HANDOUT 2. SUSPENSION TYPES & DESIGN ISSUES

HANDOUTS 2 & 3 go together, and they will:

1. DESCRIBE SOME COMPONENTS, TERMS AND OBSERVATIONS USING A PHYSICAL MODEL.
2. ADOPT A DESIGN VIEWPOINT AND STATE THE NEEDS AND SOME SPECS FOR ANY SUSPENSION
3. IDENTIFY AND CLASSIFY VARIOUS SUSPENSION TYPES BASED UPON THE PREVIOUS DESIGN VIEWPOINT.
4. IDENTIFY SOME PACKAGING ISSUES REGARDING THE COMPONENTS THAT MUST INTERACT.
5. TRACE FORCES FROM THE BOTTOM OF THE TIRES TO THE CHASSIS USING FREE BODY DIAGRAMS
6. IDENTIFY SOME DETAILS REGARDING CONSTRUCTION DETAIL AND FORCE TRANSMISSION.
7. DESCRIBE THE "ANTI-SQUAT" SUSPENSION IN TERMS OF FORCE TRANSMISSION.

WE will NOT describe the geometric/KINEMATIC PROPERTIES of suspensions at this time, But will do so in later presentations.

WE will focus upon 4 wheeled vehicles with REAR wheel DRIVE AND FRONT wheel steer, (with comments on 3 wheel (solarcars) and WONDER About 4 wheel STEER for improved Response for the FSAE CAR — BOTH LATER)

1. Physical MODEL – Double A-ARM

Components:
- Kingpin / Knuckle / spindle
- Upper & Lower A-arms – "short/Long"
- STEERING ARM & TIE ROD
- Hub / BEARING / AXLE
- Pivots – 3D capability: ROD ENDS AND SPHERICAL BEARINGS

TERMS:

CAMBER (ANGLE)
Kingpin axis
KINGPIN inclination (ANGLE)
CASTER (ANGLE)
SCRUB RADIUS

OBSERVE:
- Double-A-ARMS AND SPINDLE ⇒ 4-BAR LINKAGE (PLANAR? 3D?)
- Bump/REBOUND TRAVEL
- Bump STEER
- Path of wheel while STEERING
- HEIGHT CHANGE while STEERING

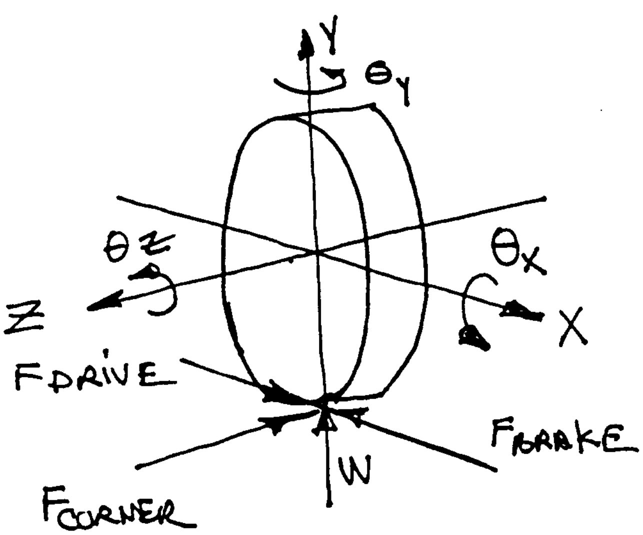
MEANING OF "MODEL"
- MISSING?
  - SPRING/shock?
  - "REAL" LOADS?
  - NO BRAKE / DRIVE elements
  - NOT DESIGNED FOR STRENGTH?

2. "NEEDS" – A DESIGN VIEWPONT – STATED AS FUNCTIONS

WE EXAMINED A SIMPLE VEHICLE MODEL THAT WASS ACCELERATING, BRAKING AND CORNERING AND WROTE EXPRESSIONS TO ESTIMATE THE CORRESPONDING FORCES AT THE TIRE CONTACT PATCH. FIGURE 1 SHOWS A TIRE-WHEEL WITH THOSE FORCES ALONG WITH A COORDINATE SYSTEM TO DESCRIBE THE 3 TRANSLATIONAL AND 3 ROTATIONAL MOVEMENTS OF THE TIRE-WHEEL. THE FUNCTION OF A SUSPENSION WILL BE DESCRIBED IN TERMS OF THESE FORCES AND MOTIONS. FIGURE 1 DOES NOT SHOW THE FORCES FROM THE SUSPENSION, SO IT IS NOT A FB DIAGRAM – THE LOCATION OF THESE REACTIONS WOULD DEPEND UPON THE SPECIFICS OF THE SUSPENSION TYPE, SO ARE NOT SHOWN. GENERALLY, THE FUNCTION OF A SUSPENSION SHOULD INCLUDE:

1. TRANSFER FORCES FROM THE BOTTOM OF THE TIRE (CONTACT PATCH) TO THE CHASSIS, USUALLY VIA SPRING/SHOCK UNITS AND SUSPENSION MEMBERS
2. PROVIDE MEANS TO APPLY OR TRANSFER DRIVE TORQUE TO PRODUCE FORCE F_{DRIVE}
3. PROVIDE MEANS TO APPLY OR TRANSFER BRAKE TORQUE TO PRODUCE F_{BRAKE}
4. CONTROL WHEEL MOVEMENT TO:
   - (a) ACCOMODATE BUMP/REBOUND TRAVEL (SPECIFIED)
   - (b) ACCOMODATE STEERING ANGLES AT FRONT (SPECIFIED)
   - (c) PRODUCE LITTLE OR NO CHANGE IN CAMBER, CASTER AND STEER ANGLES INDUCED BY BUMP AND REBOUND MOTION.

DIRECTION	MOTION FUNCTION	FORCES – PATH TO CHASSIS
Y	Bump & REBOUND TRAVEL: PROVIDE SPECIFIED VALUES	TRANSMITTED VIA SPRING-SHOCK UNITS & SUSP. MEMBERS
X	FORE-AFT MOVEMENT: CONTROLLED TO BE ZERO OR SMALL	TRANSMITTED VIA SUSP. MEMBERS.
Z	LATERAL MOTION: CONTROLLED TO BE ZERO OR SMALL	TRANSMITTED VIA SUSP MEMBERS.
θ_Y	"STEER" MOTION: PROVIDE SPECIFIED RANGE AT FRONT, AND CONTROLLED TO BE ZERO OR SMALL CHANGES DURING BUMP AND REBOUND ("BUMP STEER")	TRANSMITTED VIA STEERING ARM & TIE ROD AND SUSP. MEMBERS.
θ_X	CAMBER CHANGE: CONTROLLED TO BE SMALL DURING BUMP & REBOUND	TRANSMITTED VIA SUSP. MEMBERS.
θ_Z	ROLLING MOTION: MUST DO THIS! USUALLY WITH HUB, BEARING & AXLE OPTIONS	DRIVE & BRAKE TORQUES PROVIDE F_{DRIVE} & F_{BRAKE}; REACTIONS VIA SUSP. MEMBERS



ALSO: SOME FORCES MAY NOT ACT AT THE CENTER OF THE CONTACT PATCH AND CAUSE ADDL. TORQUES ABOUT THE X AND Y AXES

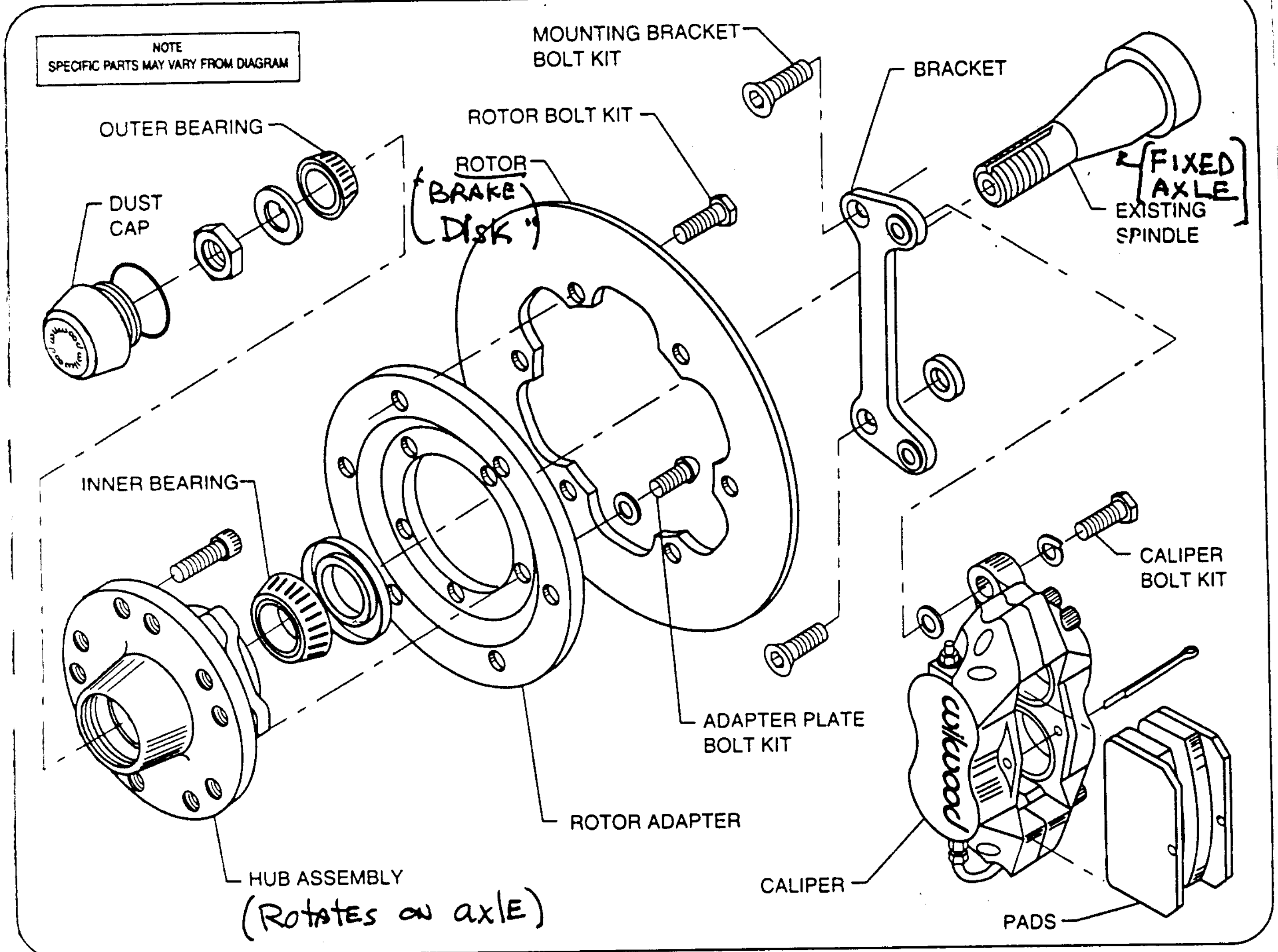
FIGURE 1 – FORCES & MOTIONS

3 Classification of suspension types –

THERE ARE MANY suspension systems that have been utilized on 4 wheeled vehicles. The purpose here is not to survey All types, but to show representative systems that illustrate the ways FORCE AND motion NEEDS ARE met, AND to emphasize the systems commonly USED in FSAE, Baha AND Solarcars. The manner in which the drive torque is provided also introduces levels of complexity, so the various means of providing this torque will be described as a classification. Similarly, the manner in which the brake torque is provided also introduces a distinction regarding which members take the load, so this will also be pointed out.

All systems provide some means of holding the wheel itself, usually consisting of a rotating hub assy. on a fixed axle OR a rotating axle within a housing. Figure 2 shows these two options in a generic manner. On some suspensions, the "housing" is a part of an axle assy., as shown in the figure, and on others, the "housing" is part of an "upright" or kingpin/knuckle/spindle unit

The follow table classifies suspensions as "AXLE" or "INDEPENDENT" according to drive components and location needs. Figures illustrate some options.



