Column Buckling:

$$S_{CR} = \frac{\pi^2 E}{S_R^2}$$

	A.) Link AB	B.) Link CK	C.) Link CN	D.) Link IJ
Material Type:	7075-T6 Aluminum	7075-T6 Aluminum	7075-T6 Aluminum	4130 Steel Tubing with Outer Diameter 1.000 in. and wall thickness of 0.035 in.
Nominal Length (in.):	13.53	18.22	14.68	18.18
Column Endpoint Conditions:	pinned-pinned	pinned-pinned	pinned-fixed	fixed-fixed
Effective length (in.):	13.53	18.18	11.74	11.82
Cross-section type:	solid square	solid square	solid square	circular tube
Cross-sectional area (in^2):	0.25	0.25	0.25	0.1061
Cross-sectional moment of inertia (in^4):	0.00521	0.00521	0.00521	0.0123
Radius of gyration, ρ (in.):	0.1443	0.1443	0.1443	0.3405
Compressive Load (lbf):	817.19	104.82	223.96	457.49
Resulting Stress, S_{ACTUAL} (psi):	3269	419.3	895.8	4312
Decision Coefficient, $S_R = L_E/\rho$:	93.74	125.95	81.36	34.72
Decision Criteria Coefficient, $S_{R/D} = ((2\pi^2E)/S_Y)^{0.5}$:	53.03	53.03	53.03	88.41
Buckling Type:	Since $S_R > S_{R/D}$, Euler Buckling	Since $S_R > S_{R/D}$, Euler Buckling	Since $S_R > S_{R/D}$, Euler Buckling	Since $S_R < S_{R/D}$, Johnson Buckling
Critical Stress, S_{CR} (psi):	11681	6470	15505	69218
Safety Factor, N = S_{CR}/S_{ACTUAL}:	3.57	15.43	17.31	16.05

E.) Swing Arm (Tubing Portion of Link EFIM)

The tubing of the swing arm experiences the greatest amount of stress at point I. At this point, tensile, bending, and torsional stresses exist. Each stress component will be calculated, and then the equivalent mean stress at point I will be determined. The actual stress will be compared to the yield strength of 4130 Steel in a safety factor calculation.

The tubing of the swing arm has the following geometric properties:

Cross-sectional area (in^2):	0.3441
Cross-sectional moment of inertia (in^4):	0.1223
Polar moment of inertia (in^4):	0.2446
Outer Diameter (in.):	1.75
Wall Thickness (in.):	0.065

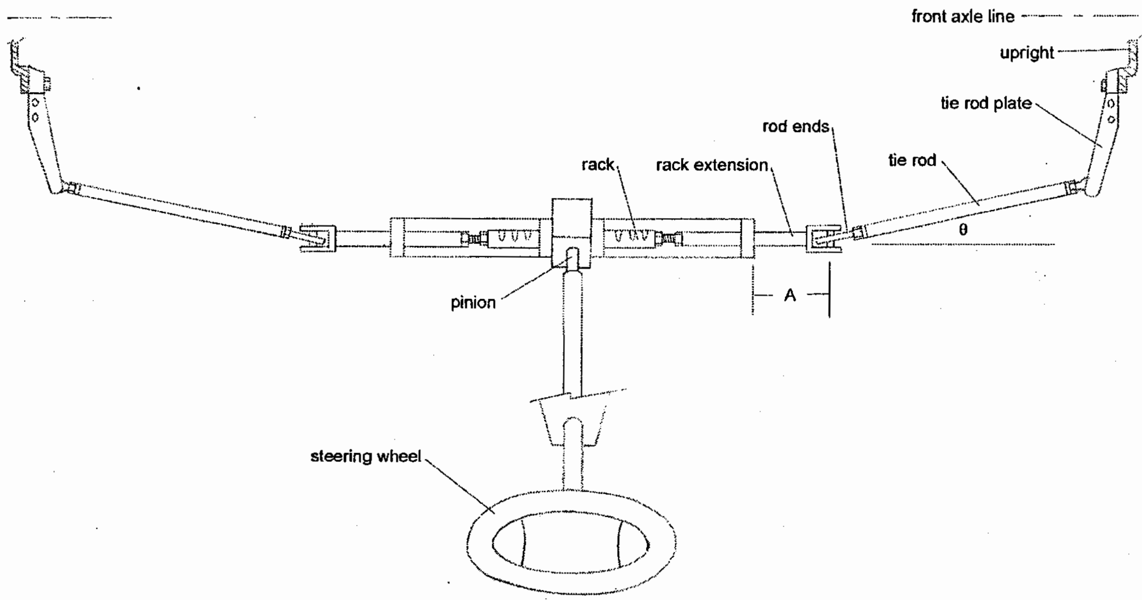
Torsion at point I (in-lbf):	4907.74
Tensile Force, R_{MZ} (lbf):	802.19
Bending Moment at point I (in-lbf):	1294.10
Torsional Stress, τ (psi):	17556
Tensile Stress, σ_T (psi):	2331
Bending Stress, σ_B (psi):	9259
Equivalent Axial Stress, $\sigma_M = \sigma_B - \sigma_T$ (psi):	6927
Equivalent Stress, $\sigma_{EQ} = (\sigma_M/2)+((\sigma_M/2)^2+\tau^2)^{0.5}$ (psi):	21358
Safety Factor = S_Y/σ_{EQ} :	3.51

