BUMP STEER HANDOUT

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(Handout 6 in the 2004 Mech. Team "Bible" refers to a "special topic" to address the details of Bump Steer – This handout shows how).

This handout describes a method to locate the inner position of the steering tie rod pivot in order to minimize "Bump Steer" in a double A-arm suspension. The notion of Bump Steer is described, followed by an overview of a theory-based algebraic method to locate the inner tie rod pivot, and then details of the approach and an example. Results from the applicable theory will simply be stated and references provided for the interested reader.

Figure 1 shows the front view of a double A-arm suspension with the pivots projected on a lateral plane through the axle line. The issue of Bump Steer requires one to visualize the movement of the suspension upright as it traverses its range of motion from full rebound to full bump. The following pivot points are presumed to be specified:

- OA is the pivot point on the chassis for the upper A-arm. It is actually an axis of rotation and perpendicular to the plane of figure 1.
- OB is the pivot point on the chassis for the lower A-arm, and is actually an axis of rotation and perpendicular to the plane of Figure 1. It's the origin of the xy coordinates.
- Points A and B are the pivots of the outer ends of the A-arms, and must be spherical bearings ("ball joints") to allow angular motion in the plane of Figure 1, and to allow the upright to rotate around the line between A and B, (which is called the "kingpin axis") to implement steering.
- Point C is the projection of the steering arm pivot upon the plane of Figure 1. It is where the tie rod attaches to the upright, and must be a spherical bearing to accommodate steering and bump/rebound motions.

Point OC is to be determined. It is the inner pivot of the tie rod, and with a rack and pinion steering, it corresponds to the ball joint at the end of the rack when the steering is in the straight-ahead position.

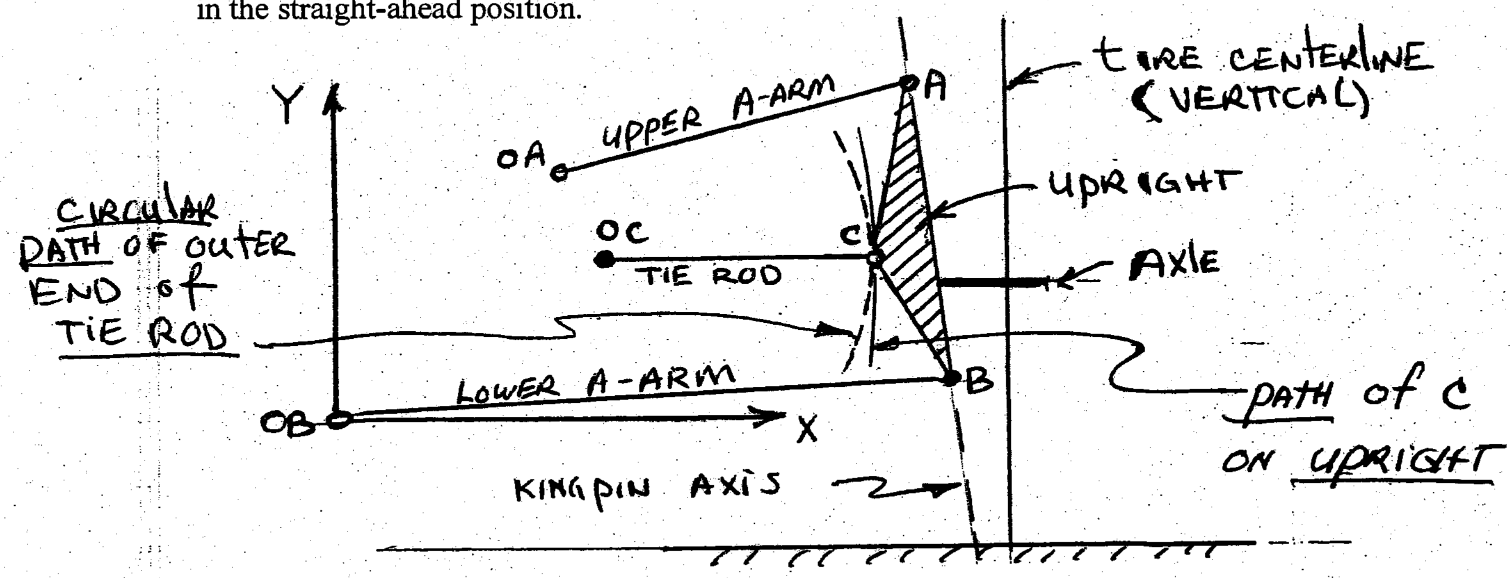


Figure 1

As the suspension moves through the bump-rebound positions, point C on the upright is guided along some path as determined by the geometry of the A-arms, as shown in Figure 1. Also, the outer end of the tie rod is guided on a circular arc which is centered on point OC, as shown in Figure 1. When the outer end of the tie rod is attached to the upright at C, then point C on the upright must follow the circular arc of the end of the tie rod. If that arc does not match the path of C as guided by the A-arms, the upright will be forced to rotate around the kingpin axis, which causes the "Bump Steer". So the issue is to reduce bump steer by locating point OC in such a way that the circular path of the outer end of the tie rod will closely match the path of point C as guided by the A-arms.