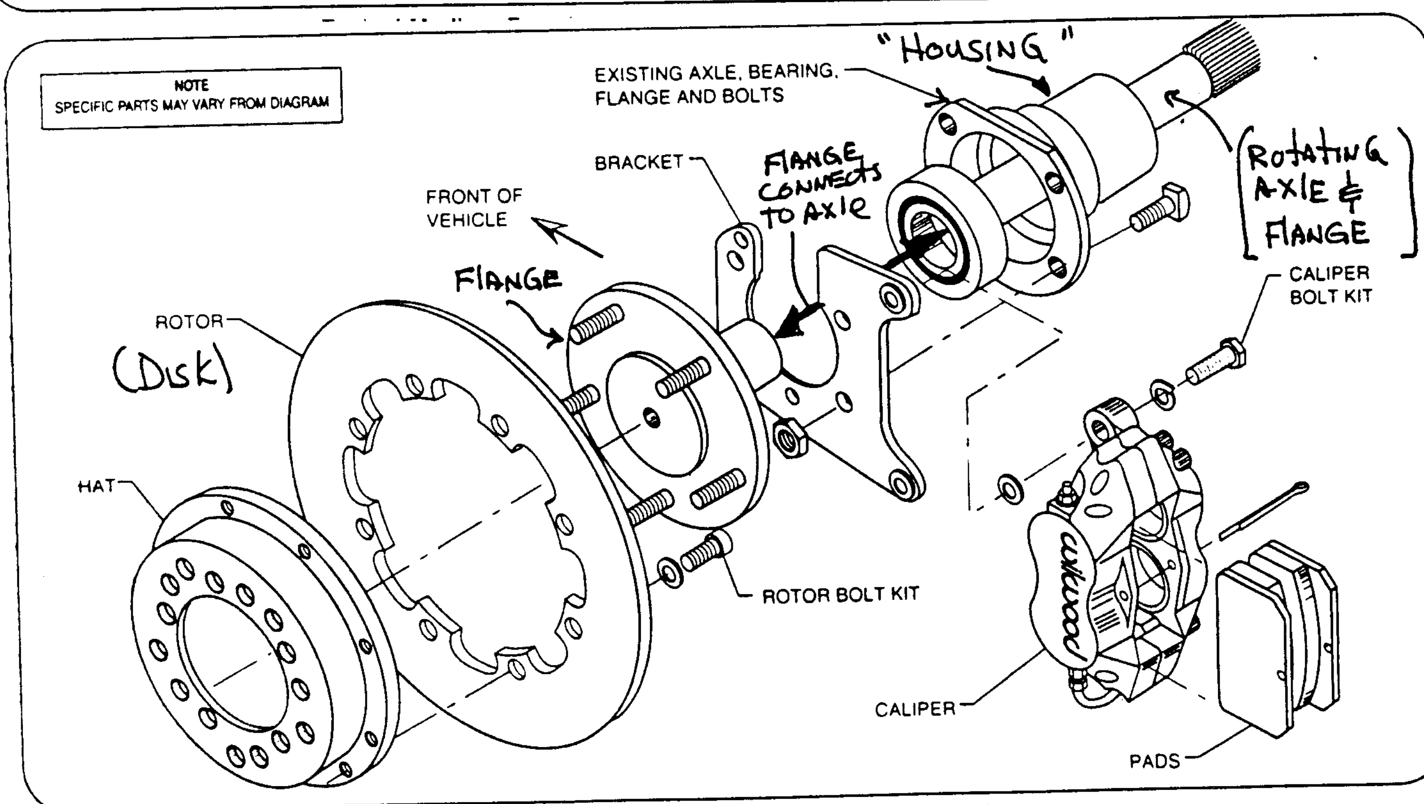


FIGURE 2 - HUB & AXLE options - AND Disc Brake components

DRIVE OPTIONS	RING/PINION/DIFF LOCATION	IN AXLE, UNSPRUNG	ON THE CHASSIS, SPRUNG				
	U-JOINTS PER WHEEL		TWO JOINTS PER WHEEL				ONE JOINT PER WHEEL
	DRIVE AXLE LENGTH		LENGTH CHANGES with SUSP. TRAVEL			LENGTH FIXED (usually ↓)	
DETAILED CLASSIFICATION		LIVE AXLE	DEDEION	SEMI-TRAILING ARM	MULTI-LINK	MULTI-LINK: AXLE IS A LINK	SWING AXLE
SOME EXAMPLES:		OLDER PASSENGER CARS, Most Trucks, NASCAR, OVAL TRACK	"DEAD AXLE" FIG (3) VERY FEW CARS, SOME OLD RACE CARS	MANY PASSENGER CARS FIG (4)	MOST RACE CARS – U of M FSAE car (DOUBLE A-ARM Geometry) FIGS (5, 6, 7)	Corvette, JAGUAR, OLD Lotus – (Double-A arm Geometry) FIG (8)	VW Beetle, OFFROAD CARS, U of M Baha cars
GENERAL CLASSIFICATION		"AXLE TYPE"		"INDEPENDENT"			
REQUIRED MOTION & FORCE-TORQUE CONTROL	FORE/AFT	YES	YES	YES	YES	YES	YES
	INDIVIDUAL WHEEL CAMBER CONTROL	NO	NO	YES	YES	YES	By DESIGN
	LATERAL LOCATION	YES	YES	YES	YES	YES	NOT IN BAHA STYLE
	DRIVE TORQUE REACTION	YES	NO	NO	NO	NO	NO
	BRAKE TORQUE REACTION	YES	← NO IF BRAKES MOUNTED "INBOARD" →				
			← YES IF BRAKES MOUNTED AT WHEELS →				

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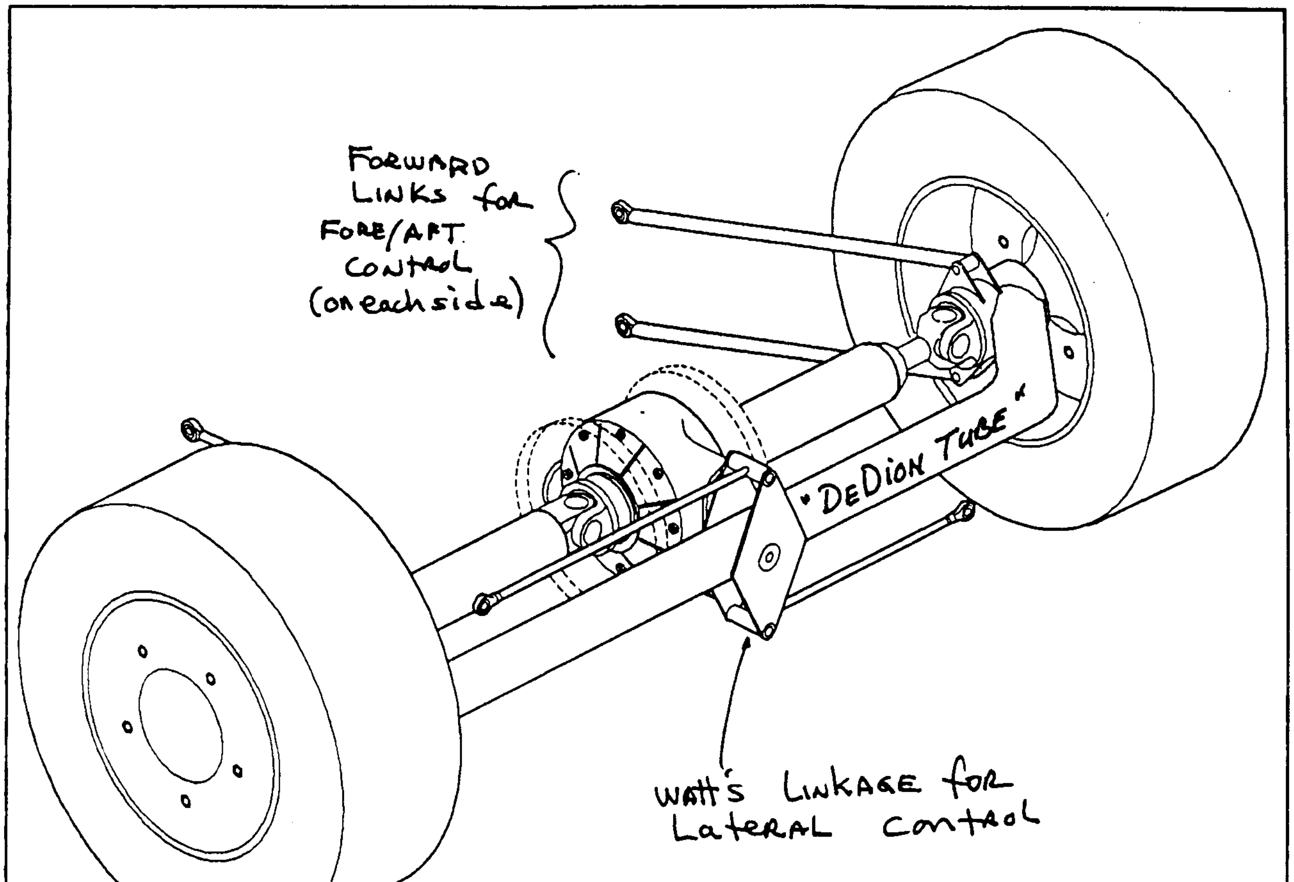


FIGURE 3. DeDion Suspension

De Dion suspension uses an axle beam to connect the rear wheels together and to control their camber. The power is routed from a chassis-mounted center section through half axles with U-joints. A splined section on each half axle allows the axle to change length as the wheels move.

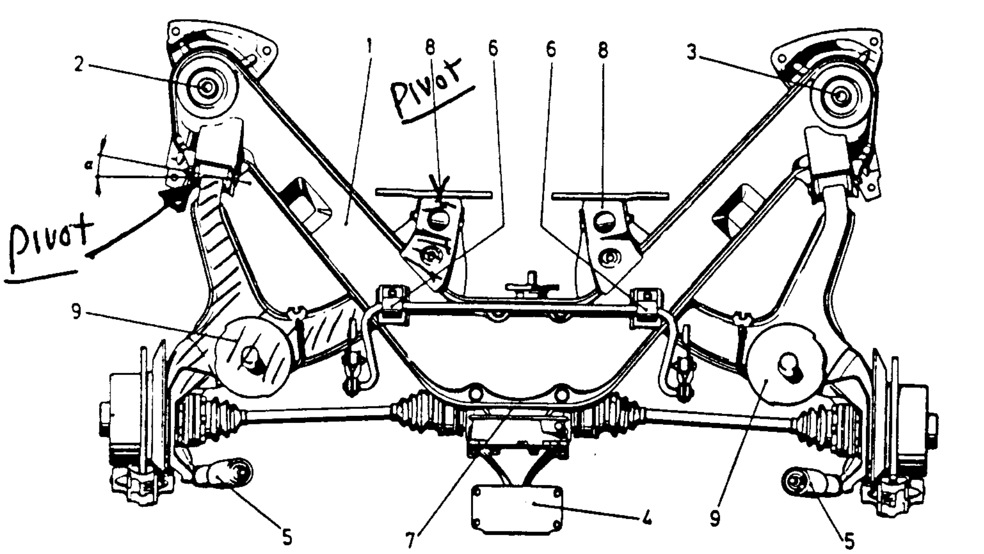
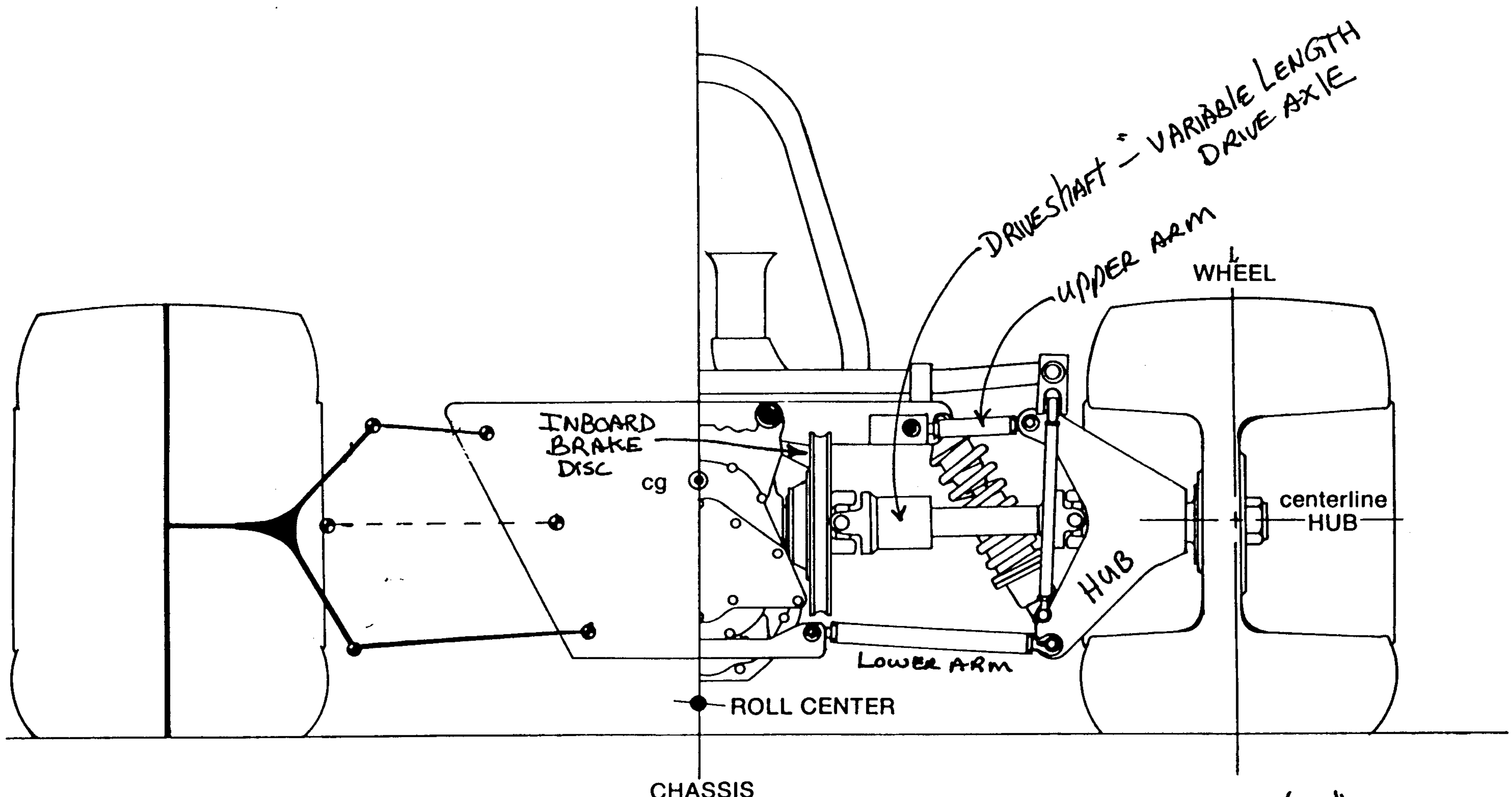


FIGURE 4 SEMI-TRAILING ARM SUSPENSION

Fig. 1.9 Top view of the rear axle on the Vauxhall Omega; the final drive 7 is screwed firmly to the subframe 1, which also carries the back of the anti-roll bars (in bearings 6). The entire assembly is held on the body by the specially-designed rubber bearings 2, 3 and 4. The two outriggers 8 are used to take the inner link bearings, and the barrel-shaped coil springs sit on the spring seats 9. In order to achieve a flat boot floor, parts 9 were moved in front of the drive shafts; at 1.5, the ratio i_{sp} (wheel to spring) is relatively high. The shock absorbers 5 are located behind the axle centre, at $i_D = 0.86$ the ratio is good. Furthermore, the top view angle also amounts to $\alpha = 10°$, the rear view angle to $\beta = -1°20'$ (Fig. 3.29), the camber with two persons in the vehicle to $\epsilon_{W,pl} = -1°40'$ (and with a permissible axle load to $\epsilon_{W,lo} = -2°45'$) and the level of the body roll centre to $h_{Ro,r} = 100$ mm.



