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SPACE FLIGHT HANDBOOKS

Volume 2

Lunar Flight Handbook

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

SPACE FLIGHT HANDBOOKS

Volume 2

Lunar Flight Handbook

PART 3 - MISSION PLANNING

Prepared for the
GEORGE C.
MARSHALL SPACE FLIGHT CENTER
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FOREWORD

This handbook has been produced by the Space Systems Division of the Martin Company under contract NAS8-5031 with the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration. The Lunar Flight Handbook is considered the second in a series of volumes by various contractors, sponsored by MSFC, treating the dynamics of space flight in a variety of aspects of interest to the mission designer and evaluator. The primary purpose of these books is to serve as a basic tool in preliminary mission planning. In condensed form they provide background data and material collected through several years of intensive studies in each space mission area, such as earth orbital flight, lunar flight, and interplanetary flight.

Volume II, the present volume, is concerned with lunar missions. The volume consists of three parts presented in three separate books. The parts are:

Part 1 - Background Material
Part 2 - Lunar Mission Phases
Part 3 - Mission Planning

The Martin Company Program Manager for this project has been Jorgen Jensen; George Townsend has been Technical Director. Fred Martikan has had the direct responsibility for the coordination of this volume; he has shared the responsibility for the generation of material with Frank Santora.

Additional contributors were Robert Salinger, Donald Kraft, Thomas Garceau, Andrew Jazwinski and Lloyd Emery. The graphical work has been prepared by Dieter Kuhn and Elsie M. Smith. John Magnus has assisted in preparing the handbook for publication. William Pragluski, Don Novak, James Porter, Edward Markson, Sidney Roedel, Wade Foy and James Tyler have made helpful suggestions during the writing of this book.

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Volume II, Part 3 - Mission Planning

The preceding contents are Part 3 of Volume II. The remaining two parts of Volume II contain the following:

Volume II, Part 1 - Background Material

Volume II, Part 2 - Lunar Mission Phases

CHAPTER XI

MISSION PLANNING

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March 1963

Chapters VI and IX have catalogued a large quantity of generalized lunar trajectory data. It is the purpose of this chapter to present the methods whereby this generalized material can be interpreted and used for specific mission dates.

Section A presents the equations for converting the selenographic coordinates of a particular lunar site or area to coordinates relative to the moon's orbital plane (MOP) and the moon-earth line ($\overrightarrow{MEL}$). In addition, the equations determine the required orientations (i_m, θ_M) of the possible circular orbits around the moon that pass over the site in question. These equations are very important and should be mechanized on a computer as soon as possible, since the $i_m - \theta_M$ relationship is essentially a boundary condition imposed by the particular mission that must be satisfied by the generalized trajectory data.

In Section B, the lunar terminator is defined and equations giving the orientation of the terminator in both the selenographic and MOP-$\overrightarrow{MEL}$ coordinate systems are presented. It is also shown how the lunar libration equations (Chapter III, Section C) are readily adapted to the calculation of the orientation of the terminator.

Section C records the declination, radial distance and phases of the moon from 1963 to 1971. This data forms the connection between the generalized material of the trajectory catalogues and specific mission dates, as will be illustrated in the sample missions, while the conversion between the two can be accomplished by means of the figures in Section D.

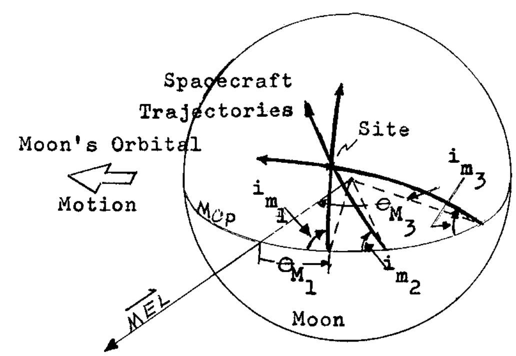
Section E presents empirical equations that extend the catalogued data to other injection altitudes and flight path angles (also given in Chapter VI, Section A), while still other equations are given that improve the accuracy of converting the generalized data to specific dates. This section also includes the energy requirements for controlling return flight time and changing the transearth trajectory inclination.

Section F, also used extensively for mission planning, compiles the generalized geometrical relationships (i_{VTL}, i_{VTE}, i_m, θ_M) for a given set of mission constraints ($R_{\oplus☾}$, h_{PL} or t_p) onto one chart. These summary charts are referred to as "mission planning envelopes" and are provided for both the circumlunar and approach class trajectories. These envelopes are used primarily to determine what combination of generalized trajectory parameters satisfy the boundary conditions (required $i_m - \theta_M$) generated by Section A.

Finally, Section G presents two sample missions that illustrate the procedures and steps to be followed when evaluating specific missions. The specific missions selected are a manned lunar landing and return mission and an unmanned photo-reconnaissance mission. The choice of these two missions necessitates an evaluation of material from both catalogues.

A. REQUIRED ORIENTATION OF TRANSLUNAR AND TRANSEARTH TRAJECTORIES RELATIVE TO THE MOP

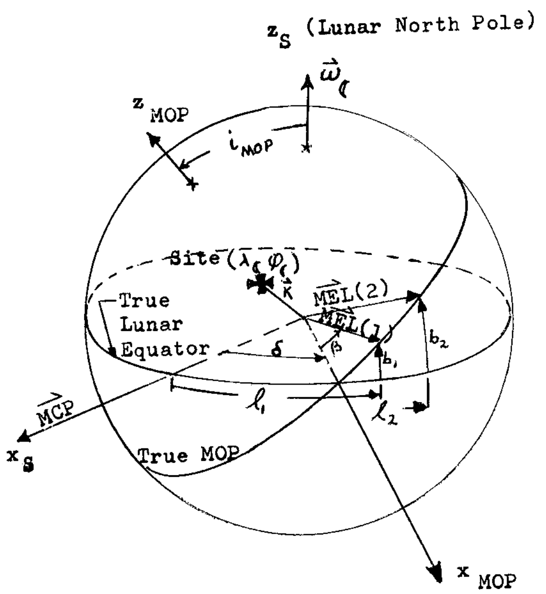
In Chapters VI and IX, where translunar and transearth trajectory data were catalogued, it was seen that on arrival at the moon from a translunar trajectory, a certain trajectory orientation (i_m, θ_M) relative to the moon existed, and that the orientation varied as the mission constraints (h_{PL}, i_{VTL}, i_{VTE}, etc.) changed. In like manner, transearth trajectories departing from the moon also have an unlimited number of possible orientations. Thus, the question is, what combination of i_m, θ_M is required to orbit over, land at, or launch from a specific lunar site? In other words, trajectories arbitrarily selected will result in i_m, θ_M relationships other than the required combination, and therefore the mission specification of acquiring a specific lunar site will not be met. The required combination or relationship is easily visualized through use of the following sketch.