This problem can be addressed in many ways:

- Trial and error with a CAD system,, in which point OC is guessed then the path of point C on the upright is compared to that on the tie rod (using some measure of differences in the paths), and then keep choosing OC positions until the paths closely match.
- Graphical Methods, as described in reference 1 pages 276-281.
- Algebraic methods, using 4-Bar Linkage Theory, from reference 2, chapter 7.

We will utilize the latter method. It will not be necessary to understand the "theory", but only to apply certain algebraic results.

Overview of 4-Bar Linkages

Students in Mechanical Engineering Programs may take a course on "Linkages and Mechanisms", and would study a common device known as the "planar 4-bar linkage", as shown generally in Figure 2.

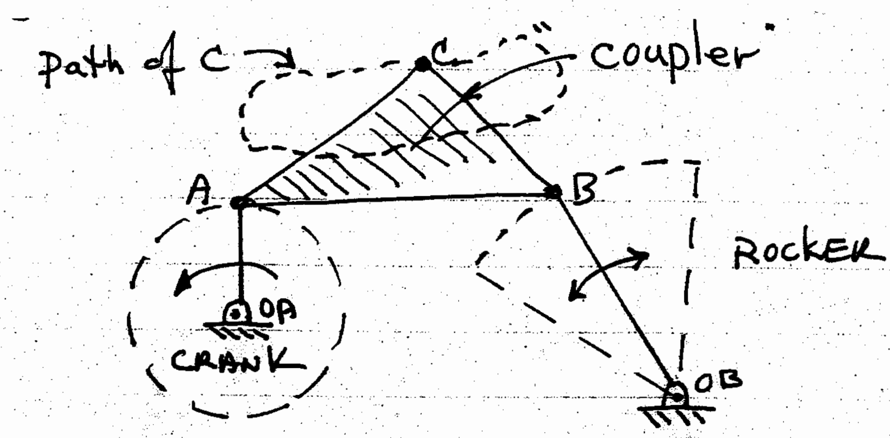


Figure 2 4-Bar Linkage

The four "bars" refer to the following elements:

- Link OA-A is attached to the ground and the coupler with pivots that allow rotation.
- Link OB-B is attached to the ground and the coupler with pivots that allow rotation.

- Link ABC is the "coupler" (it couples or joins the grounded links) and contains a point of interest denoted as C.
- The fourth link is the "ground" which connects OA and OB.

Four bar linkages exhibit a wide range of motion possibilities, depending upon the link lengths, ground locations and coupler shapes. Figure 2 shows a "crank-rocker" linkage, with link OA-A able to rotate about OA (the "crank"), and link OB-B rocking between two extremes as the crank rotates through 360 degrees. Simultaneously, point C will trace out some path. The path of point C depends upon the geometry of the linkage, and many designs are used in machinery. The path of point C usually has curved portions, and the "radius of curvature" at points on the path can be found from the theory of planar motion.

The double A-arm suspensions is a four bar linkage, with the upper and lower A-arms being the links that pivot on the ground, and the upright being the coupler link. The tie rod attachment, point C in Figure 1, is the coupler point of interest, and it will trace out some path as the suspension moves through the bump-rebound motion. To minimize bump steer, we want to find the radius of curvature of point C. Then by placing point OC at the center of this radius of curvature, the outer end of the tie rod will closely match the movement of point C as guided by the A-arm, thereby reducing bump steer.

Overview of Algebraic Method and the Results of Theory

One element of the method is the instant center of the upright with respect to the chassis, represented by points OA and OB. Figure 3 shows the suspension of Figure 1, along with the instant center, denoted as I. Instant center I is located at the intersections of the lines through A-OA and B-OB. The horizontal distance from I to the tire centerline is termed the "swing arm length", and is used to classify various Double A-arm suspensions. One "result from theory" is that point OC will be on the line connecting I and C.