"Double A-Arm Geometry"

FIGURE 5

Explanation of diagrams to be used to illustrate suspension geometry—right side shows rear suspension in end view—left side shows links, link pivots centerlines and instantaneous centers only:

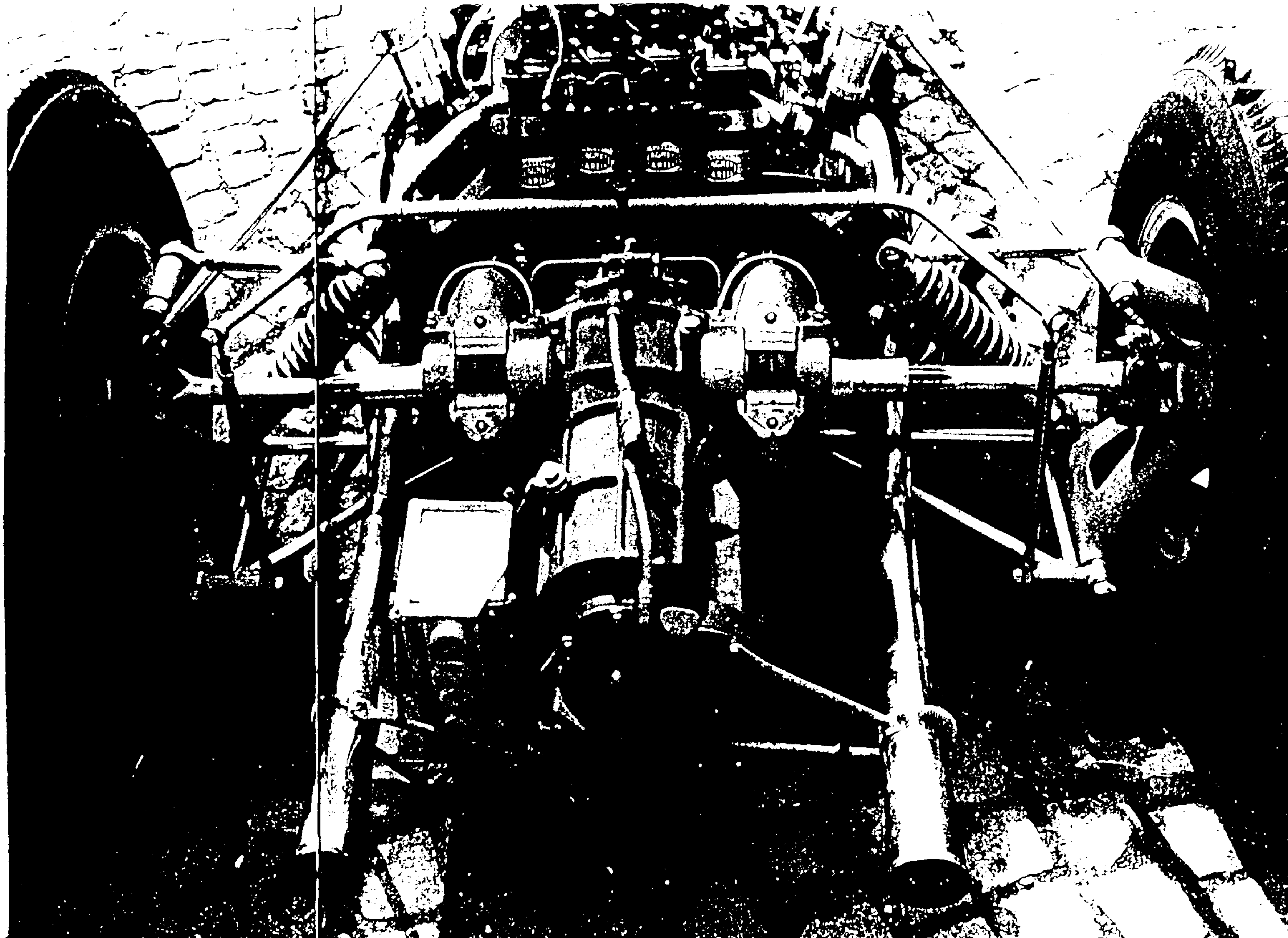
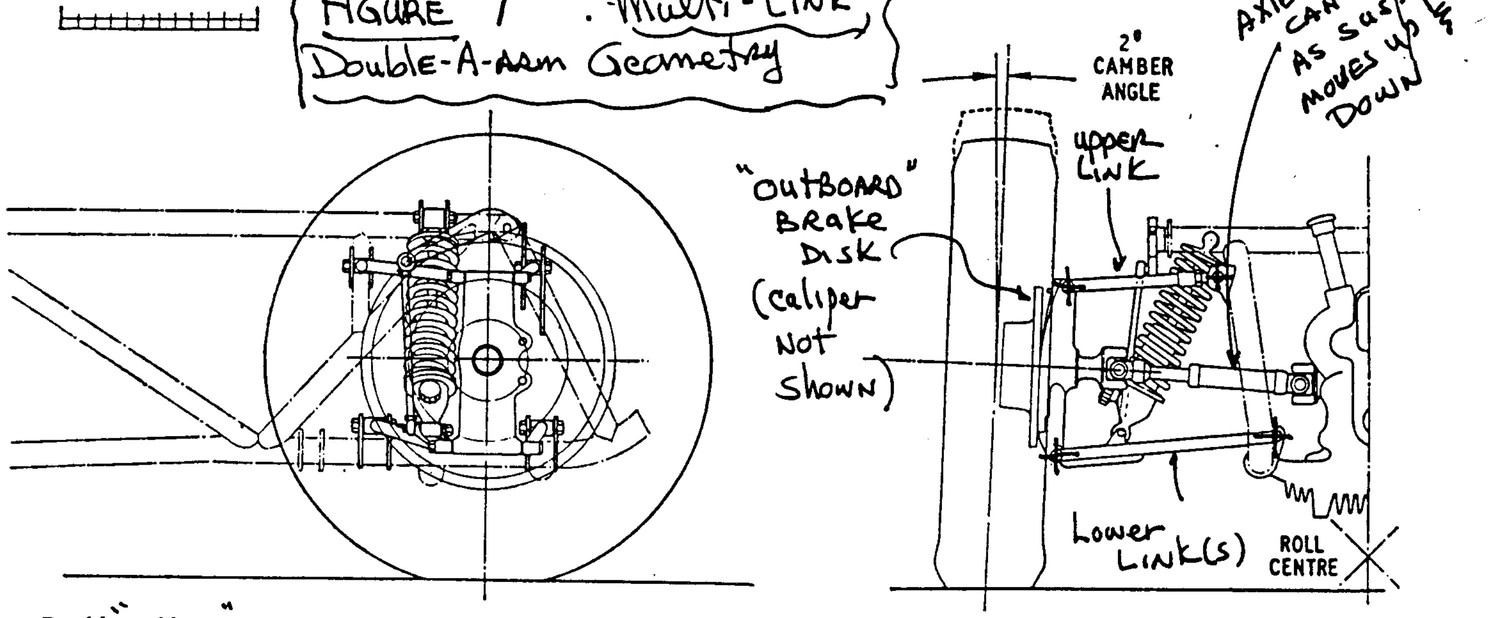


FIGURE 6

- "INBOARD" BRAKES — MOUNTED ON RING/PINION/DIFF. HOUSING.
- AXLE LENGTH VARIES — NOTE SPLINED SHAFTS
- FORWARD LINKS (RADIUS ARMS), AND SINGLE UPPER LINK PLAYS ROLE OF FIXED LENGTH AXLE IN FIGURE 8.
(LOWER ONE NOT VISIBLE)

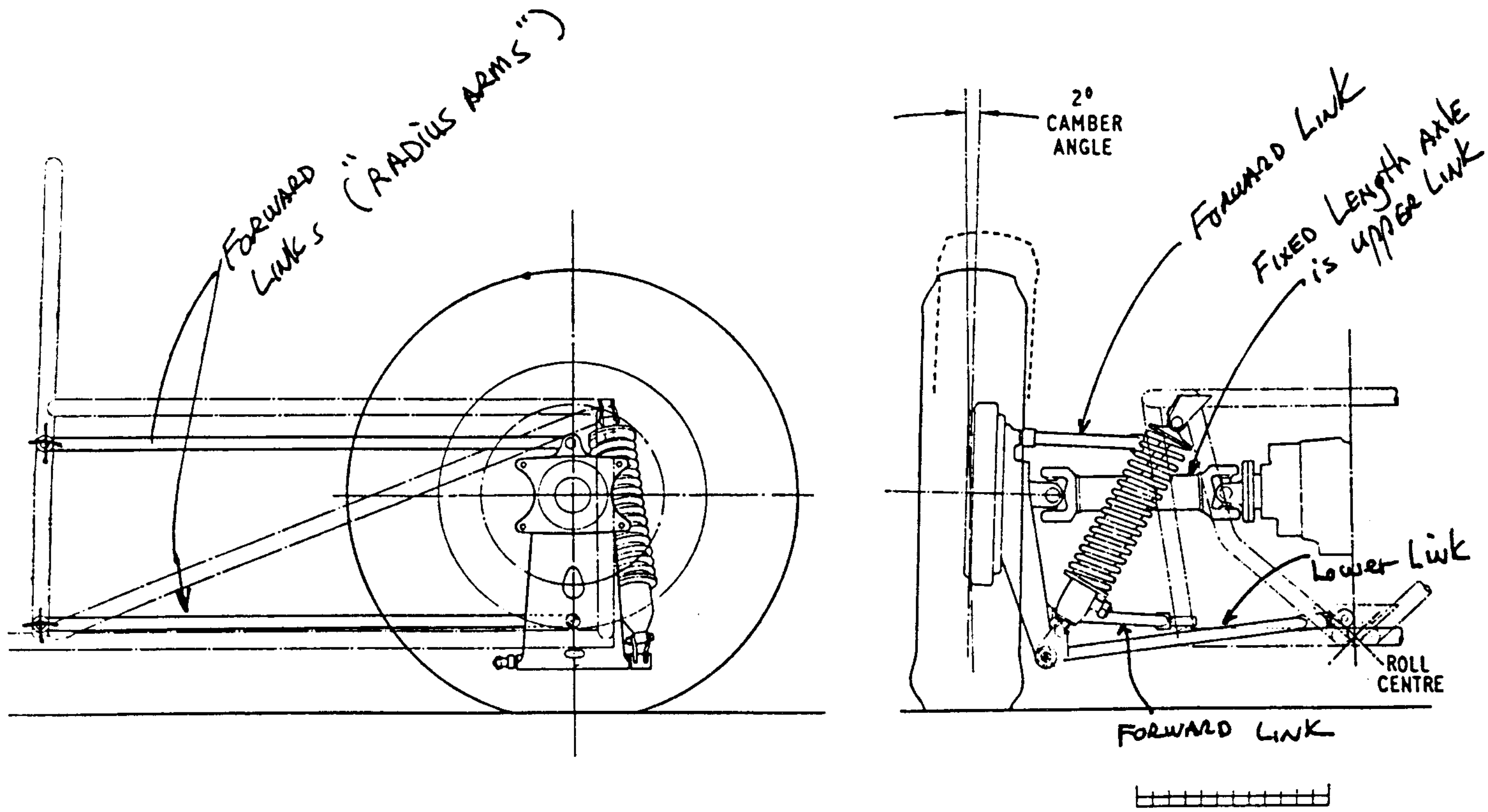
This early Ferrari design is another typical example of a four-link rear suspension. It used long forward-facing radius arms combined with unequal length top and bottom transverse links to form a classical wishbone layout

FIGURE 7 . Multi-Link, Double-A-arm Geometry



Double "wishbone" rear suspension exemplified by the 1960 Formula One Cooper. The wishbone links provide both lateral and fore-and-aft location; no suspension loads are taken out by the splined drive shafts. An anti-roll bar, housed in the top transverse chassis member, is connected to the lower wishbone

"WISHBONE" => A-ARM



42 *Double transverse link rear suspension, as used on the 1960 Formula Junior Lotus. Lateral location is provided by the two-piece lower link and the fixed-length drive shaft, and fore-and-aft location is by parallel radius arms. All loads are taken out over an extremely wide base and their magnitude is therefore much reduced. (This drawing is of the original prototype; straight members were used in the rear chassis frame on all subsequent examples). A similar layout is used on the 1960 Formula One Lotus and the Lotus Nineteen*

FIGURE 8 Multi-link, Double A-arm Geometry
w/ FIXED AXLE LENGTH

4. PACKAGING ISSUES

THE NEXT FEW PAGES ARE FROM:
"RACE CAR VEHICLE DYNAMICS" by Milliken & Milliken, SAE, 1995 — pages 624 – 627 —

THE FOCUS IS FRONT SUSPENSIONS — SOME APPLIES TO REAR ALSO —

17.5 Front Suspensions

Introduction

Many types of front suspensions have been used over the years. They include various beam type axles with steering via kingpins at each end of the axle, the parallel trailing arm type such as the VW, the Morgan sliding pillar type, and the Chevrolet Dubonnet. In recent history, passenger car designs have come down to basically two types: the MacPherson Strut and the SLA (Short-Long-Arm). ← Double A-ARM

This chapter will deal only with the last two mentioned as these make up the majority of front suspensions that will be encountered. The other types suffer from either high bending loads, poor geometry, high friction, or a combination of these problems. The best way to discuss each type is to go through the design process step by step. For each step a decision has to be made that is often a compromise. By discussing these decisions, hopefully a feeling for the limitations of the design will develop.