F.) Crosstube (Link LMO)

The crosstube of the rear suspension experiences the greatest amount of stress at point M. The torque transmitted by the swing arm induces a bending moment in the XY-plane. Due to the tensile force, R_{MZ} there is another bending moment produced in the XZ-plane. These bending moments will be combined and a bending stress will be calculated. The resulting bending stress value will be compared to the yield strength of 4130 Steel in a safety factor calculation.

Moment in the XY-plane, M_{XY} (in-lbf):	4908
Moment in the XZ-plane, M_{XZ} (in-lbf):	2759
Equivalent Moment, $M_{EQ} = (M_{XZ}^2+M_{XZ}^2)^{0.5}$ (in-lbf):	5630
Bending Stress at point M, σ_B (psi):	40281
Safety Factor = S_Y/σ_B :	1.86

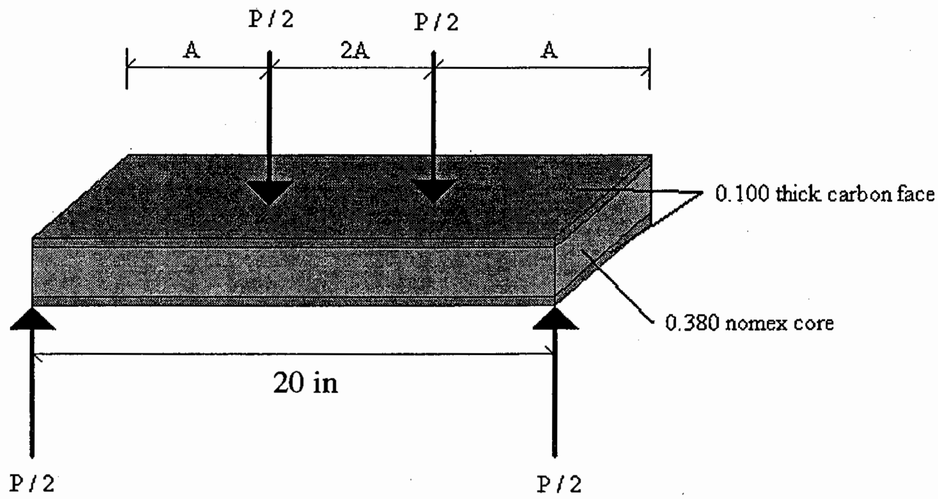
Appendix C – Steering



2.12

Appendix D – Chassis

Material properties of Hexcel Fibrelam 2000 paneling:



Four point bending test diagram and data

P = 260 lbs   A = 5 in.