Two coordinate systems are defined:
System x, y is centered on OB ("on the car")
System X, Y is centered on point I

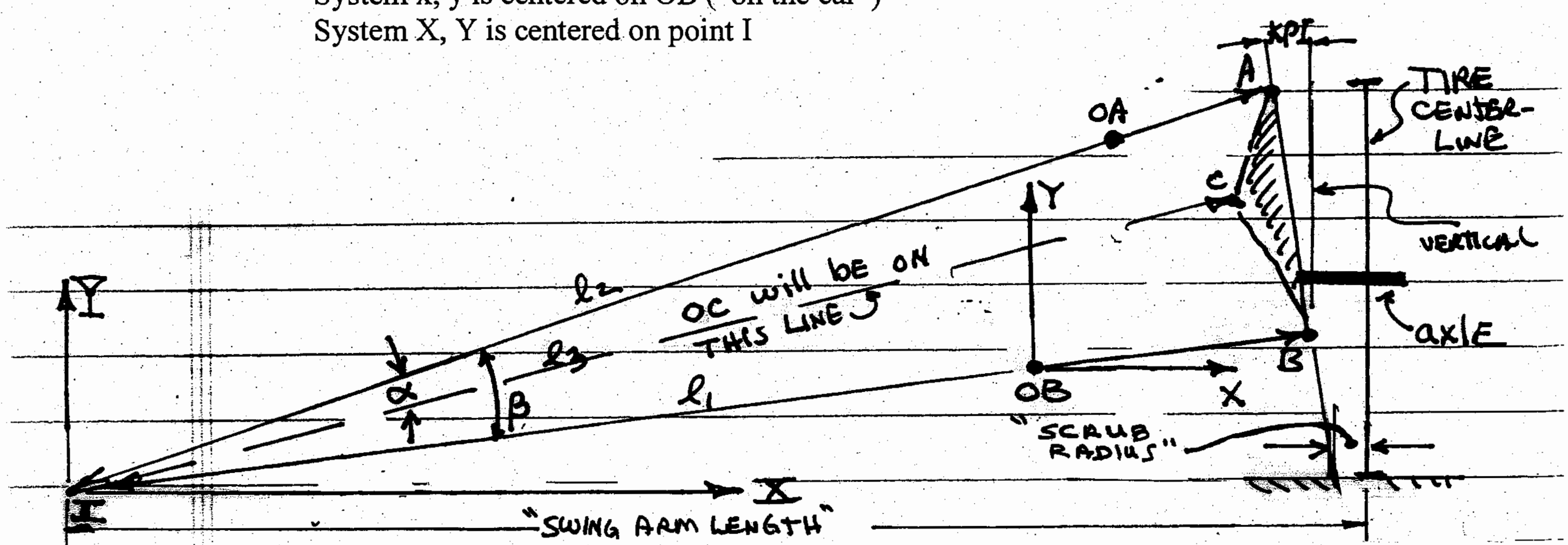


Figure 3

The approach will be to first locate point OC in the X, Y coordinates, and then translate it to the xy coordinates on the chassis. Distances from I to A, B and C will be needed, and are identified in the figure as l_2, l_1 *and* l_3 respectively. Angles α *and* β are also defined on the figure, as is the "kingpin inclination angle", KPI.

Now, another set of distances are introduced. They are part of the theory of 4-bar linkages and extend the lines from A, B and C through I to the left as shown in Figure 4, and their endpoints lie on a circle. The endpoints are denoted as J_A, J_B *and* J_C. Point I is also on the circle which is called the "inflection circle". The final part of the theory that we will use is known as the Euler-Savary equation, which is a relationship among the various distances and link length for any point on the coupler, and will allow the finding of point OC.