Front Suspension Design Issues—General

The first task in designing a front suspension of any type is to establish the packaging parameters that are fixed, or absolutely cannot be changed for whatever reason (see Figure 17.17). These should be listed so that they are not overlooked. The next task is to package the wheel, tire, brakes, and bearings. This is done in car position, so the track width has to be known. If it is not yet established, it should be made as wide as practical. This sounds evasive, but there are trade-offs in everything, even things as simple as choosing the track width. For example, what do the rules allow? What is the predominant race

{ wheel, TIRE, BRAKES, BEARINGS, TRACK

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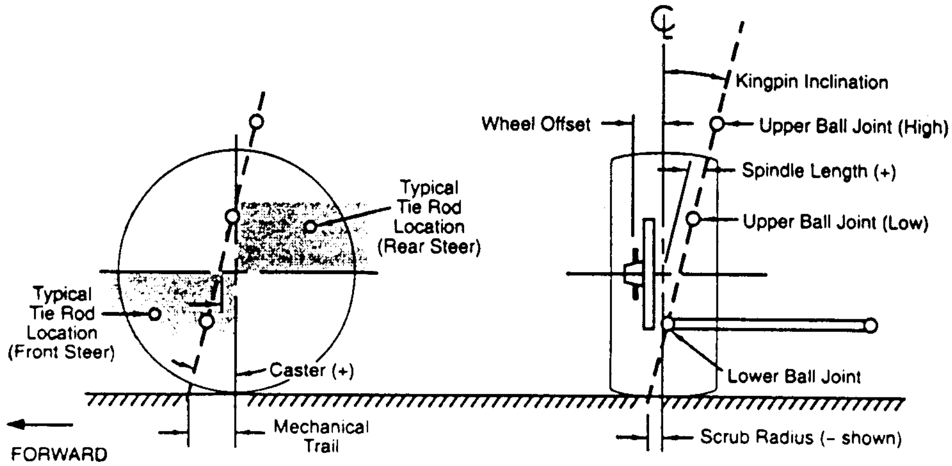

Figure 17.17 Front suspension packaging.

track type on which the car will run? Is top speed, thus low frontal area important? Are slow-speed tight street circuits of concern? All these issues can affect the decision on the basic track width!

TIRES & WHEELS

Tire size and rim diameter and width must be settled. The specific wheel manufacturer needs to be known and a cross section of the wheel is desirable for optimizing the use of that wheel. Tire sizes are usually limited by the sanctioning body rules. In general, use all the tire they will let you get away with. Another point is to always design for the latest sizes being developed by the suppliers; this guarantees that the latest compounds and constructions will fit your car. Remember, the tire is the single most important chassis component on the car.

BRAKES,

The wheel offset is worked out in conjunction with fitting the brake caliper to clear the inside surface of the wheel. Once the caliper is located, this automatically locates the brake rotor. With the rotor location comes the absolute farthest outboard location for the lower ball joint. Wheel bearings need to be looked at soon, as ideally they should be located such that the tire center is between the two rows of balls or rollers (to minimize loads on the bearings).

UPRIGHT/KINGPIN/SPINDLE

Now that the lower ball joint cross car boundary (lateral position) has been set, the height of the lower ball joint comes next. In production cars it must be above a 5-in. wash rack clearance requirement, but on race cars it should be made as low as possible for structural reasons. Usually there is no rule but some practical considerations such as deflated tire ground clearance might be in order. If it is totally inside the wheel all it has to do is clear the wheel and the brake rotor under all travel and load conditions.

The decision about the kingpin angle in the front view is the next order of business. The issues here become scrub radius, spindle length, and kingpin angle. They are interrelated

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AKA "Kingpin offset"

and a compromise is needed. If you want a certain scrub radius you now have two points established, i.e., the lower ball joint and the ground contact point of the kingpin (set by the scrub radius)—the kingpin angle becomes fixed automatically. If you want a certain kingpin angle then the scrub radius will not necessarily be what you want. Basically, on rear-wheel-drive cars push the lower ball joint out as far as possible and run a fairly low kingpin angle, less than 8°, and accept the scrub radius that results. If you are dealing with a front-wheel-drive car you **must** minimize the spindle length and have a negative scrub radius. This may result in a kingpin angle as high as 16°, but you will have to accept it or find another clever way around it.

Kingpin angle affects the performance of the car when the wheels are steered. One concept that should be understood is that the more the kingpin angle the more the car is lifted when it is steered. This is one source of steering returnability, the weight of the car returns the steering to center. The amount the car is lifted is also a function of the spindle length where a longer spindle means more lift.

Shown on model

angle ← [word missing in text]

The camber of the wheels when steered is a function of the kingpin angle and the caster angle. With no kingpin angle (and no caster angle) there is no camber change with steer lock. As kingpin is added (but still no caster) the wheel will "lose" camber with steer lock, or in other words it will change in a direction giving positive camber on the outside wheel.[60] As caster is added this modifies the effect of kingpin. With positive caster and no kingpin angle, the wheel gains negative camber on the outside wheel and positive camber on the inside wheel. Thus caster can add favorable camber angle to the effects of kingpin angle. In other words, the reason that low kingpin angles are desirable is that kingpin angle subtracts from the negative camber gain due to caster on the outside wheel.

GOOD

The decision on a rack location depends on several packaging factors such as engine location and orientation, front-wheel drive vs. rear-wheel drive, whether it is to be high- or low-mounted, etc. In addition there are performance reasons for choosing the rack location. First we must assume that every structure is a spring and should be treated as such. As an example the rack mounting stiffness versus the upper or lower control arm mounting stiffness to the chassis will not necessarily be the same. Therefore, when a cornering force is applied, any difference in the lateral displacement of the ball joints in relation to the tie rod outer pivot will cause a steer angle. To assure stability it is better to have lateral force deflection toe-out (lateral force understeer) rather than toe-in. We can assure that this happens by the proper location of the rack. If a high-mounted rack is required it must be behind wheel center and if it is low-mounted then it must be ahead of wheel center as shown by the shaded areas in Figure 17.17.

Steering Rack Location

must? Depends on loading

Structural requirements for the suspension design must always be considered when packaging each element of the total system. Control arms that have one leg straight across from the ball joint are superior in system stiffness to arms that are splayed. Establishing

Like on our solarcars

[60] That is, the wheel further from the turn center.

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linkage ratios for the spring, shock, and stabilizer bar as close to 1:1 as possible will provide more direct load paths thus improving system stiffness while providing a lighter overall design.

Front Suspension Design Issues—SLA

The Short-Long Arm (SLA) suspension is the choice of designers without question for its ability to meet desired performance objectives with minimum compromise.

The design starts with the basic package as described above. The details of the track width, the wheel size, the tire, the brakes, etc., bring about the location available for the lower ball joint. The upper ball joint is located either via kingpin angle requirements or by scrub radius requirements. There is a little more freedom with the SLA that is not available to the strut design and that is the choice of a short knuckle or a tall knuckle.

• The *short knuckle* means the upper ball joint is located basically within the diameter of the wheel. With high offset and large-diameter wheels the kingpin angle can be kept small (while achieving small spindle lengths and scrub radius) by tucking the upper ball joint into the wheel.

Like our FSAE cars (Front)

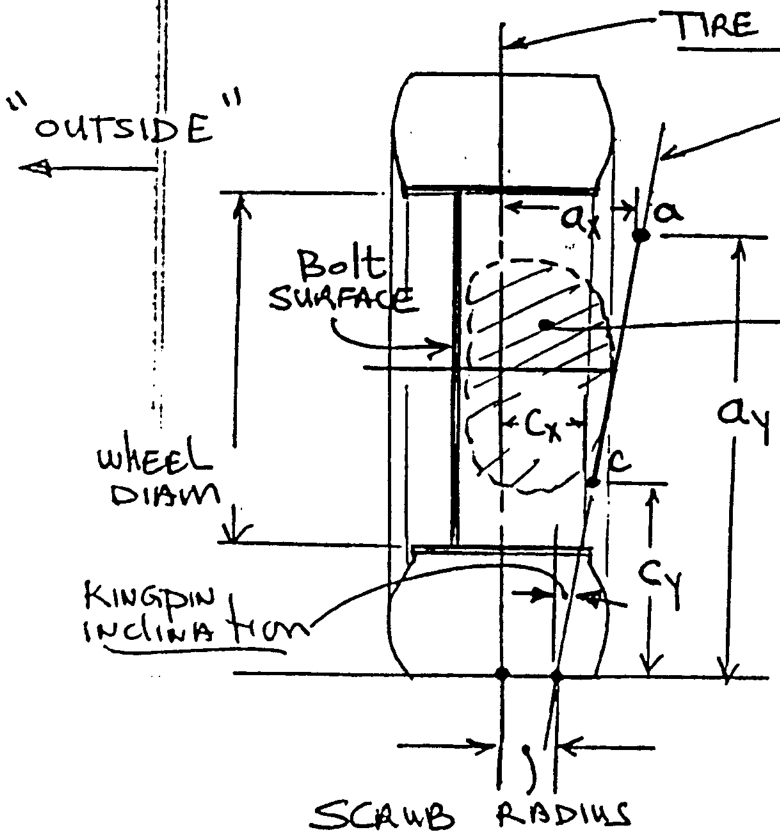
To reduce the loads on the control arms and other suspension components, it is desirable to have a long kingpin length, that is, separate the upper and lower ball joints as much as possible. Depending on details of the installation, the short knuckle may yield less than optimum kingpin length. The other alternative is the *tall knuckle* where the upper ball joint is above the tire. In the tall knuckle design the ball joints naturally have a very large span and thus reduce reaction loads. This option also allows reasonable kingpin angles while achieving desired spindle length and scrub radius. Another advantage for the tall knuckle is that build errors will result in smaller geometry errors than with short knuckle designs. Some negatives to the tall knuckle, of course, are the added structural requirements of the knuckle, and the limitation of never changing tire size or width without widening the track and increasing the spindle length and scrub radius after the design is completed.

Like the 99 BAJA CAR

With the upper and lower ball joint locations established, the tie rod outer point should also be set per the requirements established in Chapter 19 on steering geometry.

(K/P)

FRONT UPRIGHT ("KINGPIN") CONSIDERATIONS



ENOUGH SPACE FOR THE FRONT HUB, BEARING, BRAKE ASSY, STEERING ARM (if forward) and KINGPIN MATERIAL!

NOTE Bolt SURFACE of wheel may NOT be ON the TIRE CENTER LINE ℄

Points a & c are outer a-arm pivots

Scrub Radius ~ try to make small, but usually want some value (not zero) for steering "feel"

KINGPIN inclination ~ Try to get between 5° and 10° if possible

If point c is "INSIDE" the wheel, be sure there's enough room around it (below it) for the ball joint hardware.

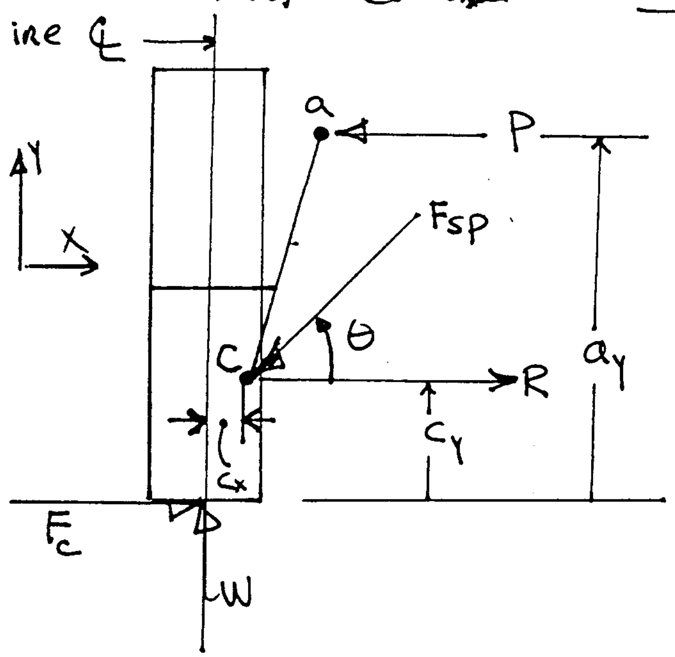
HANDOUT 3. LOADS WITHIN SUSPENSIONS

FORCE TRACING, supporting the VEHICLE, CONSTRUCTION DETAILS, BEARING LOADS AN A look at ANTI-SQUAT GEOMETRY.

FORCE TRACING – FROM the TIRE contact patch to the chassis using FB Diagrams. Some ideas and RESULTS are shown here, and others will be IN the homework. —

Double A-ARM FRONT SUSP IN static (bump) and CORNERING AND braking –

FIRST CONSIDER Bump (static) and CORNERING



ASSUME – • A-ARMS ARE horizontal (makes treatment easier)

- F_{sp} = SPRING force OR pushrod force, and acts at pt. c at angle θ.
- F_c = CORNERING FORCE "inputs"
- W = Vertical FORCE "inputs"

FORCES F_{sp}, P and R can be expressed in terms of F_c, W as:

$$F_{sp} = W / \sin\theta \qquad (1)$$

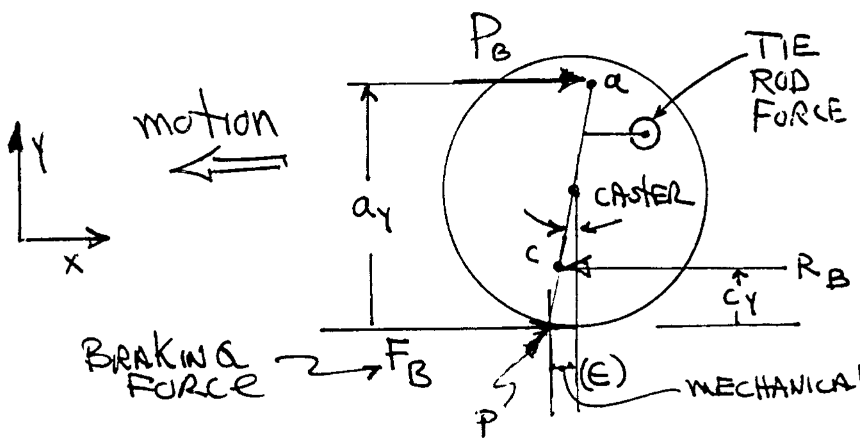
$$P = W\left[\frac{C_x}{l}\right] - F_c\left[\frac{C_Y}{l}\right] \qquad (2)$$

$$R = W\left[\frac{C_x}{l} + \cot\theta\right] - F_c\left[\frac{a_Y}{l}\right] \qquad (3)$$

WHERE $l = a_Y - C_Y$ (4)

OBSERVE:
- Role of ℓ, a_Y, c_Y
- Role of (−) sign
- Steering torque due to F_c is ignored (see eqn (7))

Now, **BRAKING:** examine the side view:

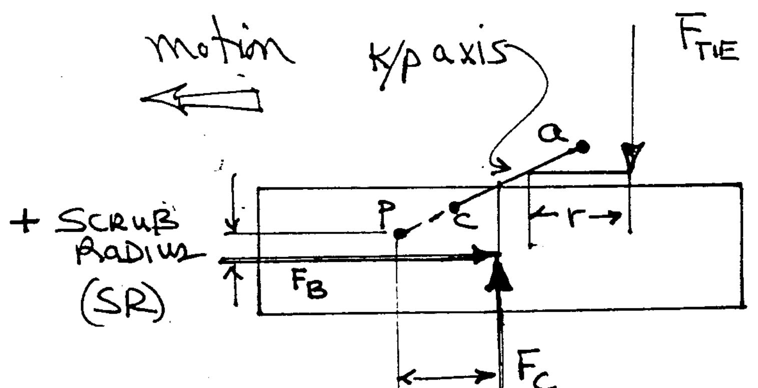


$$P_B = \left(\frac{c_Y}{\ell}\right) F_B \quad (5)$$

$$R_B = \left(\frac{a_Y}{\ell}\right) F_B \quad (6)$$

OBSERVE: Role of ℓ, a_Y, c_Y

AND, **STEERING:** Top view

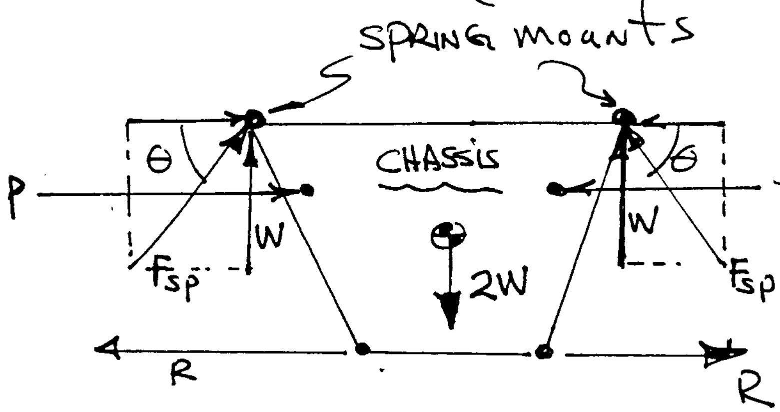


If we regard the caster angle & kingpin inclination angles as small, and sum torques about the kingpin axis, then:

$$F_{TIE} = \frac{F_B(SR) + F_c(\epsilon)}{r} \quad (7)$$

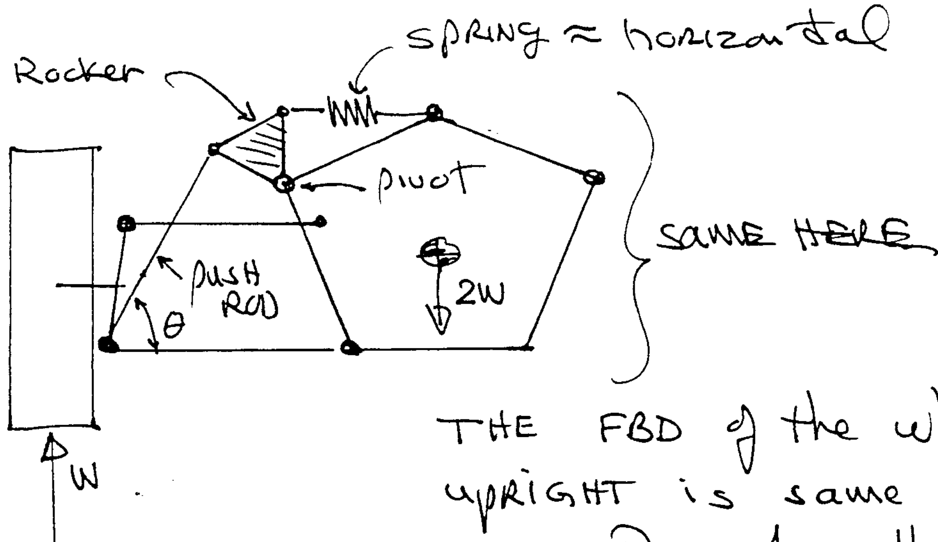
Note that F_{TIE} will also change the P & R values in eqns (2) & (3)

What Holds the car up? Examine the chassis — (use notation from P.1, wt. "W" PER WHEEL)

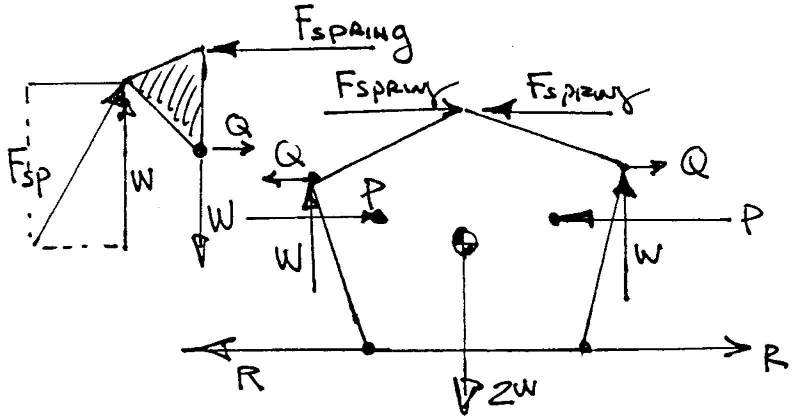


With A-ARMS "about horizontal", the only vertical forces act via the springs.

How about pushrod susp., ala 1999 FSAE?

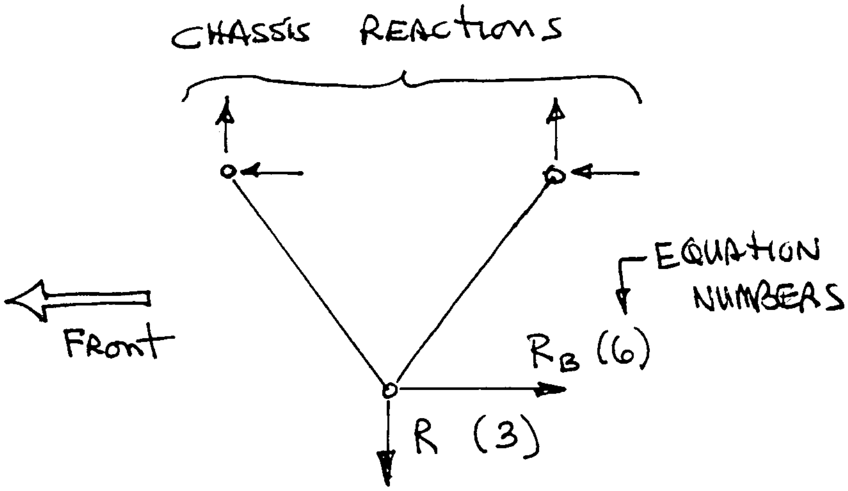


THE FBD of the wheel & upright is same as on PAGE 1, with F_{sp} being the force upon the pushrod.

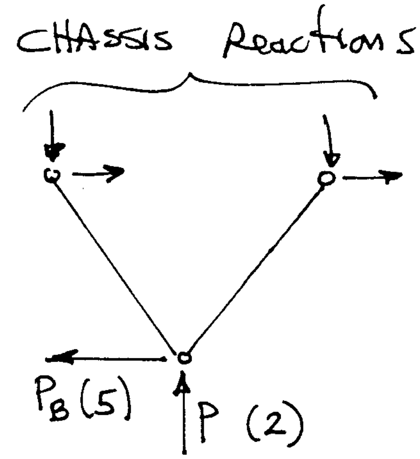


NOW – The ROCKER pivots "hold up" the car — supporting W on each side. You will see some marginal supports of these pivots on FSAE CARS.

The FORCES At the upper AND lower Ball joints (ROD ENDS) ARE TRANSmitted to the A-ARMS -



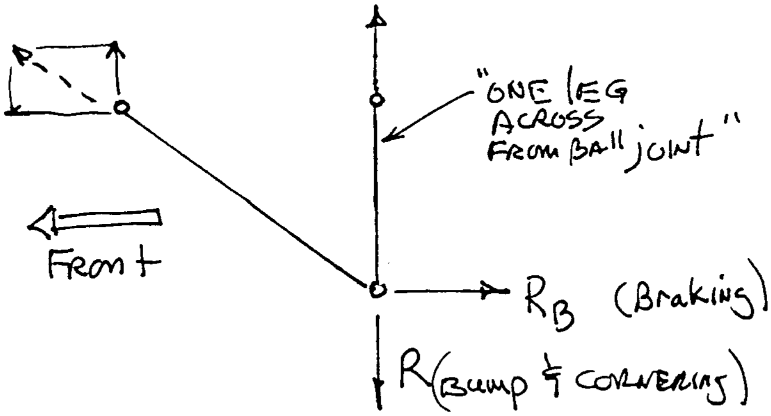
LOWER A-ARM, Top View



UPPER A-ARM, Top View

ON page 14 of HANDOUT 3, Mr. Milliken comments about the stiffness of "control arms that have one leg across from the ball joint are superior in stiffness to arms that are splayed".

THE ABOVE DIAGRAMS show "splayed arm's" in which Both legs React to the FORE/AFT and lateral forces at the outer pivot.



THE REAR LEG TAKES All the load from R, AND some load from R_B.

THE FORWARD LINK is only LOADED IN BRAKING.

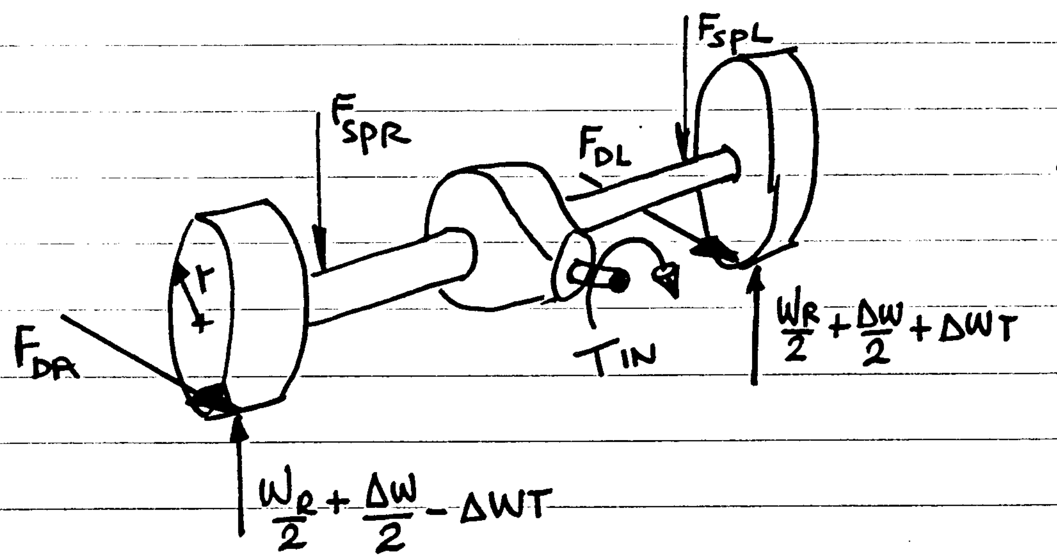
Milliken states this is "superior in STIFFNESS"

FORCE TRACING AT THE REAR ~ F_{DRIVE} & TORQUE REACTIONS. The following will look at the "LIVE AXLE" CASE AND the INDEP. Rear Susp "case" with a frame mounted differential assy. PROCEDURES similiar to the front susp on pages 1-4 can be used to examine the vertical and cornering forces on an INDEP REAR SUSPENSION.

Shown Below is A partial F.B. Diagram of a Live axle suspension with input torque T_{IN} and drive forces F_{DR} & F_{DL}

F_{DR} ← RIGHT, F_{DL} ← LEFT

where $F_{DRIVE} = F_{DR} + F_{DL}$ (8)



NOTE

- $\frac{W_R}{2}$ = Static wt. on each Rear wheel
- $\frac{\Delta W}{2}$ = wt. Transfer due to ACCELERATION to Each wheel.
- ΔWT = WT. Transfer from RIGHT to LEFT Due to T_{IN}.
- F_{SPR} & F_{SPL} are Right & Left SPRING FORCES.

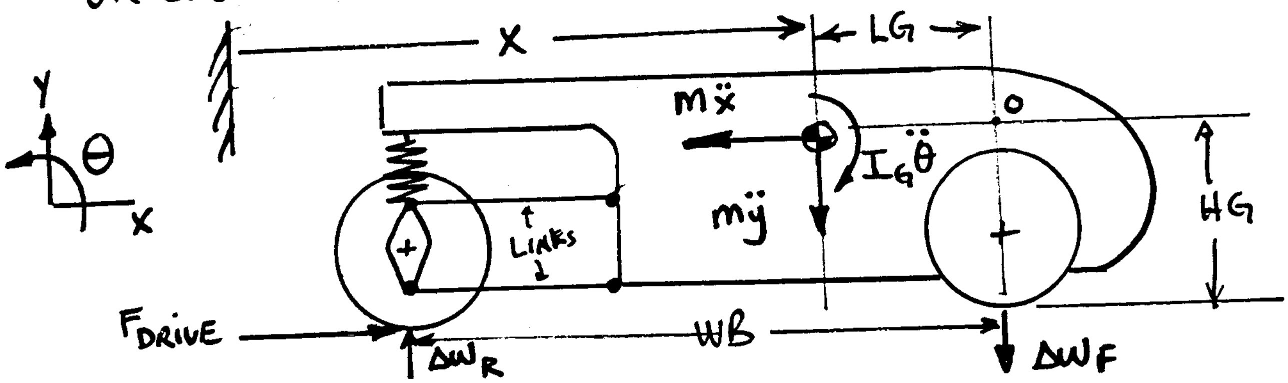
This system is not in Equilibrum:

1) NEED fore/aft control to counteract F_{DR} & F_{DL}.
2) NEED Rotation control about the axle line to counteract torque $(F_{DR} + F_{DL})r$

(This is new)

It is possible for the spring forces to balance the vertical ground reaction and other effects of T_{IN}.
We'll go back to side view and show a simple, effective means of providing the needed controls, and introduce other motions

The Figure below shows a planar vehicle with a rear trailing link suspension, with links parallel to ground (for EZ analysis) and mounted on Brackets on the axle and pivots on each end. (W = vehicle wt.