In this sketch, the moon-earth line $\overrightarrow{MEL}$ is shown at the time the spacecraft trajectory is established, which may occur at the time of pericynthion of the translunar trajectory, at lunar liftoff, or at departure from the moon. The desired landing site or reconnaissance area as indicated by the cross can be located relative to the MOP fundamental plane and the $\overrightarrow{MEL}$ direction. An unlimited number of spacecraft trajectories that acquire the site are possible; however, a unique relationship exists between the inclination i_m and orientation θ_M of which only three possibilities are shown in the sketch (i_{m_1}, θ_{M_1}; i_{m_2}, θ_{M_2}; i_{m_3}, θ_{M_3}).

Before this i_m, θ_M relationship can be determined, the position of the site relative to the $\overrightarrow{MEL}$ and MOP on the selected mission date must be determined from its location as given by its selenographic coordinates on a lunar map. The following discussion presents a method for determining the i_m, θ_M relationship if the selenographic coordinates of the site and the mission date are known.

In Chapter III, Section C, it is pointed out how a site on the moon continually changes its position relative to the MOP because of the lunar librations. It was shown there how the total lunar librations in latitude (b) and longitude (ℓ) of the $\overrightarrow{MEL}$ may be determined by means of lunar ephemeris data. Lunar librations are measured relative to the mean center point (MCP) of the moon, which is the intersection of the prime meridian and the true lunar equator as shown in the next sketch, with positive values for ℓ and b indicated:



Define a coordinate system x_{MOP} y_{MOP} z_{MOP}, with the z_{MOP}-axis normal to the MOP in the general direction of the lunar angular velocity vector $\omega_☾$, the x_{MOP}-axis defining the line of intersection between the true MOP and the true lunar equator and in the general direction of the $\overrightarrow{MCP}$, and the y_{MOP}-axis completing the right-handed Cartesian coordinate system in the MOP.

In order to find the inclination of the true MOP with respect to the true lunar equator, let T be the Julian date on which the spacecraft trajectory is established, and ℓ_1 and b_1 the total librations of the $\overrightarrow{MEL}(1)$ at this date. Next, let $\overrightarrow{MEL}(2)$ be directed to the earth's position (ℓ_2, b_2) at an earlier date, but as seen from the moon at T (ℓ_2 and b_2 may be found by using the equations presented in Chapter III, Section C, for the date T but with the position coordinates of the moon $x_{\oplus☾}$, $y_{\oplus☾}$, $z_{\oplus☾}$ for one-half day earlier). The two vectors $\overrightarrow{MEL}(1)$ and $\overrightarrow{MEL}(2)$ define the true earth orbital plane (EOP) about the moon. This plane may also be interpreted as the true MOP whose descending node with the true lunar equatorial plane is given by the x_{MOP}-axis.

The vector $\vec{z}_{MOP}$, which is normal to the MOP and hence identical with $\vec{z}_M$, is then defined in the selenographic coordinate system by the following determinant (see preceding sketch):

$$\vec{z}_{MOP} = \begin{vmatrix} \hat{x}_S & \hat{y}_S & \hat{z}_S \\ \cos\ell_1 \cos b_1 & \sin\ell_1 \cos b_1 & \sin b_1 \\ \cos\ell_2 \cos b_2 & \sin\ell_2 \cos b_2 & \sin b_2 \end{vmatrix} \tag{1}$$

The inclination i_{MOP} of the true MOP with respect to the true lunar equator can now be determined from Eq (1):

$$i_{MOP} = \cos^{-1}\left(\frac{\vec{z}_{MOP} \cdot \hat{z}_S}{z_{MOP}}\right) \tag{2}$$

Next, the angle δ, measured from the $\overrightarrow{MCP}$ to the x_{MOP} axis, is given by:

$$\delta = \cos^{-1}\left(\frac{\hat{z}_S \times \vec{z}_{MOP} \cdot \hat{x}_S}{|\hat{z}_S \times \vec{z}_{MOP}|}\right) \tag{3}$$

and β, measured from the x_{MOP} axis to $\overrightarrow{MEL}(1)$, by:

$$\beta = \cos^{-1}\left(\frac{\hat{z}_S \times \vec{z}_{MOP} \cdot \overrightarrow{MEL}(1)}{|\hat{z}_S \times \vec{z}_{MOP}|}\right) \tag{4}$$

By rotating the x_S y_S z_S coordinate system through the three positive angles δ, i_{MOP} and β, in that order, the unit vector in the direction of a specific lunar feature, $\hat{K}$, may be found. Denote the components of $\hat{K}$ relative to the $\overrightarrow{MEL}$ and MOP coordinate system by K_1 in the $\hat{x}_{MOP}$ direction, by K_2 in the $\hat{y}_{MOP}$ direction and by K_3 in the $\hat{z}_{MOP}$ direction. The components K_1, K_2 and K_3 in terms of selenographic longitude and latitude are given by the following matrix multiplication:

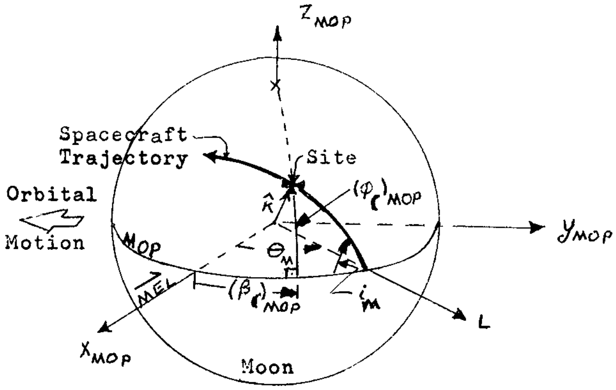
$$\begin{Bmatrix} K_1 \\ K_2 \\ K_3 \end{Bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i_{MOP} & \sin i_{MOP} \\ 0 & -\sin i_{MOP} & \cos i_{MOP} \end{bmatrix} \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \cos\phi_☾ \cos\lambda_☾ \\ \cos\phi_☾ \sin\lambda_☾ \\ \sin\phi_☾ \end{Bmatrix} \tag{5}$$

where:

$$\hat{K} = K_1 \hat{x}_{MOP} + K_2 \hat{y}_{MOP} + K_3 \hat{z}_{MOP}$$

and $\lambda_{☾}$ and $\phi_{☾}$ are the selenographic coordinates of the lunar site.