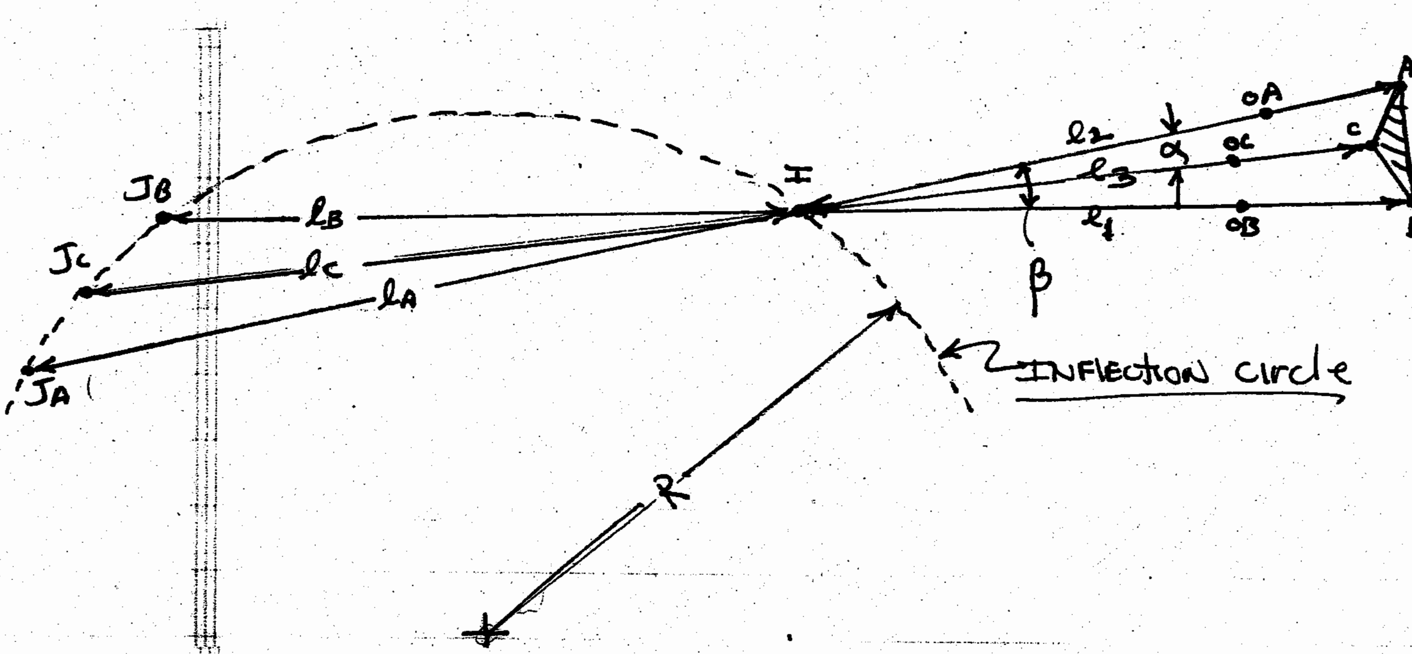


Figure 4

The Euler Savary equations for A, B and C are:

Point A: $$\frac{OA-A}{l_2}=\frac{l_2}{l_2+l_A} \tag{1}$$

Point B: $$\frac{OB-B}{l_1}=\frac{l_1}{l_1+l_B} \tag{2}$$

Point C: $$\frac{OC-C}{l_3}=\frac{l_3}{l_3+l_C} \tag{3}$$

To find point OC, we will use Equation 3, where the length OC-C is the tie rod length in the plane of the Figures. Length l_3, the distance from I to C is found from the known layout of points A, B, C, OA and OB and applying of some trigonometry. (Details in the next section) The unknown is l_C. Using the theory that points I, J_A, J_B *and* J_C all lie on the inflection circle, it is possible to express the distance l_C as follows:

$$l_C = l_A \cos\alpha - \frac{\sin\alpha}{\sin\beta}[l_A \cos\beta - l_B] \tag{4}$$

This equation expresses l_C in terms of l_A, l_B and angles α *and* β. The angles can be found from the layout of the A-arms, upright and point C, and the l_A *and* l_B can be found from equations (1) and (2). Once l_C is found, then equation (3) is used to find distance OC-C, which locates point OC.

<u>Locating Point OC: The Details</u>

The following will describe the <u>details</u> of starting with the location of points A, B, C, OA, OB and then locating point OC to minimize bump steer. Figure 3 is repeated below, and renamed as Figure 5, and includes the definition of more angles to be used in the development. Data needed will be the coordinates of point OA, A, C and B in the xy system which is centered at OB. Those coordinates will be denoted as:

$x_{OA}, y_{OA}, x_A, y_A, x_C, y_C,$ *and* x_B, y_B, corresponding to points OA, A, C and B respectively. The "unknown" point OC is also shown in Figure 5. Angles θ_A, θ_B *and KPI* are then:

$$\theta_B = \arctan(y_B / x_B)$$
$$\theta_A = \arctan\left(\frac{y_A - y_{OA}}{x_A - x_{OA}}\right) \tag{5}$$

$$KPI = \arctan\left(\frac{x_B - x_A}{y_A - y_B}\right) \tag{6}$$

NOTE COORDS

Angle θ_C will be determined when point C is located in the X, Y system.