With rear suspension, this vehicle can move vertically and rotate during acceleration, so intertia effects $m\ddot{y}$ and $I_G\ddot{\theta}$ are shown in addition to $m\ddot{x}$. Static vertical weight and reactions are cancelled and not shown.

Without suspension, there was no need to identify ΔW_F and ΔW_R individually, since the weight taken off the front was the same as that transferred to the rear — no so here.

WE WANT to SEE how F_{DRIVE} Gets to the chassis, but we'll make a few observations along the way.

$$\sum F_x: \quad F_{Drive} \equiv m\ddot{x} = f_a W \quad \text{(as before)} \qquad (9)$$

$$\sum F_y: \quad \Delta W_R - \Delta W_F = m\ddot{y} \quad \left\{ \begin{array}{l} \text{if } \ddot{y}=0, \text{ then} \\ \Delta W_R = \Delta W_F \end{array} \right. \qquad (10)$$

$$\sum T_o: \; -\Delta W_R (WB) + m\ddot{y}(LG) + F_{DRIVE}(HG) - I_G \ddot{\theta} = 0 \qquad (11)$$

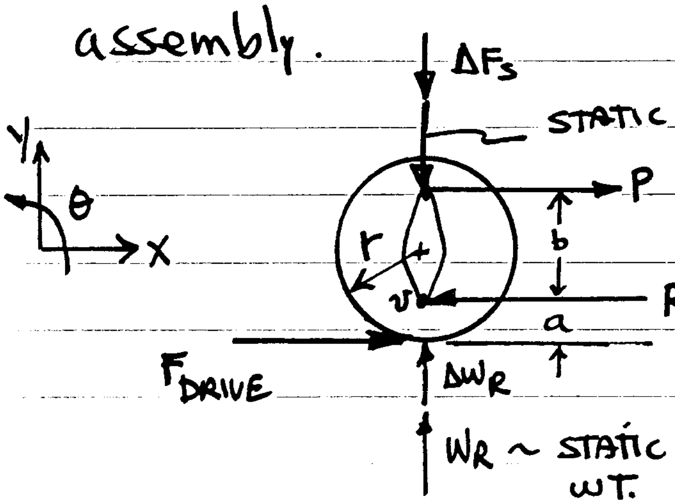
EXAMINE ΔW_R with this SET-UP:

$$\Delta W_R = \underbrace{f_a W \left(\frac{HG}{WB}\right)}_{\text{SAME TERM AS "NO SUSP" CASE}} + \underbrace{m\ddot{y}\left(\frac{LG}{WB}\right) - \left(\frac{I_G}{WB}\right)\ddot{\theta}}_{\text{EFFECTS of } \ddot{y} \text{ and } \ddot{\theta} \text{ accelerations}} \qquad (12)$$

NOTE

- ΔW_R occurs due to the inertia forces and Torque on the whole system.
- ΔW_R will be present no matter what suspension is on the vehicle
- "SQUAT" implies $\ddot{y} \neq 0$ and/or $\ddot{\theta} \neq 0$.
- If there is "NO SQUAT", then $\ddot{y} = 0$ and $\ddot{\theta} = 0$ and ΔW_R is the same as without suspension.

NOW TRACE FORCES – EXAMINE A FBD of the axle assembly.



- INTRODUCE LINK FORCES P & R
- CANCEL static WT. W_R and the static spring force.
- ΔF_s = CHANGE IN SPRING FORCE due to "SQUAT" DURING ACCELERATION.

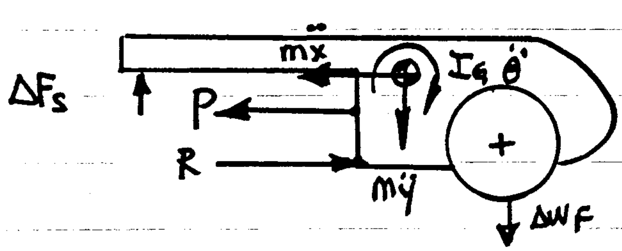
$$\Sigma F_x: \quad F_{DRIVE} + P - R = 0 \tag{13}$$
$$\Sigma T_\upsilon: \quad F_{DRIVE}(a) - P(b) = 0 \tag{14}$$
$$\Sigma F_y: \quad \Delta W_R - \Delta F_s = 0 \tag{15}$$

← NOTE: ΔW_R is only balanced by ΔF_s. The spring must compress from its static position to balance wt. Transfer ΔW_R (16)

squat must occur with this SET-UP

$$\boxed{\text{show: } P = \left(\frac{a}{b}\right) f_a W \qquad R = \left(\frac{b+a}{b}\right) f_a W}$$

What moves the car?



$$\Sigma F_x: \quad R - P = m\ddot{x} = f_a W \tag{17}$$

"THE DIFFERENCE IN LINK FORCES accelerates the car" (The links better be up to it!)

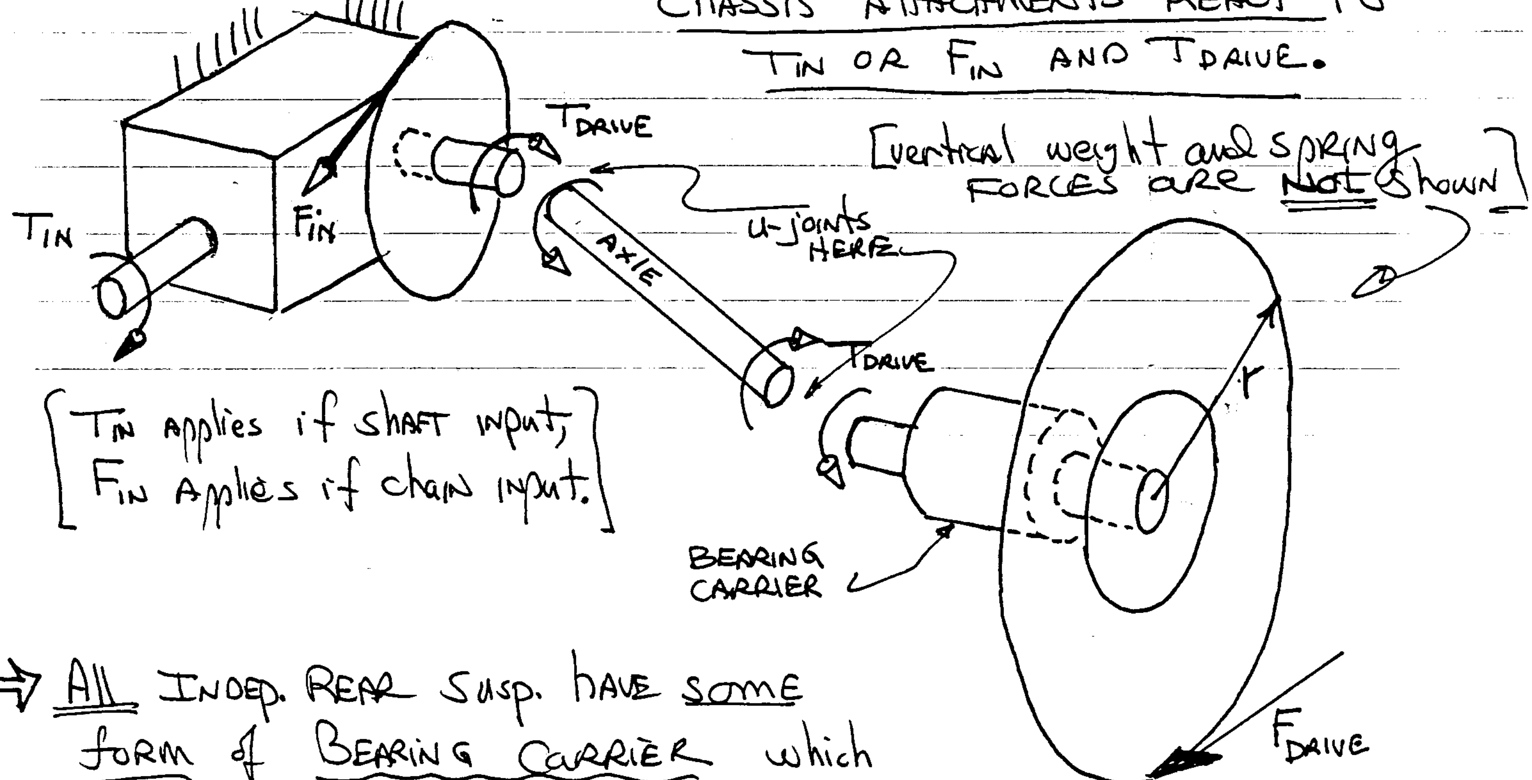
DEFINE the DRIVING TORQUE as $(F_{DRIVE})(r)$. (18)
r ~ Tire radius

FROM (13) WE SEE $(F_{DRIVE})(r) = \underbrace{(R-P)r}$ (19)
"DRIVING TORQUE" applied to chassis.

NOW TRACE FORCES FOR AN INDEPENDENT REAR SUSP:

GEARBOX/DIFF. UNIT MOUNTED ON CHASSIS: CHASSIS ATTACHMENTS REACT TO T_{IN} OR F_{IN} AND T_{DRIVE}.

[vertical weight and spring forces are NOT shown]



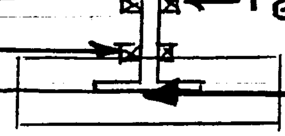
[T_{IN} applies if shaft input; F_{IN} applies if chain input.]

⇒ All INDEP. REAR SUSP. HAVE SOME form of BEARING CARRIER which must be constrained by susp. MEMBERS.

The wheel/TIRE/BEARING CARRIER is NOT IN EQUILIBRIUM AS shown: FORCE F_{DRIVE} CANNOT balance TORQUE T_{DRIVE}.

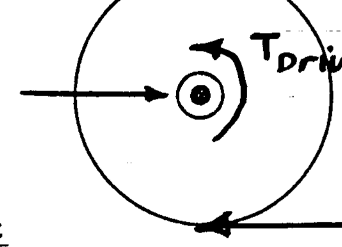
TAKE A closer look AT THIS ASSEMBLY to SEE what constraints are needed:

TOP VIEW: WHEEL, AXLE, BEARINGS.



{T_{DRIVE} NOT shown}

SIDE VIEW



{BEARINGS DO NOT TRANSMIT TORQUE to the BEARING CARRIER}

ΣF_x:

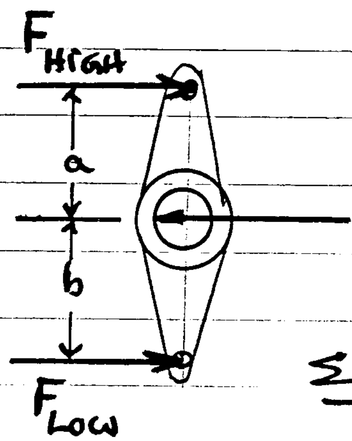
$F_{BO} - F_{Bi} = F_{DRIVE}$

(20)

EXAMINE THE CONSTRAINT REQUIREMENTS of the BEARING CARRIER — only FORCES F_{Bi} AND F_{BO} Act upon the carrier due to F_{DRIVE}.

SIDE VIEW

FRONT ⟸



{ Suppose the brg. carrier has upper & Lower Link attachment points: }

$F_{BO} - F_{Bi} = F_{DRIVE}$ { NO TORQUE IS APPLIED. }

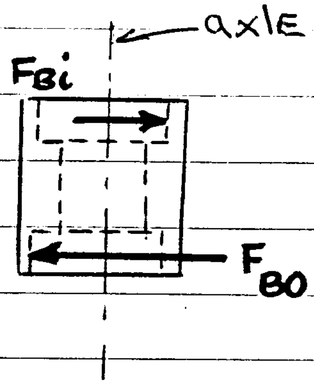
{ $F_{HIGH} + F_{LOW}$ "make" the vehicle accelerate }

ΣF_x: $F_{HIGH} + F_{LOW} = F_{DRIVE}$ (21)

NOTE assuming $a>0, b>0$, then Both F_{HIGH} and F_{LOW} will be smaller than F_{DRIVE}.

This is one result of not having the susp. members "take" the drive Torque. (compare to equations (16) for P and R)

TOP VIEW



{ SINCE $F_{BO} > F_{Bi}$ there is a Torque applied to the Bearing carrier which must BE Resisted by susp. members. }

EXAMINE figures 6,7,8 from Handout 3 and the FSAE cars to SEE how this RESISTANCE WAS PROVIDED.

Braking forces with indep. susp.:

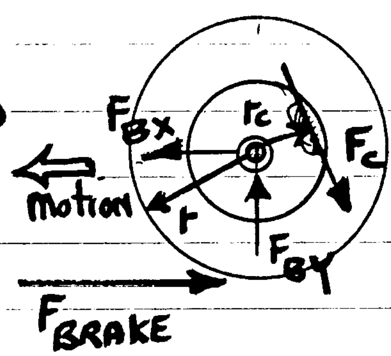
If brakes are "inboard" (mounted on the Gearbox/Diff unit), then the previous analysis holds with F_{BRAKE} acting at the bottom of the tire in the opposite direction to F_{DRIVE} and generating a T_{BRAKE} acting on the Gearbox/diff unit in the opposite direction to T_{DRIVE}. The bearing carrier forces would all change direction, but no torque would be transmitted to the bearing carrier.

If the brakes were outboard, at the BRG. carrier, its a different story:

SIDE VIEW

FBD of wheel, axle, BRGS, BRAKE DISK.

[wt. & spring forces not shown]



F_c = Force on brake disk due to pads in caliper.

r_c = radius to F_c.