Critical stress:

Maximum moment is constant and equals $\left(\frac{P}{2}\right) \bullet (A)$

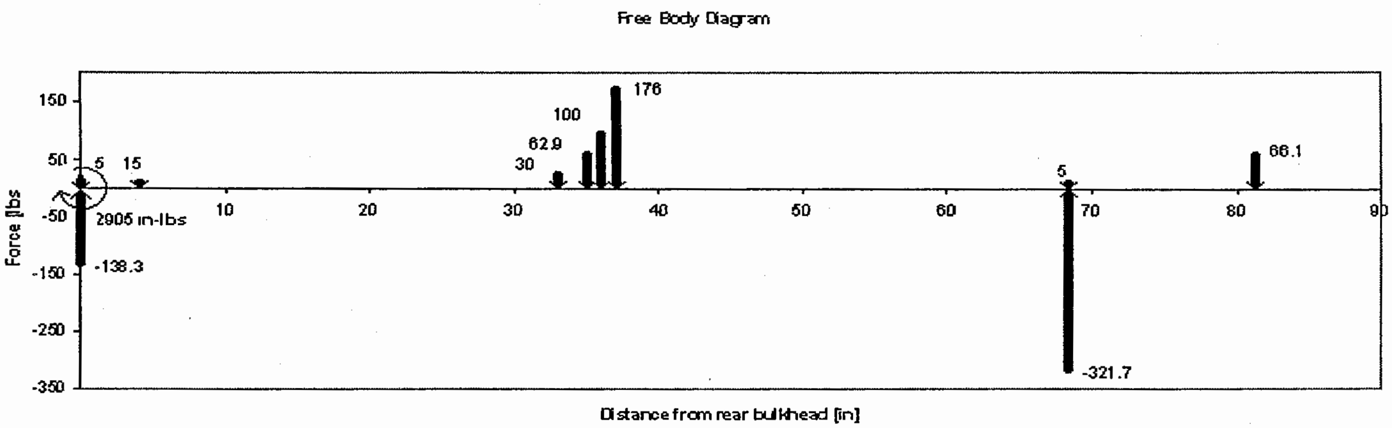
M_{max} = P / 2A = (260 / 2)5 = <u>650 in lbs</u>

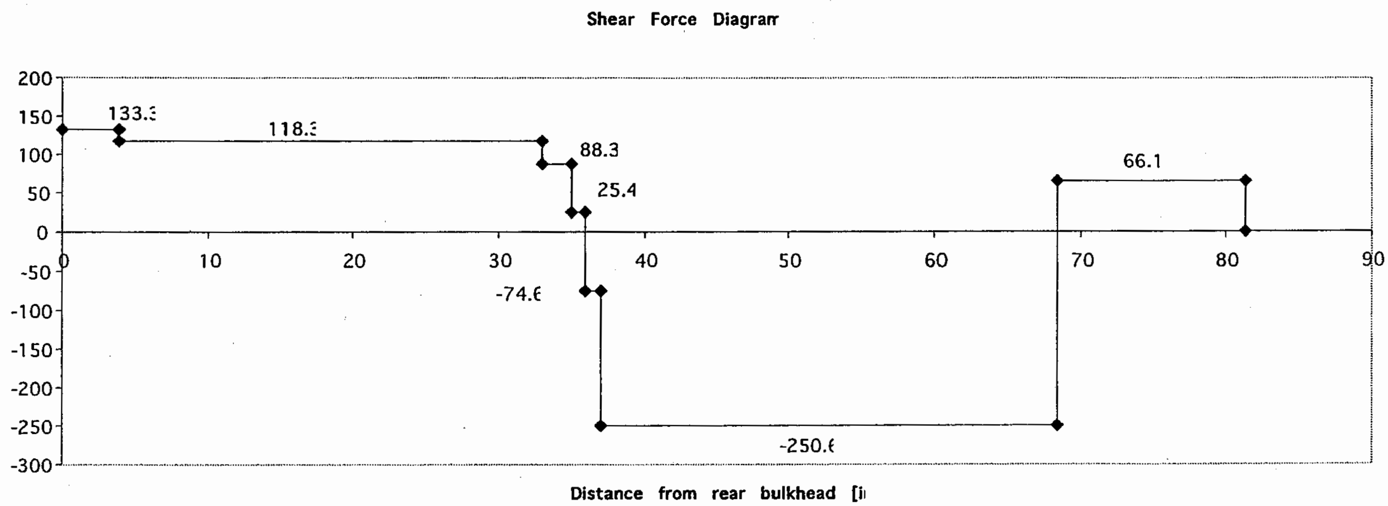
$I_x = I_{0.4} - I_{0.380} = .00228\ in^4$