$$Angle\ \beta\ is:\ \beta = \theta_A - \theta_B \tag{7}$$

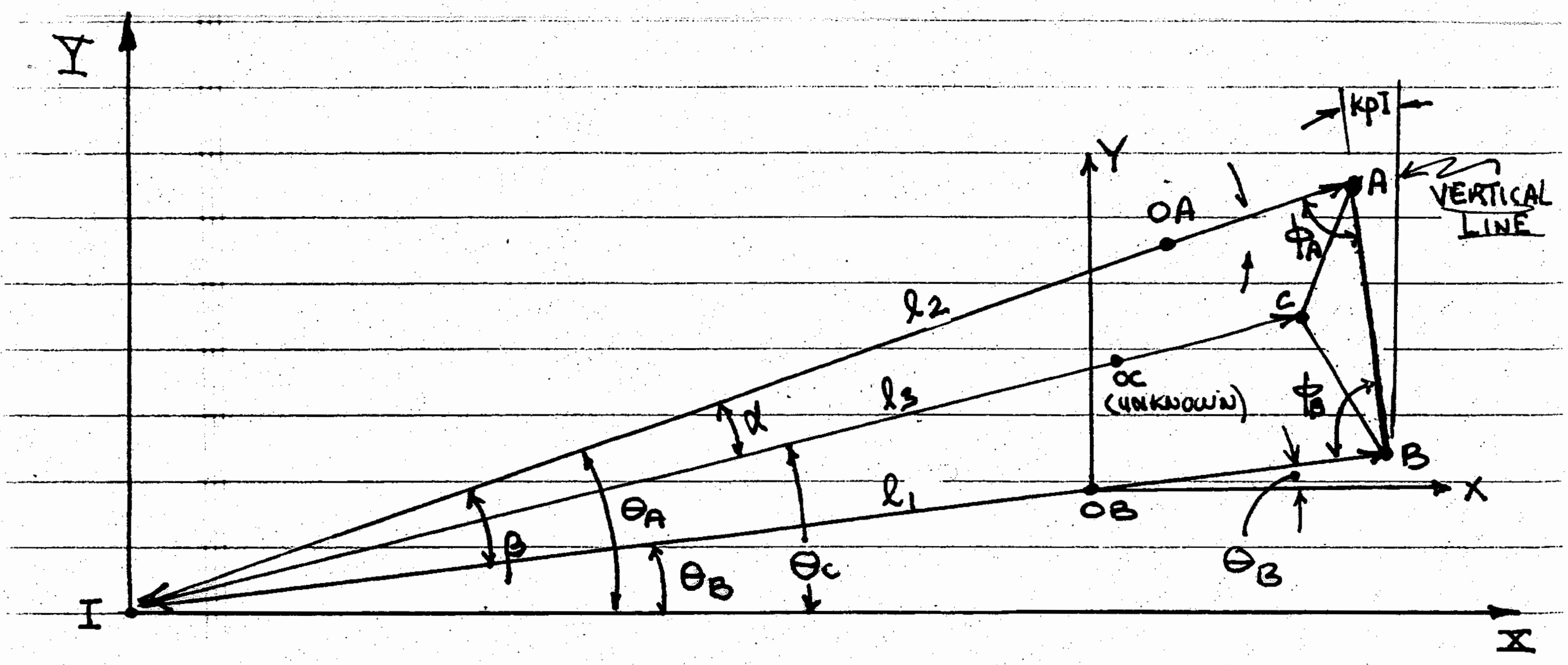


Figure 5

Distance l_1 *and* l_2 can be found in many ways, and we use the sine law:

$$\frac{l_2}{\sin\phi_B} = \frac{l_1}{\sin\phi_A} = \frac{AB}{\sin\beta} \tag{8}$$

Angles ϕ_A *and* ϕ_B are:

$$\phi_A = 90^\circ + KPI - \theta_A \tag{9}$$

$$\phi_B = 90^\circ + \theta_B - KPI \tag{10}$$

Length AB is known, so from Equation 8.

$$l_1 = \left(\frac{AB}{\sin\beta}\right)\sin\phi_A \tag{11}$$

$$l_2 = \left(\frac{AB}{\sin\beta}\right)\sin\phi_B \tag{12}$$

To find l_3, we locate point C in the X, Y system by first locating point OB and then adding the coordinates of C in the xy system.

$$\begin{aligned} X_{OB} &= (l_1 - OB - B)\cos\theta_B \\ Y_{OB} &= (l_1 - OB - B)\sin\theta_B \end{aligned} \tag{13}$$

then:
$$X_C = X_{OB} + x_C \qquad Y_C = Y_{OB} + y_C \tag{14}$$

so:
$$l_3 = \left(X_C^2 + Y_C^2\right)^{1/2} \tag{15}$$

and:
$$\theta_C = \arctan\left(Y_C / X_C\right) \tag{16}$$

and:
$$\alpha = \theta_A - \theta_C \tag{17}$$

Now we use Euler Savary equations (1) and (2) to find l_A and l_B. Rewrite these equations as:

$$l_A = l_2\left[\frac{l_2}{O_A A} - 1\right] \tag{18}$$

$$l_B = l_1\left[\frac{l_1}{O_B B} - 1\right] \tag{19}$$

Now, l_A and l_B, with angles α and β go into equation (4) (not repeated here) to find l_C. Finally, knowing l_C, and l_3 from equation (15) we can find the distance OC-C by rewriting the Euler Savary equation for point C (equation 3) as:

$$OC - C = \frac{l_3^2}{l_3 + l_C} \tag{20}$$

This locates point (OC) by finding the length of the tie rod (in the lateral plane), so working back from point C along line l_3, we find (OC). The last step is to locate OC in the xy coordinate system ("on the car") Point OC in the X, Y system is located as:

$$X_{OC} = X_C - (OC - C)\cos\theta_C \qquad Y_{OC} = Y_C - (OC - C)\sin\theta_C \tag{21}$$