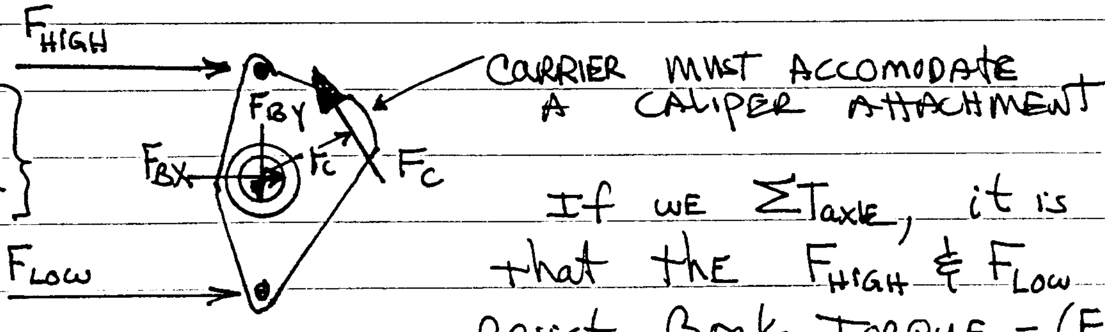
F_{BX}, F_{BY} = BEARING REACTIONS DUE TO F_{BRAKE} AND F_c.

DEFINE Brake Torque $= (F_{BRAKE})(r) = (F_c)(r_c)$ (22)

[From ΣT_{axle}]

The brake caliper is held by the BRG. CARRIER ASSY:

FBD of CARRIER

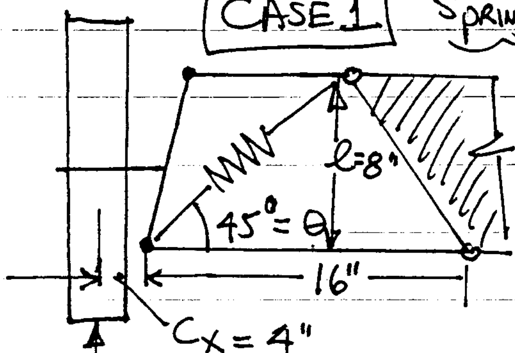


If we ΣT_{axle}, it is clear that the F_{HIGH} & F_{LOW} must also resist Brake Torque $= (F_c)(r)$.

Some "Construction Detail"s —

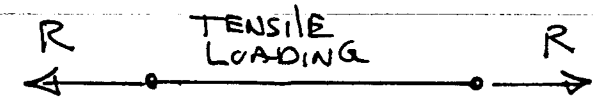
Spring location with Double-A arm susp: Page 1 shows F_{sp} acting at the Lower rod end. Here we examine that case and another mounting position and the sizes of members needed.

CASE 1 Spring acts at outer pivot



$W = 800$ Lbs — "200 Lb static & 4G Bump"

From page 1 analysis, we'll examine the Lower A-arm as a Beam.



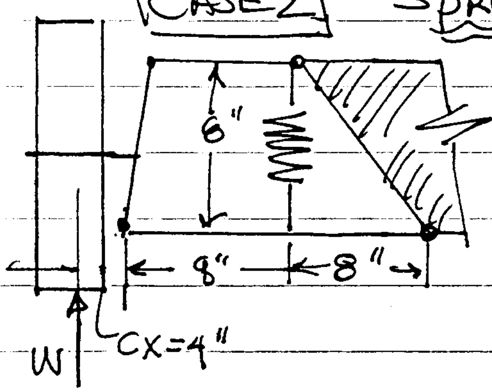
$$R = W\left[\frac{C_x}{\ell} + \cot\theta\right] \quad \text{(EQN 3)}$$

$$R = 800\left[4/8 + 1\right] = 1200 \text{ Lbs}$$

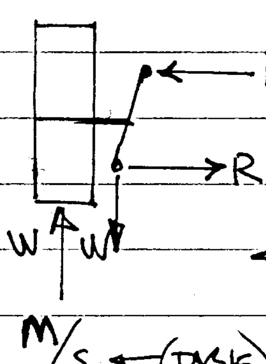
TENSILE STRESS in the ARM $= R/\text{AREA}$ (AREA FROM TABLE) : Suppose we choose 1" O.D. × 0.035" wall

$$\text{STRESS} = \frac{1200}{.1061} = 11{,}310 \text{ psi}$$

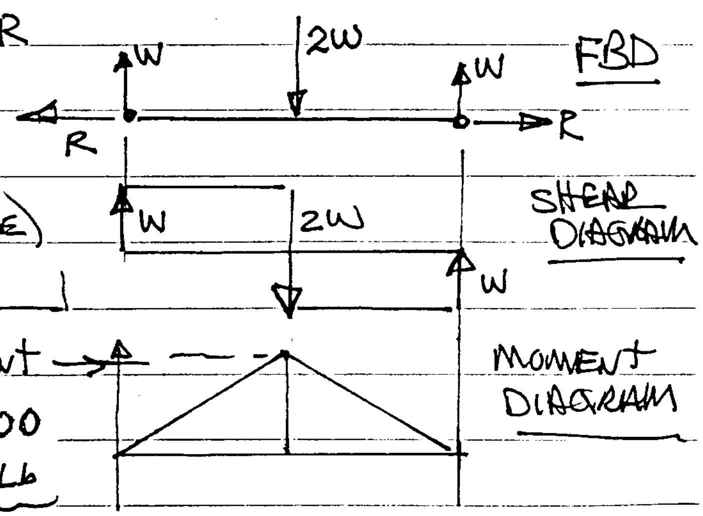
CASE 2 Spring mounts to A-arm (in middle)



Page 1 Results do not apply here - show the FBD's of upright & Lower A-arm:



$$P = R = W\left(\frac{C_x}{\ell}\right) = 400 \text{ Lbs}$$



- IGNORE THE TENSILE LOAD
- Bending Stress $= \frac{Mc}{I} = M/S$ ← (TABLE)
- Use 1" x 0.035:

BENDING STRESS $= \frac{6400}{0.0247} = 259{,}109$ psi !!

Max moment →

$$M = (W)8" = 6400 \text{ INCH-LB}$$

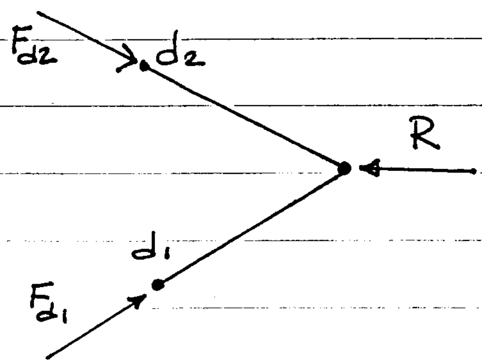
THIS IS OVER 20 TIMES THE ABOVE TENSILE STRESS!

MECHANICAL PROPERTIES—SHELBY AIRCRAFT TUBING—ROUND—Continued

STANDARD SIZES AS LISTED IN ARMY-NAVY AERONAUTICAL DESIGN STANDARDS—REVISED OCT. 1, 1942

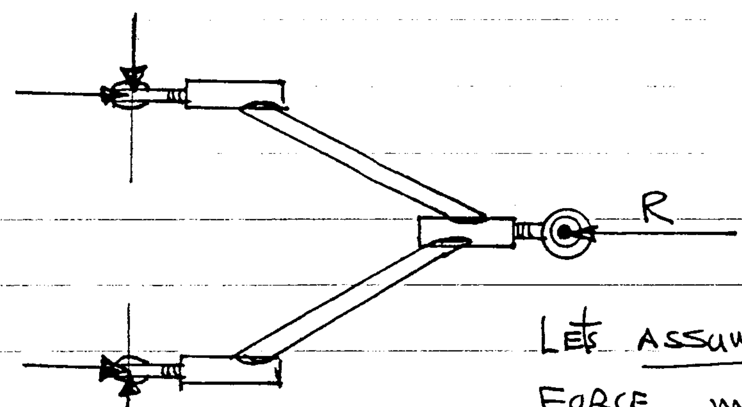
Outside diameter inches	Thickness		Weight per foot, pounds	Area of metal square inches	Moment of inertia	Section modulus	Radius of gyration
	Decimal of inch	B.W.G. or fraction	W	A	I	$S=I/y$	$r=\sqrt{I/A}$
1	.035	20	.3607	.1061	.0123	.0247	.3414
	.049	18	.4977	.1464	.0166	.0332	.3367
	.058	17	.5835	.1716	.0191	.0382	.3337
	.065	16	.6491	.1909	.0210	.0419	.3314
	.083	14	.8129	.2391	.0253	.0507	.3255
	.095	13	.9182	.2701	.0280	.0559	.3217
	.120	11	1.128	.3318	.0327	.0654	.3140
	.156	5/32	1.406	.4142	.0381	.0762	.3033
	.188	3/16	1.630	.4786	.0415	.0831	.2948
	.219	7/32	1.827	.5369	.0441	.0883	.2868
	.250	1/4	2.003	.5890	.0460	.0920	.2795
1 1/16	.049	18	.5306	.1560	.0201	.0378	.3588
	.065	16	.6928	.2037	.0254	.0479	.3534
	.095	13	.9821	.2888	.0341	.0642	.3437
	.109	12	1.111	.3265	.0376	.0708	.3393
	.120	11	1.209	.3553	.0401	.0755	.3359
	.156	5/32	1.511	.4449	.0470	.0885	.3251
1 1/8	.028	22	.3280	.0964	.0145	.0258	.3879
	.035	20	.4074	.1199	.0178	.0317	.3856
	.049	18	.5631	.1656	.0240	.0427	.3808
	.058	17	.6609	.1944	.0277	.0493	.3777
	.065	16	.7359	.2165	.0305	.0542	.3755
	.072	15	.8097	.2382	.0332	.0590	.3732
	.083	14	.9237	.2717	.0371	.0660	.3696
	.095	13	1.045	.3074	.0411	.0731	.3657
	.120	11	1.288	.3789	.0485	.0863	.3578
	.156	5/32	1.614	.4755	.0572	.1017	.3469
	.188	3/16	1.881	.5522	.0630	.1121	.3380
	.219	7/32	2.119	.6228	.0676	.1202	.3296
	.250	1/4	2.336	.6872	.0711	.1265	.3217
1 3/16	.028	22	.3469	.1020	.0172	.0289	.4101
	.049	20	.5961	.1753	.0284	.0479	.4029
	.065	16	.7796	.2292	.0362	.0610	.3975
	.083	14	.9795	.2880	.0442	.0744	.3916
	.095	13	1.109	.3261	.0490	.0826	.3877
	.109	12	1.256	.3693	.0542	.0914	.3833
	.120	11	1.369	.4024	.0581	.0978	.3798
1 1/4	.035	20	.4542	.1336	.0247	.0395	.4297
	.049	18	.6285	.1849	.0334	.0534	.4250
	.058	17	.7384	.2172	.0387	.0619	.4219
	.065	16	.8226	.2420	.0426	.0682	.4196
	.095	13	1.172	.3447	.0579	.0926	.4097
	.109	12	1.328	.3907	.0642	.1027	.4052
	.120	11	1.448	.4260	.0688	.1100	.4018
	.134	10	1.597	.4698	.0742	.1187	.3974
	.156	5/32	1.823	.5369	.0819	.1310	.3906
	.188	3/16	2.132	.6259	.0910	.1457	.3814
	.219	7/32	2.411	.7087	.0984	.1575	.3727
	.250	1/4	2.670	.7854	.1043	.1669	.3644
1 5/16	.065	16	.8664	.2547	.0497	.0757	.4417
	.120	11	1.529	.4496	.0807	.1230	.4238
	.156	5/32	1.928	.5676	.0966	.1472	.4125
	.188	3/16	2.259	.6627	.1078	.1642	.4032
	.219	7/32	2.559	.7517	.1169	.1781	.3944
1 3/8	.035	20	.5009	.1473	.0331	.0481	.4739
	.049	18	.6939	.2041	.0449	.0653	.4691
	.058	17	.8158	.2400	.0521	.0758	.4661
	.065	16	.9094	.2675	.0575	.0837	.4637
	.083	14	1.145	.3369	.0706	.1027	.4577
	.095	13	1.299	.3820	.0787	.1144	.4538
	.120	11	1.608	.4731	.0940	.1367	.4457
	.156	5/32	2.031	.5983	.1129	.1642	.4344
	.188	3/16	2.383	.6995	.1263	.1838	.4250
	.219	7/32	2.704	.7946	.1375	.2000	.4160
	.250	1/4	3.004	.8836	.1467	.2134	.4075
1 7/16	.049	18	.7269	.2137	.0516	.0718	.4912
	.095	13	1.363	.4007	.0907	.1262	.4758
	.156	5/32	2.136	.6289	.1310	.1822	.4564
	.250	1/4	3.172	.9327	.1717	.2389	.4291

A-ARM DETAIL - WE'VE SHOWN A-ARMS SCHEMATICALLY AS LINES WITH JOINTS AT THE ENDS, IMPLYING THAT EACH LEG COULD BE VIEWED AS A <u>TWO-FORCE MEMBER</u>, EITHER IN TENSION OR COMPRESSION LIKE THIS:



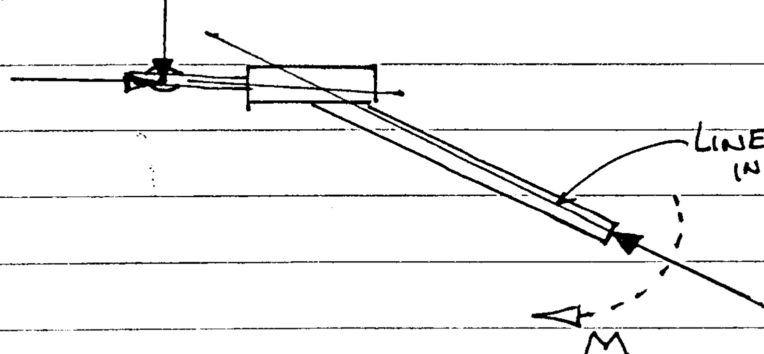
IN ACTUAL CONSTRUCTION, IT TAKES SOME <u>DESIGN</u> TO ACHIEVE THE TWO-FORCE MEMBER CONCEPT.