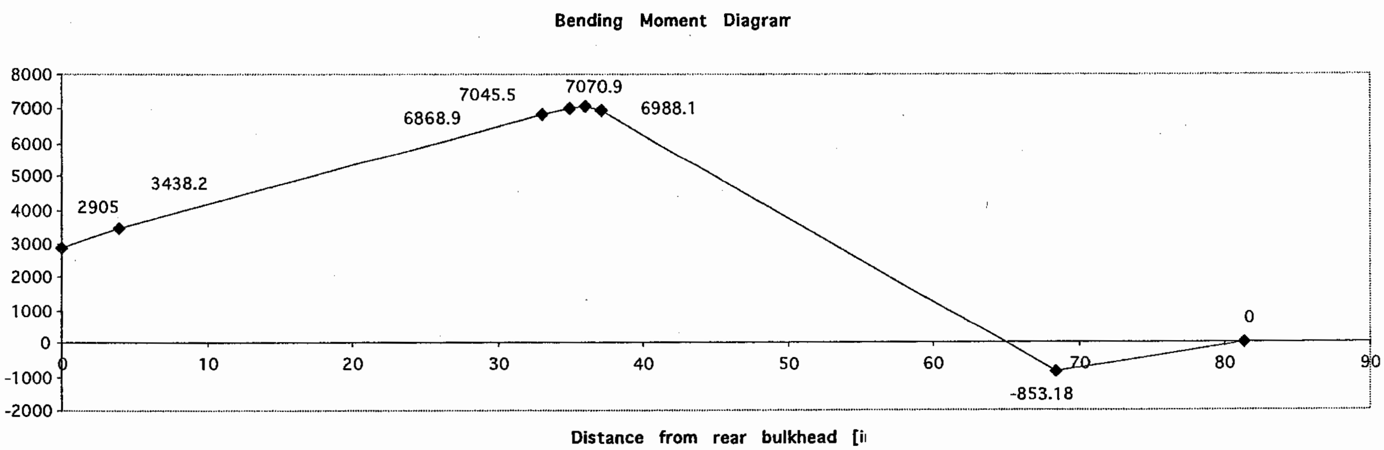
$\sigma_{crit} = -\ M_{max} * y / I_x = 650 * .2 / .00228$
$\sigma_{crit} = 57{,}020$ psi

57,000 psi will be used in the computations.