Then point OC in the xy system is located at:

$$x_{oc} = X_{oc} - X_{OB} \qquad y_{oc} = Y_{oc} - Y_{OB} \tag{22}$$

References

1. J. Reimpell and H. Stoll, The Automotive Chassis: Engineering Principles, SAE, 1996.

2. R. Hartenberg and J. Denavit, Kinematic Synthesis of Linkages, McGraw Hill, 1964.

Appendix A

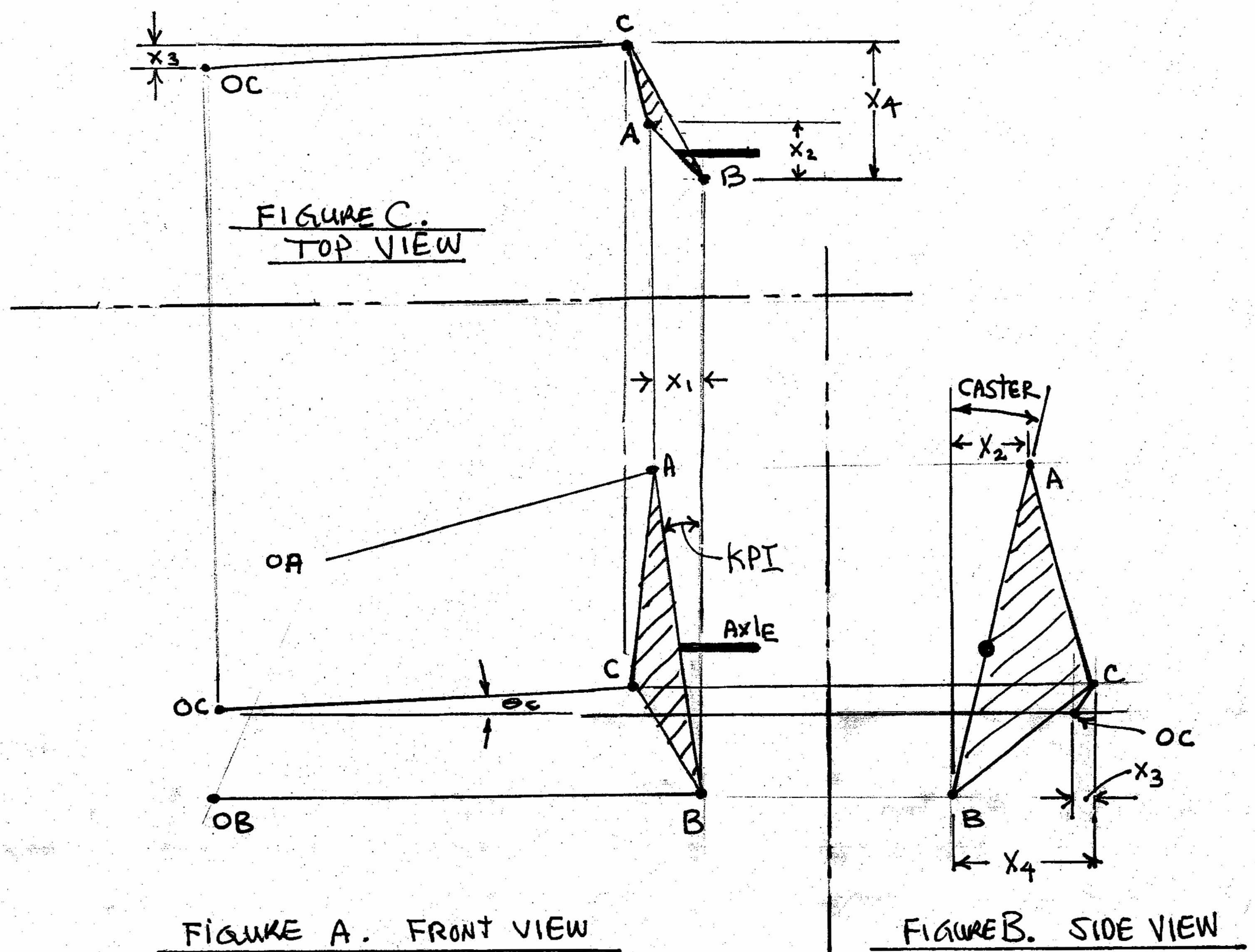
The bump steer analysis utilizes the dimensions of the double A-arm suspension and tie rod as seen from the front, with all points projected on a lateral plane through the front axle line. The chassis mounting points for the A-arms were actually pivot axes that were perpendicular to the lateral plane, but the other points may not be readily transferred to the lateral plane without some further analysis. The following figures show 3 views of the suspension upright and tie rod in order to convey how the caster angle and fore-aft rack location, (which determines point OC), play a role in determining the projected points A, B, C and OC in the lateral plane.