A COMMON A-ARM CONSTRUCTION LOOKS LIKE:



• ROD ENDS CANNOT PRODUCE MOMENTS ABOUT ANY PIVOT AXIS, ONLY FORCES —

LET'S <u>ASSUME</u> THE LEGS <u>ARE</u> TWO FORCE MEMBERS AND SEE <u>IF</u> EQUILIBRIUM COULD BE ACHIEVED. EXAMINE THE UPPER MOUNT:



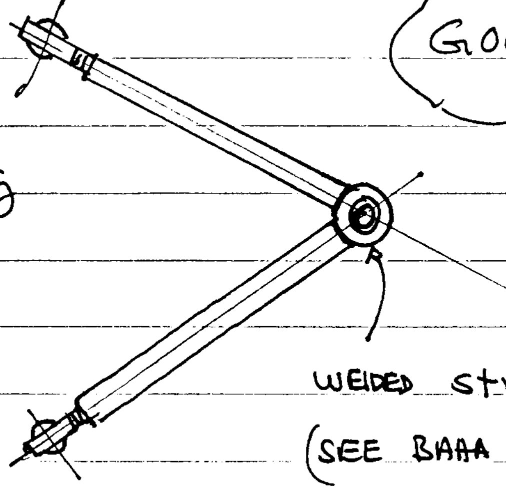
LINE OF ACTION OF FORCE IN THE LEG <u>IF</u> ITS A TWO FORCE MEMBER —

<u>BUT</u> THIS FBD <u>CANNOT</u> BE EQUILIBRIUM UNLESS A BENDING MOMENT, M, IS INTRODUCED.

SO, THIS A-ARM DESIGN <u>WILL INTRODUCE</u> MOMENTS INTO THE LEGS AND WELDS.

A Good A-ARM DESIGN will have the outer joint axis IN-LINE with the legs. This can be achieved by mounting A spherical Bearing (the "INSIDE" of A ROD END) IN A housing which is welded or machined into the legs. Then the legs are TWO FORCE MEMBERS.

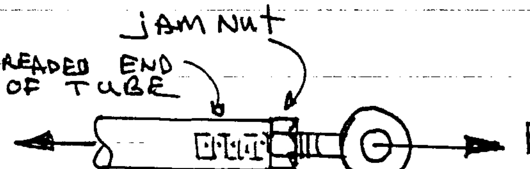
GOOD



WELDED STYLE — (SEE BAJA CAR)

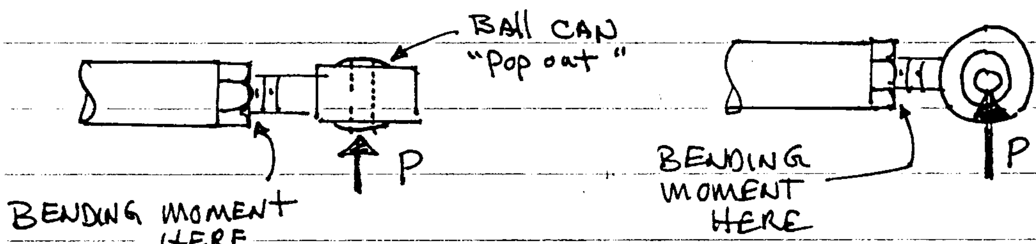
LOADING ROD ENDS

O.K.



THE DESIGNED LOADING is this: RATED LOAD EXPECTS THIS LOAD DIRECTION.

IN MANY applications, ONE SEES ROD ENDS IN BENDING LIKE THESE:

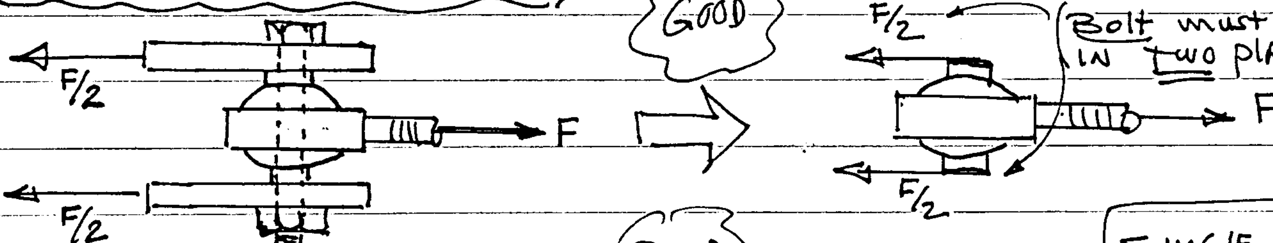


POOR — O.K. for SMALL LOADS

THE LOAD P is around 10% of F, AND THE THREADED SHANK must be sized at the ROOT DIAM. to take the MOMENT.

MOUNTING ROD ENDS

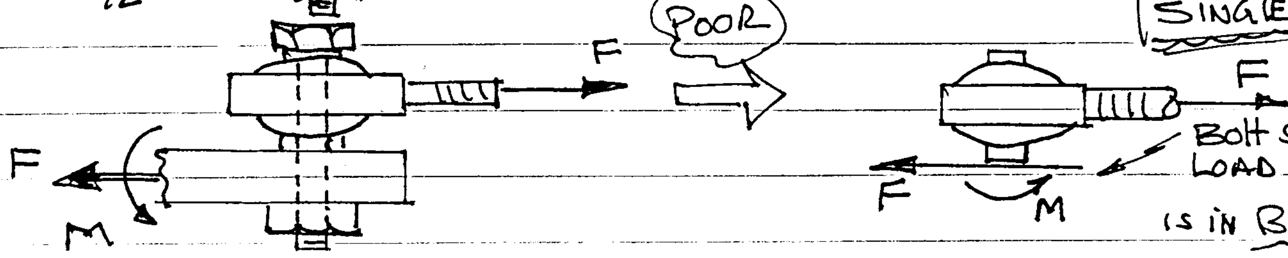
GOOD



DOUBLE SHEAR

Bolt must shear IN TWO places to fail

POOR



SINGLE SHEAR

Bolt shear LOAD = F and is in BENDING

WHEEL BEARING LOADS — SOME OBSERVATIONS FROM FRED Puhn IN "HOW TO MAKE YOUR CAR Handle", p. 128. His "Positive & NEGATIVE OFFSET wheels" REALLY REFER to the Location of the TIRE CENTER-LINE with RESPECT to the BEARING placement. His diagrams show the RESULT of superposition of REACTIONS due to WEIGHT (W) and CORNERING FORCE (C). I ADDED dimensions r and BS.

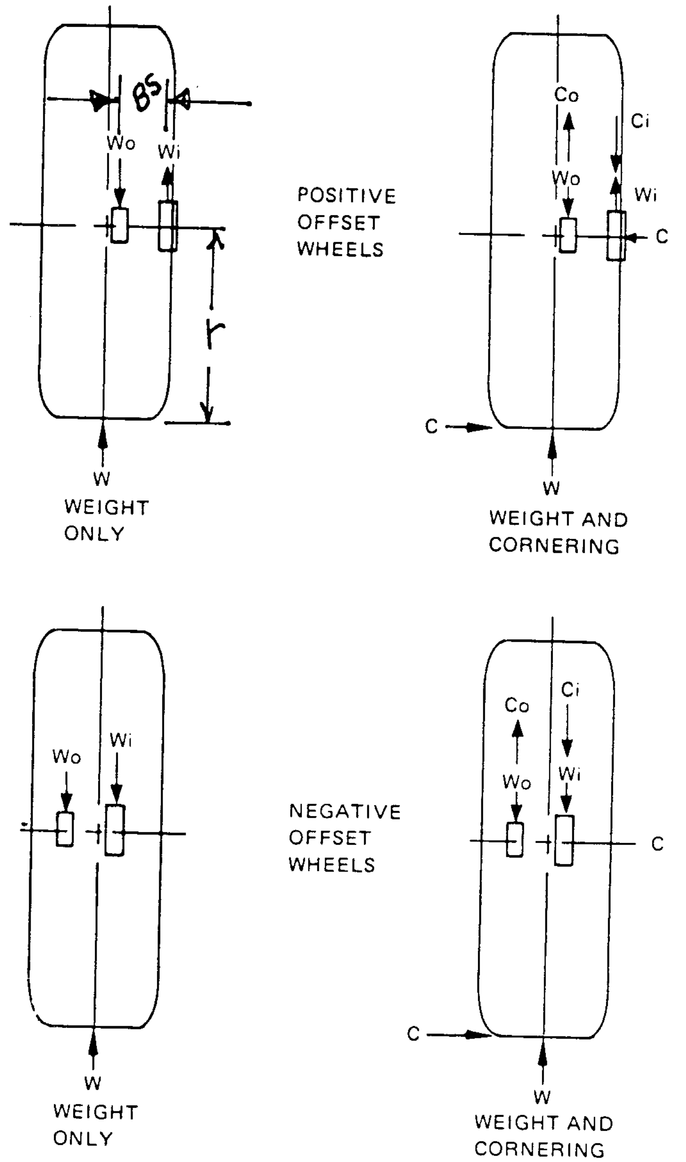
OBSERVE: $(BS)(C_i) = (r)(C)$

SO: $C_i = \left(\frac{r}{BS}\right) C$

Typical values: $r = 10''$, $BS = 2''$

so $C_i \approx 5C \Rightarrow$ CORNERING FORCE is amplified at the BEARINGS.

Also observe: $C_o = C_i$

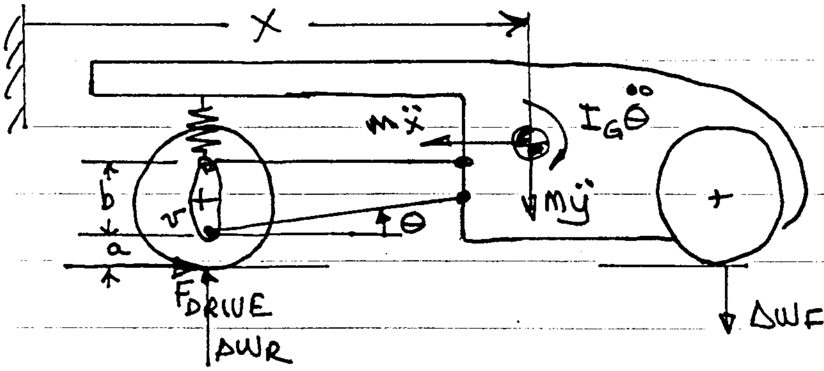


W = WEIGHT
Wo = WEIGHT ON OUTBOARD WHEEL BEARING
Wi = WEIGHT ON INBOARD WHEEL BEARING
C = CORNERING FORCE
Co = CORNERING FORCE ON OUTBOARD BEARING
Ci = CORNERING FORCE ON INBOARD BEARING

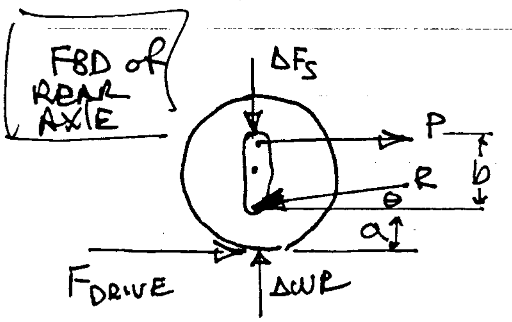
Figure 35/Wheel bearings carry weight of vehicle, "W", during straight-line running. They also carry cornering forces, "C", in turns.
With negative offset, weight loading is well distributed between inner and outer bearin However, cornering loads upset this balance by increasing force on the inside bearing and decreasing force on the outside bearing. This makes cornering much harder on the inside bearing than straight-line driving.
With positive offset, the situation is reversed. In straight-line driving, bearing loads are uneven but the addition of cornering forces tends to even them out. Thus cornering loads are less severe than those of straight-line driving.
In these sketches, offsets are exaggerated for clarity. Small changes in offset don't affe bearing loads enough to worry about. In any event, wheel-bearing life will probably be affected more by your driving than anything else.

SINCE OUR FSAE CAR CORNERS At $f_c = 1.3$, then $C > W$, AND its amplified. So BEARING LOADS DUE to CORNERING FAR EXCEED those DUE to vehicle WEIGHT, Wo & Wi. ← DISCUSS Bump LOADS too.

ANTI-SQUAT SUSPENSION EXAMPLE – USE LIVE AXLE AND A SIMPLE ANTI SQUAT GEOMETRY TO ILLUSTRATE (Pages 6-8)



ANTI-SQUAT EXAMPLE: SEE FIGURE & NOTATION ON PAGE 8. HERE, THE LOWER TRAILING LINK IS ANGLED UPWARD AT ANGLE θ



NOW, FORCE R HAS A DOWNWARD COMPONENT. DEVELOP EQNS SIMILAR TO (13, 14, 15):

$$\left.\begin{aligned} &\Sigma F_x: \quad F_D + P - R\cos\theta = 0 \\ &\Sigma F_y: \quad \Delta W_R - \Delta F_S - R\sin\theta = 0 \\ &\Sigma T_v: \quad F_{DRIVE}(a) - P(b) = 0 \end{aligned}\right\} (23)$$

If $\Delta F_S = 0$, THERE WILL BE NO "SQUAT" → AND THE $R\sin\theta$ will then CANCEL the ΔWR. But if no squat occurs, then $\ddot{\theta} = 0$ and $\ddot{Y} = 0$ and $\Delta W_R = f_a W(HG/WB)$ ← EQN (12)

SO SET $\Delta W_R = R\sin\theta$ and see if it makes sense. (24)

$$\Delta W_R = f_a W\left(\frac{HG}{WB}\right) \stackrel{?}{=} R\sin\theta = \underbrace{\left[\left(\frac{a+b}{b}\right)\frac{f_a W}{\cos\theta}\right]}_{\text{"R" → From with EQNS (23)}}\sin\theta \qquad (25)$$

OR $$\tan\theta = \left(\frac{HG}{WB}\right)\left(\frac{b}{a+b}\right) \qquad (26)$$

IF THE HG, WB, b, a+b satisfy this (and make sense for the vehicle) there will be NO SQUAT if the Lower Link is arranged at angle θ.

ARTICLES REFER to an "anti-squat line", as shown below, from points ① → ②. We'll interpret this in terms of EQUATION (26).