Define a latitude and longitude relative to the MOP as a fundamental plane and the x_{MOP}, z_{MOP} plane defining the prime meridian. In terms of:



K_1, K_2, K_3, the latitude $(\phi_{☾})_{MOP}$ of the site relative to the $\overrightarrow{MEL}$ and MOP is given by:

$$(\phi_{☾})_{MOP} = 90° - \cos^{-1}(K_3),$$

$$-90° \leq (\phi_{☾})_{MOP} \leq +90° \qquad (6)$$

and its longitude by

$$(\lambda_{☾})_{MOP} = \cos^{-1}\left(\frac{K_1^2}{K_1^2 + K_2^2}\right)^{1/2},$$

$$-180° \leq (\lambda_{☾})_{MOP} \leq +180° \qquad (7)$$

It remains to find the i_m, θ_M relationship for a given lunar site and for a given date T. First define a unit vector $\hat{L}$ in the MOP by assuming a required value of θ_M.

$$\hat{L} = \cos\theta_M \hat{x}_{MOP} + \sin\theta_M \hat{y}_{MOP} \qquad (8)$$

Then the corresponding i_m required is:

$$i_m = \cos^{-1}\left(\frac{\hat{K} \times \hat{L} \cdot \hat{z}_{MOP}}{|\hat{K} \times \hat{L}|}\right)$$

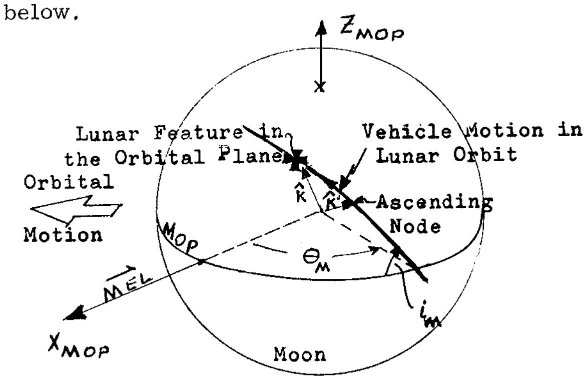
Equations (8) and (9) give the desired i_m θ_M relationship.

This approach is best suited for lunar landing missions, lunar orbit missions (reconnaissance and surveillance), and lunar departure missions that do not require many lunar orbits or rendezvous with other vehicles already in lunar orbit.

For missions where lunar rendezvous is required, such as the lunar ascent phase of the "shuttle" concept (Chapter VIII) or for missions where the spacecraft spends some time in orbit around the moon prior to leaving, the approach of Eqs (8) and (9) must be modified because lunar orbits for these missions will most likely be computed in a selenocentric or selenographic coordinate system. This implies that instead of having a range of required i_m, θ_M relationships, there is only one required i_m-θ_M combination which has to be satisfied for the specific value of orbital inclination i_{VM} relative to the true lunar equator, and longitude of the ascending node Ω_{MOP} relative to the MOP in the true equatorial plane of the moon.

In order to calculate this combination, it can be assumed that the lunar site, referred to above, is a specific lunar feature that lies below the orbital path or a position calculated from the known inclination (i_{VM}) and the longitude Ω_{MOP} of the ascending node.

If Eq (4) is solved for both the calculated position and the coordinates of the ascending node (Ω_{MOP}, 0), two positions relative to the $\overrightarrow{MEL}$ and MOP are obtained, namely, $\hat{K}$ and $\hat{K}'$, as shown below.



Therefore, the required i_m is:

$$i_m = \cos^{-1}\left(\frac{\hat{K} \times \hat{K}' \cdot \hat{z}_{MOP}}{|\hat{K} \times \hat{K}'|}\right)$$

and

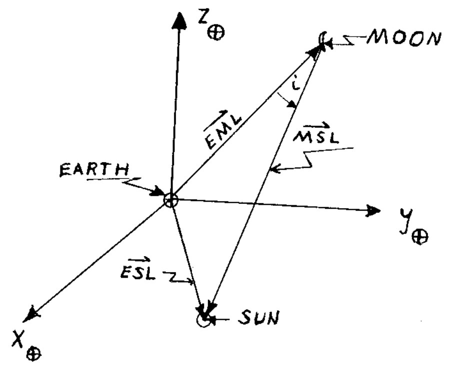
$$\theta_M = \cos^{-1}\left(\frac{(\hat{K} \times \hat{K}') \times \hat{z}_{MOP} \cdot \hat{x}_{MOP}}{|(\hat{K} \times \hat{K}') \times \hat{z}_{MOP}|}\right)$$

It is suggested that the equations given in this section and in Section C of Chapter III be mechanized on a computer so that full usefulness of the lunar handbook can be realized. The need for mechanization will become apparent in the sample missions (Section G).

B. LUNAR TERMINATOR

In the planning of lunar missions, the lighting conditions and phases (a phase is defined as the fraction of the area of the apparent disk of the moon illuminated by the sun as observed from earth) play a major role. The importance of the lighting conditions arises from the fact that daylight is desirable for exploration and that the temperature of the lunar surface depends strongly on the orientation of the particular feature relative to the sun due to the slow rotational rate of the moon around its axis (see Chapter II, Section B).

The lunar phase depends on a quantity defined as planetocentric elongation of the earth from the sun, and it is referred to as the "phase angle" which is denoted by i in the ephemeris and illustrated in the following sketch, where $x_\oplus\, y_\oplus\, z_\oplus$ is the equatorial coordinate system defined in Subsection A-1c of Chapter III.



The four cardinal phases are: (1) new moon, (2) first quarter,(3) full moon, and (4) last quarter. They occur when the phase angle, i, is equal to 180°, 90°, 0° and 90°, respectively.