Figure A shows the front view of the suspension and all mounting and pivot points as projected on the lateral plane, and was the starting point for the bump steer analysis (Figure 1 of the handout). Angles shown are KPI and θ_c and distances $x_1, x_2,$ *and* x_3 are defined for reference purposes between the views. The analysis found the location of point OC on this plane and the length of the tie rod, OC-C as projected on this plane. Distance AB on this plane may not be the measured distance between the upper and lower ball joints on the upright, because of a possible caster angle, as shown in Figure B.

The projection of the steering arm pivot on the upright is point C in Figure A, but care must be taken when locating point C with respect to some reference dimensions <u>on</u> the upright. Figures B and C show that point C is "behind" (towards the rear of the vehicle) the kingpin axis AB. Presumably, point C was chosen to achieve some steering ratio and some degree of Ackermann effect, but its projection onto the lateral plane of Figure A is what is needed for the bump steer analysis.

Point OC is actually an axis that is perpendicular to the plane of Figure A, and Figure C shows point OC being forward of point C by amount x_3. So the actual tie rod length is <u>not</u> OC-C but must be further adjusted due to distance x_3.

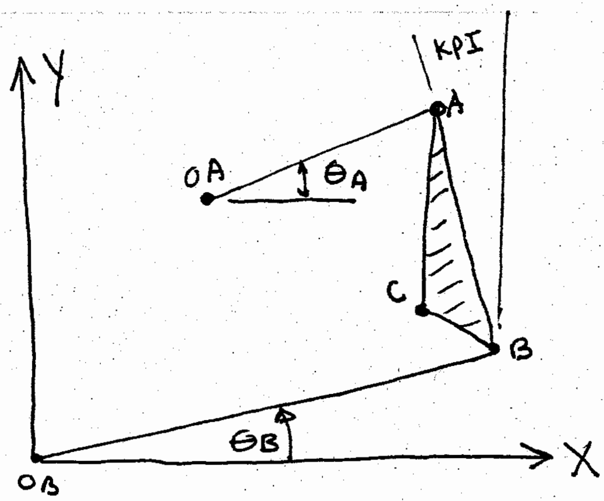
So, it is important to account for the caster, the steering arm pivot and the rack location when defining the points A, OA, B, OB and C in Figure 1, and in interpreting the results for the tie rod length.



THREE VIEWS of AN UPRIGHT

APPENDIX B – EXAMPLE

EXAMPLE, USING AURORA[3] DATA



DATA $X_B = 11.91$ $X_C = 10.962$ $X_{OA} = 2.35$ $X_A = 11.20$

$Y_B = 2.00$ $Y_C = 2.547$ $Y_{OA} = 6.10$ $Y_A = 8.2$

computed: $OB\text{-}B = 12.077$ $AB = 6.241$ $OA\text{-}A = 9.096$

EQN(5) $\theta_B = \arctan\left(Y_B / X_B\right) = \arctan\left(\frac{2}{11.91}\right) = \arctan(0.1679) \Rightarrow \theta_B = 9.5325^\circ$

EQN(5) $\theta_A = \arctan\left(\frac{Y_A - Y_{AO}}{X_A - X_{OA}}\right) = \arctan\left(\frac{8.2 - 6.1}{11.20 - 2.35}\right) = \arctan\left(\frac{2.1}{8.85}\right) \Rightarrow \theta_A = 13.348^\circ$

EQN(6) $KPI = \arctan\left(\frac{11.91 - 11.20}{8.2 - 2.0}\right) = \arctan\left(\frac{0.71}{6.2}\right) \Rightarrow KPI = 6.5328^\circ$

EQN(7) $\beta = \theta_A - \theta_B = 13.348^\circ - 9.5325^\circ = 3.816^\circ$

EQN(9) $\phi_A = 90 + KPI - \theta_A = 90 + 6.5328 - 13.348^\circ = 83.184^\circ$

EQN(10) $\phi_B = 90 + \theta_B - KPI = 90 + 9.5325 - 6.5328 = 92.999^\circ$

EQN(11) $l_1 = \left(\frac{AB}{\sin\beta}\right)\left(\sin\phi_A\right) = \left(\frac{6.241}{\sin(3.816)}\right)\left(\sin 83.184^\circ\right) = 93.11''$