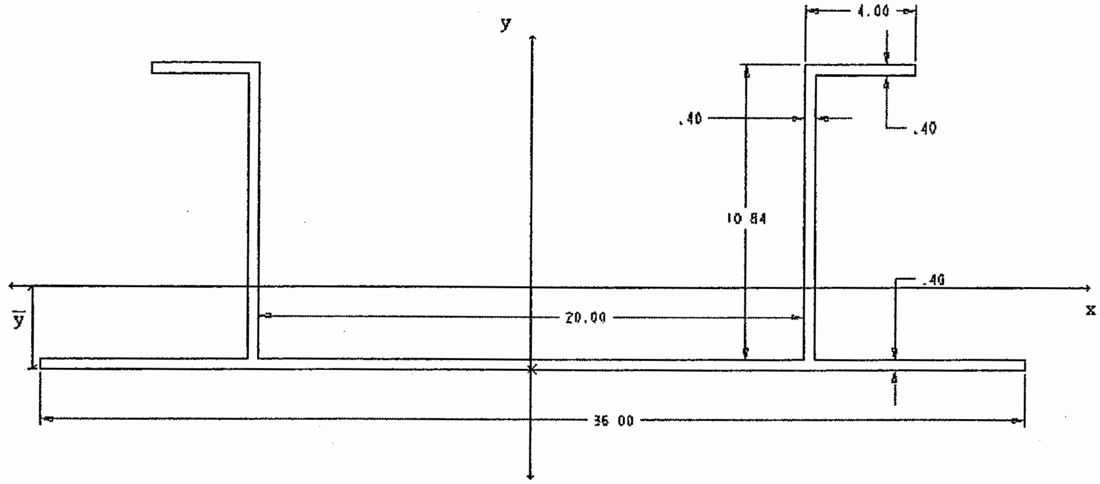
(213)


Free Body Diagram
176
100
62.9
30
5
15
66.1
5
2905 in-lbs
-138.3
-321.7
Force [lbs
Distance from rear bulkhead [in]


Shear Force Diagram
133.3
118.3
88.3
25.4
-74.6
66.1
-250.6
Distance from rear bulkhead [in


Bending Moment Diagram
7045.5
7070.9
6868.9
6988.1
3438.2
2905
0
-853.18
Distance from rear bulkhead [in

Chassis neutral axis and moments of inertia at maximum moment:



$$\bar{y} = \frac{\left[2*4*0.5(0.4+10.84+2)+2*10.84*0.4\left(0.4+\frac{10.84}{2}\right)+36*0.4*2\right]}{\left[2*4*0.4+2*10.84*0.4+34*0.4\right]}$$

$\bar{y} = 0.424\ in$

$I_x = \_\,(I_n + R^2*A)$
$I_x = [36*.01^3 / 12 + (.005 + 3.424)^2*36*.01] + [36*.01^3 / 12 + (.005 + 3.424 - .4)^2*36*.01]$
$+ 4[.01*10.84^3 / 12 + 2.396^2*.01*10.84] + 2[4*.01^3 / 12 + 7.011^2*4*.01]$
$+ 2[4*.01^3 / 12 + (7.011 + .4)^2*4*.01]$

$\underline{I_x = 22.6\ in^4}$

$\bar{x} = 36 / 2 = 18.0$ in
$I_y = \_\,(I_n + R^2*A)$
$I_y = 2[.01*18^3 / 12 + .01*18*9^2 + .01*10^3 / 12 + .01*10*5^2 + .01*7.6^3 / 12$
$+ .01*7.6(10.4 + 7.6 / 2)^2 + 10.44*.01^3 + .01*10.44 + 10.005^2 + 10.44*.01^3 / 12$
$+ 10.44*.01 * 10.405^2 + 2(.01*4^3 / 12 + .01*4*12^2)]$

$\underline{I_y = 165.6\ in^4}$

Maximum stress in chassis paneling:

$\sigma_{max} = -\,M_{max}*y / I_x = -7070.9 * 8.216 / 22.6$
$\sigma_{max} = 2570$ psi / g compression

Chassis safety factor:

C.S.F. = $\sigma_{crit} / \sigma_{max}$
C.S.F. = 57,000 psi / 2,570 psi / g = <u>22.18 g</u>

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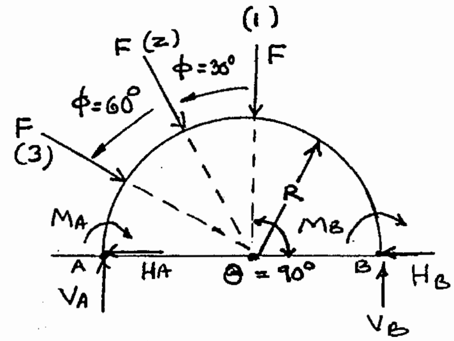
Appendix E - Roll Bar

Computation of Stresses in Roll Bar Hoop
Using Roark's Table 18, "Circular Arches", Case 5C.