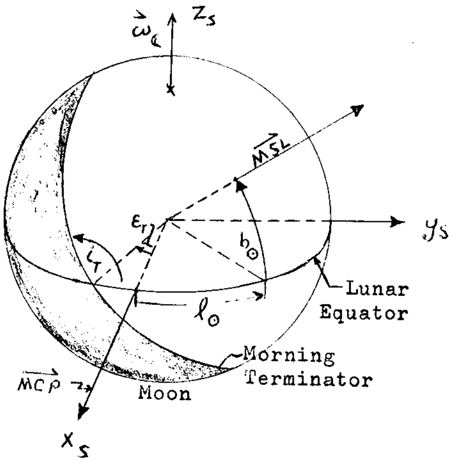
If the small triaxiality of the moon is neglected, the apparent lunar disk is circular. The terminator is defined as the orthogonal projection, onto a plane perpendicular to the line of sight, of the great circle that bounds the illuminated hemisphere of the moon. The terminator is therefore, in general, an ellipse, reducing to a straight line at i = 90° and becoming a circle at i = 0° or 180°. Although positions of the terminator can be obtained for this decade from the U.S. Naval Observatory, it may be desirable to produce data of this sort from a subroutine incorporated in numerically computed digital trajectory programs. Since these programs generally use stored positional data of the sun and moon in geocentric rectangular coordinates $x_\oplus\, y_\oplus\, z_\oplus$, the following approach can be used to determine the orientation of the lunar terminator.

In the previous sketch, the positions of the moon ($\overrightarrow{EML}$) and sun ($\overrightarrow{ESL}$) are given in geocentric mean equatorial coordinates with $x_\oplus$ in the direction of the mean equinox of date. For a given Julian date T, the moon-sun line $\overrightarrow{MSL}$ can be found through vector addition

$$\overrightarrow{MSL} = \overrightarrow{ESL} - \overrightarrow{EML} \tag{10}$$

Now if the $\overrightarrow{MSL}$ is used in place of the $\overrightarrow{EML}$ in the libration equations in Chapter III, Section C, the selenographic position of the sun can be obtained.

This position in longitude and latitude is denoted by $\ell_\odot$ and $b_\odot$ respectively, and is shown below with positive values for $\ell_\odot$ and $b_\odot$ illustrated:



The orientation of the morning terminator, or portion of the moon just to be illuminated by the sun after the lunar night, is defined by the angles i_T and ϵ_T. These angles, illustrated above, can be found by use of Eqs (1) and (2) and noting that

$$\hat{MSL} = \cos \ell_\odot \cos b_\odot \hat{x}_S + \sin \ell_\odot \cos b_\odot \hat{y}_S + \cos b_\odot \hat{z}_S \tag{11}$$

$$i_T = 90° + b_\odot \tag{12}$$

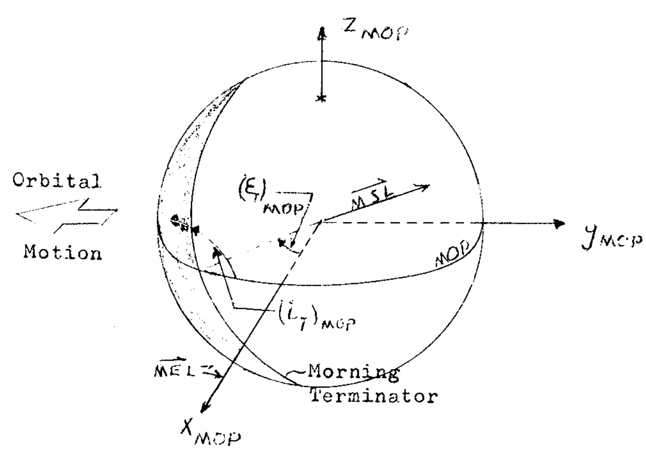
where:

$$0° \leq i_T \leq 180°$$

$$\epsilon_T = \cos^{-1}\left(\frac{\hat{MSL} \times \hat{z}_S \cdot \hat{x}_S}{|\hat{MSL} \times \hat{z}_S|}\right) \quad 0° \leq \epsilon_T \leq 360° \tag{13}$$

The orientation of the morning terminator relative to the $\overrightarrow{MEL}$ and MOP is determined by replacing the selenographic longitude and latitude of the site, $\lambda_{\text{☾}}$, $\phi_{\text{☾}}$, in Eq (5) of the previous section by $\ell_\odot$ and $b_\odot$ and obtaining the unit vector $\hat{K}$, which now is the direction of the sun as seen from the center of the moon relative to the $x_{MOP}\, y_{MOP}\, z_{MOP}$ coordinate system:

$$\hat{K} \equiv \hat{MSL} \tag{14}$$



Then

$$(i_T)_{MOP} = 180 - \cos^{-1} (\hat{z}_S \cdot \hat{MSL}),$$

$$0° \leq (i_T)_{MOP} \leq 180° \qquad (15)$$

and

$$(\epsilon_T)_{MOP} = \cos^{-1} \left(\frac{\hat{MSL} \times \hat{z}_S \cdot \hat{x}_S}{|\hat{MSL} \times \hat{z}_S|} \right)$$

$$0° \leq (\epsilon_T)_{MOP} \leq 360° \qquad (16)$$

Of course, the location of the evening terminator is given by $\epsilon_T \pm 180°$ or $(\epsilon_T)_{MOP} \pm 180°$, and it is the portion of the moon just to fall into the lunar night. The inclination i_T of the evening terminator is identical to that of the morning terminator.

C. DECLINATION, RADIAL DISTANCE, AND PHASES OF THE MOON

This section presents the lunar declination δ_M, which has been denoted by $\delta_{☾}$ up to now in the handbook, the radial distance R_M, previously designated as $R_{\oplus ☾}$, of the moon from the earth, and the phases of the moon (as previously described in Section B). The data has been taken directly from Woolston (Ref. 1), since it is in a form which permits a rapid approximate determination of the combination of declination and lighting for any calendar date for mission planning purposes.