EQN(12) $l_2 = \left(\frac{AB}{\sin\beta}\right)\left(\sin\phi_B\right) = \left(\frac{6.241}{\sin(3.816)}\right)\sin(92.999^\circ) = 93.64''$

EQN(13) $X_{OB} = (l_1 - OBB)\cos\theta_B = (93.11 - 12.077)\cos(9.5325) = 79.914''$

$Y_{OB} = (l_1 - OBB)\sin\theta_B = (93.11 - 12.077)\sin(9.5325) = 13.419''$

EQN(14) $\quad X_c = X_{OB} + x_c = 79.914 + 10.962 = 90.876''$

$\quad\quad Y_c = Y_{OB} + y_c = 13.419 + 2.547 = 15.966''$

EQN(15) $\quad l_3 = \left(X_c^2 + Y_c^2\right)^{\frac{1}{2}} = \left[(90.876)^2 + (15.966)^2\right]^{\frac{1}{2}} = (8513.36)^{\frac{1}{2}}$

$\quad\quad = \underline{92.268'}$

EQN(16) $\quad \theta_c = \arctan\left(Y_c / X_c\right) = \arctan\left(\frac{15.966}{90.876}\right) = \arctan(.17569)$

$\quad\quad = \underline{9.964°}$

EQN(17) $\quad \alpha = \theta_A - \theta_C = 13.348 - 9.964 = \underline{3.383°}$

EQN(18) $\quad l_A = l_2\left[\frac{l_2}{OAA} - 1\right] = 93.64\left[\frac{93.64}{9.096} - 1\right] = 870.35''$

EQN(19) $\quad l_B = l_1\left[\frac{l_1}{O_B B} - 1\right] = 93.11\left[\frac{93.11}{12.077} - 1\right] = 624.740''$

NOW use eqN 4 to find l_c —

$$l_c = l_A \cos\alpha - \frac{\sin\alpha}{\sin\beta}\left[l_A \cos\beta - l_B\right]$$

$$l_c = \underbrace{(870.35)(\cos(3.383))}_{868.833} - \frac{\sin(3.383)}{\sin(3.816)}\underbrace{\left[870.35 \cos(3.816) - 624.740\right]}_{243.680}$$

$l_c = 652.76''$

NOW use $\underline{\text{EQN 20}}$ for OC-C

$$OC\text{-}C = l_3^2 / l_3 + l_c = \frac{(92.268)^2}{92.268 + 652.76} = 11.43$$

EQN 21

$X_{OC} = X_c - (OCC)\cos\theta_c = 90.876 - (11.43)\cos(9.964°) = 79.618''$

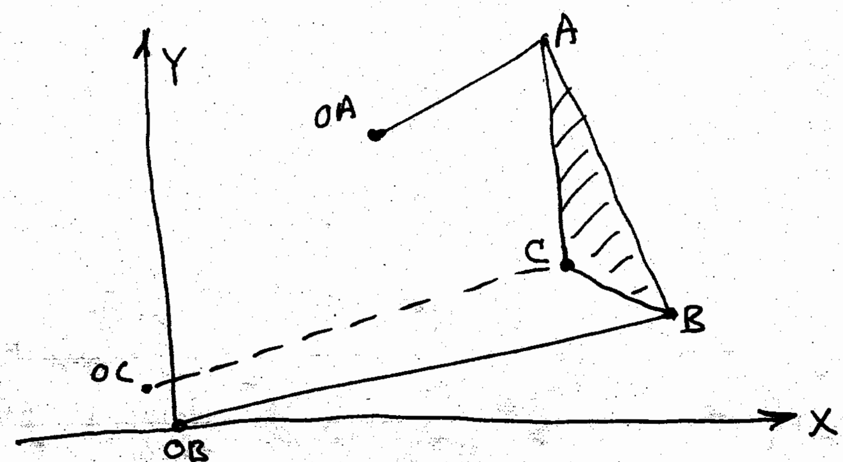
$Y_{OC} = Y_c - (OC\text{-}C)\sin\theta_c = 15.966 - (11.43)\sin(9.964°) = 13.988''$

Then Eqn 22 —

$$X_{oc} = \bar{X}_{oc} - \bar{X}_{oB} = 79.618 - 79.914 = -0.296$$
$$Y_{oc} = \bar{Y}_{oc} - \bar{Y}_{oB} = 13.988 - 13.419 = 0.569$$

so The TIE Rod length projected onto the lateral plane is $\overline{OC}C$ = 11.43"

and OC should be located as follows:



$X_{oc} = -0.296$
$Y_{oc} = 0.569$