$F = 3g$ Load $= 3(540) = 1620$ Lb
$R = 9.25$ inches



Roark's load case 5C finds the reactions at A from the following Matrix Equation:

$$\begin{bmatrix} \frac{\pi}{2} & 2 & 2/R \\ 2 & \frac{3\pi}{2} & \pi/R \\ 2 & \pi & \pi/R \end{bmatrix} \begin{bmatrix} H_A \\ V_A \\ M_A \end{bmatrix} = \begin{bmatrix} LF_H \\ LF_V \\ LF_M \end{bmatrix}$$

Matrix elements depend on angle $\theta = 90°$ and radius R. The "LF" terms depend upon load F and angle ϕ

Reactions at B are found from:

$V_B = F\cos\phi - V_A$, Lbs
$H_B = F\sin\phi - H_A$, Lbs
$M_B = FR\cos\theta - 2RV_A - M_A$, in-Lbs

Results summary for 3 load cases

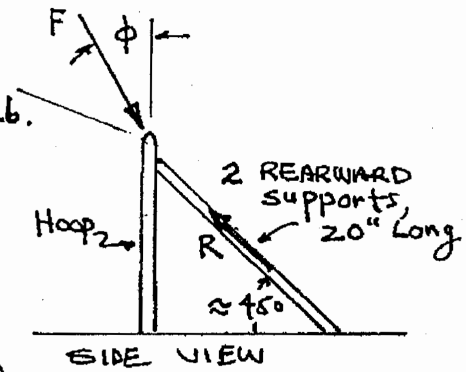
Case	ϕ	LF_H	LF_V	LF_M	H_A	V_A	M_A	H_B	V_B	M_B	σ_B, psi
1	0°	810	2892	1620	-744	810	1659	744	810	1656	31,067
2	30°	1555	3888	2430	27	928	-1590	783	475	-2605	48,764
3	60°	2236	4082	3023	1047	674	-3500	356	136	-1480	65,543

σ_B = the maximum bending stress, psi

$$\sigma_B = \frac{\text{Max}(M_A, M_B)}{I/c}$$

For 1.25" O.D. x 0.049" wall tube, $I/c = 18.726$

FORE-AFT LOADS ON Roll Hoop



CONSIDER LOAD $F = 3(540) = 1620$ Lb.
at angles $0 \le \phi \le 60°$

REActions are assumed to be EQUALLY divided between the two supports, each denoted as R.
TREAT EACH support AS A Fixed-Fixed column (due to welded ENDS) IN COMPRESSION AND find P_{crit}.
The EFFECTIVE LENGTH $L_e = 20/2 = 10$ inches.
Tube properties: r = RAD. of Gyration = $0.425''$, AREA $= .1849\ in^2$
$E = 30 \times 10^6 psi$ $\quad \sigma_y = 75 \times 10^3 psi$
Check if Johnson Theory Applies:
is $L_e/r = 23.5 < \left(\frac{2\pi^2 E}{\sigma_y}\right)^{\frac{1}{2}} = 88.8$? yes, so:

$$P_{crit} = \text{AREA}\left[\sigma_y - \frac{\sigma_y^2}{4\pi^2 E}\left(\frac{L_e}{r}\right)^2\right] = 13{,}382 \text{ Lb}$$

DEFINE 4 load cases. Compare Load R in a support to P_{crit}.
<u>DEFINE:</u> Saftey Factor $\equiv$ S.F. $= \frac{P_{crit}}{R}$, for each case:

$\phi°$	R, Lb.	S.F	comments
0°	0	—	NO FORE-AFT REAction with $\phi = 0$
45°	810	16.5	EACH support Load is (0.5)(F)
$0 < \phi < 45°$	<810	>16.5	support loads <(0.5)(F).
60°	988	13.5	support loads =(0.61)F)