The times of new moon, first quarter, full moon, and last quarter, as determined by the Nautical Almanac Office of the U. S. Naval Observatory are "the times at which the excess of the apparent longitude of the moon over the apparent longitude of the sun is 0°, 90°, 180°, and 270°, respectively." In the data presented herein, the determination of the phases has been approximated by comparing the geometric position of the sun with the apparent position of the moon. While reduction of the sun's geometric longitude to apparent longitude should take into account aberration and the precession and nutation in longitude, only the precession has been considered. For the period of interest, the combined contributions of nutation and aberration have been examined and it has been found that they should not exceed approximately 30" corresponding to a discrepancy in time of the moon's phase of about one minute.

A related approximation involves the use of a mean obliquity of the ecliptic at the beginning of each year rather than true obliquity in transforming from the sun's geocentric equatorial rectangular coordinates $x_{\oplus\odot}$ $y_{\oplus\odot}$ $z_{\oplus\odot}$ to the longitude. For present purposes, its effect is negligible.

Data is given in tabular and graphical form. Table 1 serves simply to identify a particular month and date with the corresponding number of the day in the year after January 0. It is intended to be used in conjunction with the plots of the declination and distance of the moon. Two forms are given: one for nonleap years, the other for leap years.

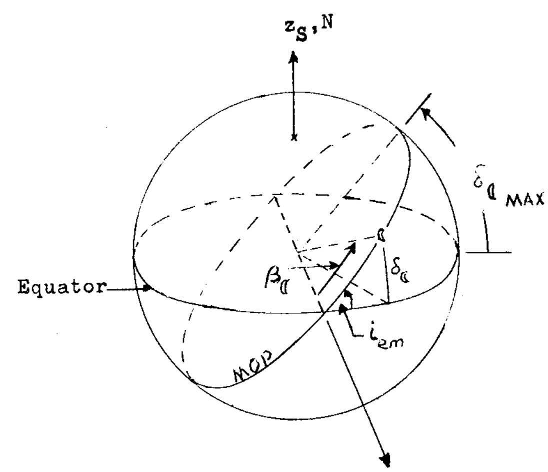
Tables 2 (a) to 2 (i) contain the calculated universal time (UT) of the phases of the moon for the years 1963 to 1971, respectively.

Figures 1 to 9 present graphs for 1963 to 1971, respectively, of the radial distance R_M (in earth radii) and declination δ_M as a function of the day of the year, numbered from January 0. Included on the graphs of the declination are symbols indicating the cardinal phases of the moon. In these tables and figures January 0, 0 hr UT corresponds to December 31, 0 hr UT of the previous year; e.g. January 9.5 is January 9, 12 hr UT.

D. CONVERSION OF GENERAL TRAJECTORY DATA TO SPECIFIC DATES

All the material in the catalogues of Chapters VI and IX is presented relative to the MOP (which, as a plane of symmetry, is independent of the orientation of the earth's equatorial plane relative to the MOP). However, for a specific date, there is a unique orientation of the equatorial plane with respect to the MOP, and the generalized trajectory data must be interpreted for that date if a mission is to be analyzed properly.

The first step is to determine the inclination of the MOP with respect to the earth's equatorial plane i_{em} and the position of the moon in its orbit. The inclination, i_{em}, can be found by means of ephemeris data, or, less accurately, by means of Figs. 1 to 9. For purposes of preliminary mission planning, the figures are adequate, and i_{em} is found by taking the maximum declination of the moon for the year and month in question ($i_{em} \equiv \delta_{☾ max}$). This is shown in the following sketch of the celestial sphere.

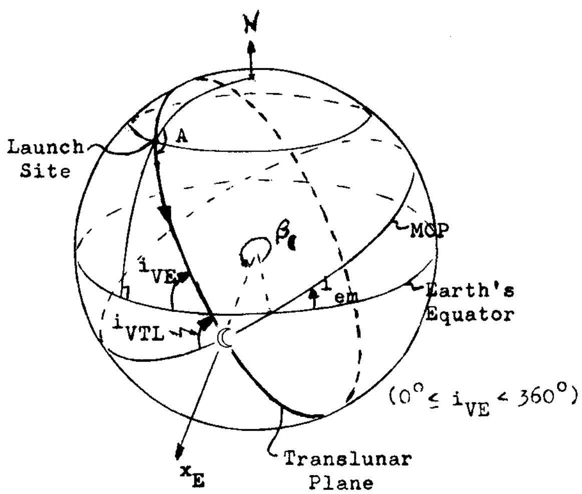


The actual declination of the moon at the date of interest is $\delta_{☾}$, and its position in orbit is given by the central angle $\beta_{☾}$ $\delta_{☾}$ is also obtained from Figs. 1 to 9, and $\beta_{☾}$ can be found in Fig. 10 or from

$$\beta_{\mathbb{C}} = \sin^{-1}\left(\frac{\sin \delta_{\mathbb{C}}}{\sin i_{em}}\right) \quad (17)$$

In Fig. 10, $\beta_{\mathbb{C}}$ is given from 0 to 360° where 0 and 180° represent the ascending and descending nodes, respectively, and 90° and 270° the maximum northerly and southerly declinations, respectively.

As was pointed out in Chapter VI, the intersection of the translunar plane with the MOP is very nearly along the earth-moon line ($\hat{EML}$) at the date of pericynthion. Assuming, for the time being, that this intersection is identical with the $\hat{EML}$, the next sketch can be drawn.



The inertial launch azimuth, A, is given by the following equation:

$$A = \sin^{-1}\left(\frac{\cos i_{VE}}{\cos \phi'_L}\right) \quad (18)$$

ϕ'_L geocentric latitude of the earth launch site

If it is assumed that launches from Cape Canaveral are restricted in azimuth (70° < A < 110°), then the resulting i_{VE} will be limited to 28.5° ≤ i_{VE} ≤ 34.5°. However, the inclination of the translunar plane, i_{VTL}, varies considerably depending on the time of lunar month, i.e., lunar position $\beta_{\mathbb{C}}$. Figure 11 presents the value of i_{VTL} as a function of $\beta_{\mathbb{C}}$ for i_{VE} = 28.5°, 31.5° and 34.5° for the years 1965 and 1966. Figure 12 shows the same information for the years 1967 to 1971. The highest i_{VTL} that can be encountered for the above azimuth restrictions is 63° and the lowest is near 0°. Thus, for a given date of pericynthion the catalogue data (Chapters VI and IX) can be entered much more discriminatingly for mission analysis, as will be demonstrated in the sample missions (Section G).

In like manner, when returning to earth from a lunar mission it is highly advantageous to be able to convert i_{VTL} to the return trajectory inclination i_{VTEQ} relative to the earth's equator, or vice versa. Figures 13 and 14 present i_{VTE} as a function of $\beta_{\mathbb{C}}$ for various i_{VTEQ}.

In the above discussion, it was assumed that the intersection of the translunar and transearth planes with the MOP is coincident with the EML at the date of pericynthion or date of transearth injection from lunar orbit. This assumption is not exactly true, since, in general, the actual intersection is ($\beta_{\mathbb{C}} + \Delta\Phi$). The next section presents empirical equations whereby $\Delta\Phi$ can be estimated. It has been found that $\Delta\Phi_{TL}$ < 1° for $|i_{VTL}|$ > 15° and $\Delta\Phi_{TE}$ < 2° for $|i_{VTE}|$ > 15°.

E. EMPIRICAL EQUATIONS AND AUXILIARY DATA FOR USE IN MISSION PLANNING

1. Empirical Equations

There are four empirical equations that are used with the catalogued data of Chapters VI and IX for mission planning purposes. Two equations are utilized for extending the catalogued material to other mission constraints not given in the catalogues, and two other equations are used to increase the accuracy of analysis of specific missions.

The first two equations were previously given in Chapter VI and are repeated here for convenience.

$$\psi'_0 = \psi_0 + 2\,\Delta\gamma_0 \quad (19)$$

Since the translunar catalogue data presented in Chapters VI and IX are given for an injection flight path angle of γ_0 = 5° and 0°, respectively, Eq (19) is used to extend the value of γ_0 to a value γ_0'. The only trajectory variable that is significantly affected by the change in γ_0 is ψ_0, the injection position. Therefore, Eq (19) gives the new value of injection position, ψ_0', for γ_0' where $\Delta\gamma_0$ is $(\gamma_0' - \gamma_0)$.

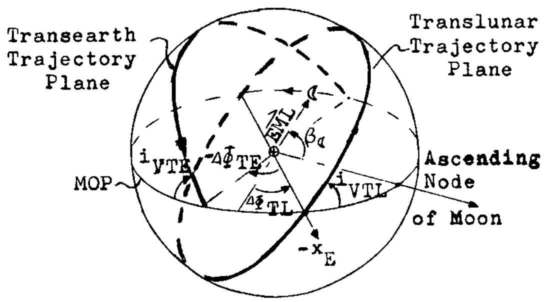
$$V_0' = \left[V_0^2 - V_p^2\left(\frac{\Delta h_0}{r_0}\right)\right]^{\frac{1}{2}} \quad (20)$$

It is also desirable to extend the translunar catalogue data to other injection altitudes h_0', since in Chapter VI the injection altitude, h_0, is constant (250 km), and likewise in Chapter IX, the catalogue data is presented for h_0 = 183 km. With a variation in h_0, the only trajectory parameter significantly affected is V_0, the injection velocity. Equation (20) gives V_0' the injection velocity corresponding to h_0' where $\Delta h_0 = h_0' - h_0$, r_0 is the radius from the earth's center to h_0 and V_p is the 2-body parabolic speed at h_0'.

The practical limits of the above two equations are $\Delta\gamma_0 \lessapprox \pm 20°$ and $\Delta h_0 \lessapprox \pm 1750$ km.

The next two empirical equations allow a more accurate evaluation of the generalized catalogue material for specific dates of interest.

It was mentioned in Section D above that the intersection of the translunar or transearth plane with the MOP is adjusted by an amount $\Delta\Phi$ to be algebraically added to $\beta_☾$. This adjustment is usually ignored for $|i_{VTL}| > 15°$, $|i_{VTE}| > 15°$ but should be included for inclinations < 15°.



For translunar trajectories $\Delta\Phi = \Delta\Phi_{TL}$.

$$\Delta\Phi_{TL} = \pm (0.22 + 0.000156\, h_{PL})\, C_0 \sin i_{VTE} + 1.0 \tag{21}$$

use + for inject south

use - for inject north

where

h_{PL} is measured in kilometers

C_0 is a constant determined from Fig. 15.

Therefore the intersection of the translunar plane with the MOP (which is the Voice x_E-axis) is located $\beta_☾ + \Delta\Phi_{TL}$ from the ascending node of the moon's orbit around the earth.

For the case of circumlunar trajectories, the intersection of the transearth plane will not in general be coincident with the x_E-axis as shown in the above sketch, but it is located at a position $\beta_☾ + \Delta\Phi_{TL} + \Delta\Phi_{TE}$, where $\Delta\Phi_{TE}$ is the adjustment required and is given by Eq (22):

$$\Delta\Phi_{TE} = \pm (0.232 + 0.0001537\, h_{PL})\, D_0 \sin i_{VTL} - E_0 \tag{22}$$

use + for inject north

use - for inject south

$E_0 = 1.7 + 0.0375\,(R_{\oplus☾} - 56.0)$

$R_{\oplus☾}$ is measured in earth radii, h_{PL} in km

and

D_0 is obtained from Fig. 16.

Equations (21) and (22) apply only to circumlunar trajectories catalogued in Chapter VI and for $i_{VTL} \gtrapprox 2°$. For the approach class of translunar injections in Chapter IX, similar equations for $|i_{VTL}| < 15°$ are not available at this time, and the data can be used with relatively good accuracy for $|i_{VTL}| \gtrapprox 10°$ in most cases.

2. Auxiliary Data

The auxiliary data consists of the characteristic velocity impulse, ΔV, that is required to conduct certain maneuvers during transearth flight. These maneuvers may be necessary for timing the perigee, for landing site acquisition, or changing the transearth inclination for tracking purposes, or both, or they may even be interpreted as abort maneuvers.

The first maneuver involves controlling the time of arrival at vacuum perigee (h_{PE} = 45.72 km) without altering the transearth plane inclination. The maneuver is executed by applying a velocity impulse in a retarding direction ($-\Delta V_{TE}$) to delay the arrival or in an accelerating direction ($+\Delta V_{TE}$) to hasten the perigee arrival time. Figure 17 presents the ΔV_{TE} requirement to delay or hasten the time of perigee for a typical transearth trajectory having an eccentricity of 0.975, which is representative of the majority of lunar return trajectories. The data are presented for three different $R_{\oplus\Delta}$--namely, 20 ER, 30 ER and 40 ER--at which the maneuver is conducted. Δt in the figure represents the time difference between the nominal arrival time and the maneuver adjusted arrival time:

$$\Delta t = t_{nominal} - t_{maneuver}$$

The data was limited to eccentricities less than 1 after this maneuver.

A second maneuver that may be performed on the return flight alters the inclination of the transearth trajectory plane, i_{VTE}. In this maneuver, the time to arrive at vacuum perigee from the maneuver point is not altered. In other words, the maneuver deflects the velocity vector $\vec{V}_{\oplus\Delta}$ and does not change its magnitude or flight path angle, γ. Figure 18 shows the impulse ΔV_i required to change i_{VTE} by an amount Δi_{VTE}. Again, the eccentricity of the transearth trajectory is assumed to be 0.975, and the distances selected, at which the maneuver is conducted, are

$R_{\oplus\Delta}$ = 20, 30 and 40 ER. However, when this maneuver is performed there is a shift $\Delta\Phi'_{TE}$ in the intersection of the transearth trajectory plane with the MOP. This corresponding shift with maneuver is found in Figs. 19, 20 and 21 and, thus, the position of the intersection is now $(\beta_{☾} + \Delta\Phi_{TL} + \Delta\Phi_{TE} + \Delta\Phi'_{TE})$.

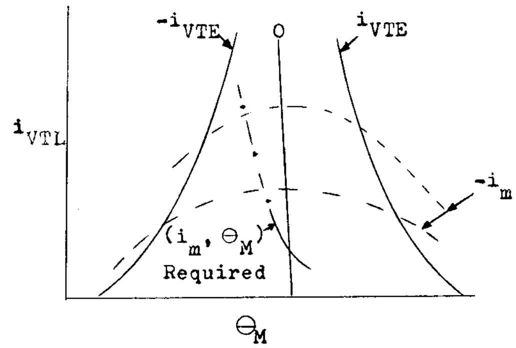
If both a time and i_{VTE} correction are required, then the magnitude of the total velocity impulse as given by vector addition is $(\Delta V^2_{TE} + \Delta V^2_{i})^{1/2}$.

F. MISSION PLANNING ENVELOPES

The catalogued data in Chapters VI and IX refers to either geometrical or "energy" parameters; however, it is the geometric properties at given times which define its usefulness. The text below discusses the importance of displaying these geometric properties in such a manner that they lend themselves readily for mission planning.

1. Circumlunar Envelopes

For circumlunar missions, the geometric parameters involved are i_{VTL}, i_m, θ_M and i_{VTE}, which can be expressed in terms of each other by spherical trigonometry. From Section A of this chapter, it is evident that the combination of i_m and θ_M required to fulfill the mission for a certain date is a major mission constraint. With this in mind, the geometric parameters of i_{VTL}, i_m, θ_M and i_{VTE} are presented in the manner shown in the sketch below and in Figs. 22 to 31. All data has been taken from the circumlunar catalogue in Chapter VI, and it is presented with i_{VTL} as a function of θ_M for the parameter of i_{VTE} in the field of the graph. Superimposed on the figures is the remaining geometrical parameter i_m. Thus the required combination of i_m and θ_M (Section A) could be easily plotted on the same figure and the required i_{VTL} and i_{VTE} relation immediately determined. Each figure is for a specific $R_{\oplus☾}$ and h_{PL} and represents an inject north case. The figures can be interpreted for the inject south case by adding 180° to θ_M and letting i_{VTE} (inject south) = $-i_{VTE}$ (inject north). Notice that to the right of i_{VTE} = 0°, if the injection direction is north, the return direction is also north and if the injection direction is south the return direction is also south. On the left of i_{VTE} = 0°, the return direction is south if the injection direction is north and north if the injection direction is south. Only the direct returns are displayed on the figures.



Below is a table listing the available mission planning envelopes for circumlunar trajectories.

Fig.	$R_{\oplus☾}$ (ER)	h_{PL} (km)
22	56	185.2
23	56	1000
24	56	5000
25	60	185.2
26	60	1000
27	60	3000
28	60	5000
29	64	185.2
30	64	1000
31	64	5000

Application of these envelopes to mission planning is clearly demonstrated in sample mission I of Section G. If desired, mission envelopes for retrograde returns can be generated from the catalogue.

2. Transearth or Translunar Envelopes

For transearth trajectories, with h_{PL} and h_{PE} constant, the geometrical parameters involved are i_m, θ_{MTE} and i_{VTE}. These parameters are plotted in envelopes which present i_{VTE} as a function of θ_{MTE} for values of i_m from 5° to 90°. The data has been obtained from the transearth trajectory catalogue of Chapter IX. For a given $R_{\oplus☾}$, an envelope was determined for each flight time, t_p, of 70, 90 and 110 hr and the envelopes are given in Figs. 32 to 40. They reflect data only for direct returns and the above t_p. If additional envelopes are desired for retrograde returns or other t_p, these cases can also be generated from the transearth catalogue. Listed below are the available envelopes for transearth trajectories.

Fig.	$R_{\oplus ☾}$(ER)	t_p(hr)	
32	56	70	h_{PL} = 185.2 km
33	56	90	h_{PE} = 183 km
34	56	110	
35	60	70	(Transearth injection--west (retrograde))
36	60	90	
37	60	110	
38	64	70	
39	64	90	
40	64	110	

Figures 32 to 40 also provide scales that allow easy interpretation for translunar trajectories. The curves and scales are established through the interpretation rules given in Chapter IX, Subsection A-2. The envelopes are used in the same manner as those representing circumlunar trajectories (Figs. 22 to 31), i.e., the i_m θ_{MTE} or θ_{MTL} required to acquire a certain site or position can be directly drawn on the figures. The use of these figures is again illustrated in sample mission II of Section G.

G. SAMPLE MISSIONS

Chapter IV described various classes of lunar trajectories together with missions that might employ them. Trajectory data has been graphically catalogued for the circumlunar class in Chapter VI and for the approach and return-to-earth class in Chapter IX.

It was noted in Chapter VI that the circumlunar trajectories were restricted to passing the moon relatively close to the MOP ($i_m \lesssim \pm 15°$). If this feature is acceptable, the use of such trajectories is most desirable because of their "safe" nature, especially if the spacecraft is manned or must be recovered.

In Chapter IX it was pointed out that two types of mission parameters cannot be achieved with the circumlunar class, but an approach trajectory is required. For approach trajectories, first, the flight time from earth to the moon or from the moon to earth can be easily varied by two days, and secondly, an area or site anywhere on the lunar surface can be surveyed or explored ($0° \le i_m \le 180°$).

Mission planning not only involves determination of launch dates, lighting conditions, mission profiles, etc., but also includes analysis of the generalized data of the catalogues to establish various parameter trends and tradeoffs and an analysis of how these parameters form or modify the operational concept and system design. For instance, summary plots such as Figs. 7, 8, and 9 of Chapter VII can be generated to determine overall ΔV requirements to enter into a circular lunar orbit. Other summary charts dealing with flight time, injection velocity, etc., are easily obtained, and pertinent aspects of the lunar mission problem can be clearly illustrated. For example, the timing problem of returning to a specific earth site from a circumlunar trajectory, by comparing total flight times with the earth's rotation during this time, indicates that the required pericynthion altitude of the trajectory will vary throughout the lunar month. Also, when the accessibility of lunar landing sites for the approach trajectory class is considered, it can be assumed that for lunar bases at a given latitude relative to the MOP the variation in t_p and ΔV for a variation in descending node of $-30° \le \Delta\theta_{MTL} \le 30°$ is insignificant. This assumption is based on a comparison of t_p and ΔV necessary to land at these lunar sites. Variations in ΔV and t_p are also very small with variation in lunar latitude, and it is apparent that the higher the value of translunar trajectory inclination, i_{VTL} (which is a direct function of the day of the lunar month), the higher the value of ΔV. However, this is not necessarily true if it is acceptable to fly the mission with a different t_p.

This section illustrates that part of mission planning wherein launch and injection dates, lighting conditions as well as mission profiles and requirements are examined. To this end, two sample missions have been presented. The first mission has been chosen to illustrate the use of the circumlunar trajectory catalogue and the iterations and interpolation procedures to follow when catalogued data is not directly applicable. The first mission also illustrates the use of the transearth catalogue of Chapter IX. In the second mission, use of the translunar catalogue of Chapter IX and the empirical equations of Section E is illustrated.

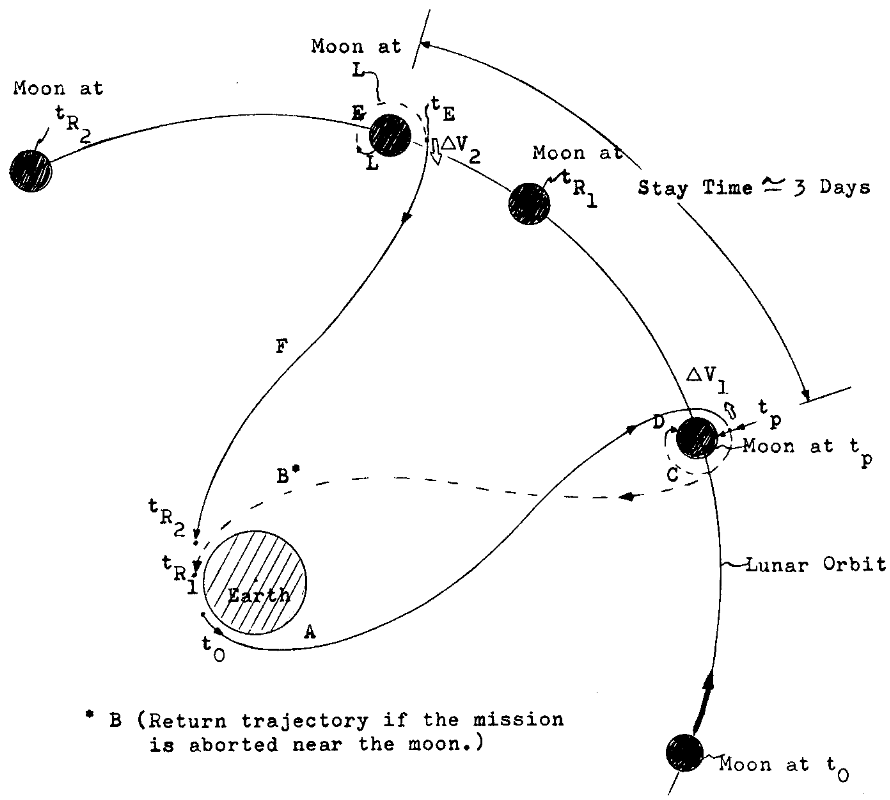
The numerical procedures presented in the sample missions require only a slide rule, thereby allowing rapid evaluation of various aspects of the mission, except for the determination of the required i_m - θ_M relationship given by the equations in Section A. For this part of the mission planning phase it is again emphasized that these equations be programmed on a digital computer, if the planning of specific missions is to be done efficiently.

Although the steps outlined in the sample missions seem lengthy at times, the actual number of arithmetical steps and data manipulations is small.

1. Sample Mission I

Mission: It is desired to conduct a manned lunar mission during October 1966, for the purpose of landing and exploring the Sinus Medii region.

For purposes of visualization, a schematic of the mission is presented below.



Injection into the circumlunar trajectory (A, B) takes place at time t_0. The spacecraft transfers ballistically along the nominal translunar trajectory A until pericynthion is attained at time t_p. At this time the velocity impulse ΔV_1 is applied to reduce the velocity of the spacecraft to the circular orbital velocity around the moon. The spacecraft continues in a circular lunar orbit C. At the proper point in this orbit a descent is made to the specified landing site along trajectory D. However, if for some reason the velocity impulse ΔV_1 cannot be applied, the transearth portion B of the circumlunar trajectory is designed to result in a safe re-entry at t_{R_1} of the spacecraft. (Trajectory B is referred to as the abort trajectory.) Although a safe re-entry is guaranteed in the event the mission is aborted at t_p, the geographical re-entry point may be over remote recovery areas. Therefore, a maneuver at some time on the transearth abort trajectory B should be performed to adjust the re-entry point.

If the mission is not aborted, it is assumed that the lunar landing is executed and that the crew spends approximately three days in exploration. At the end of this lunar stay period, an ascent from the lunar surface is conducted (trajectory L) and a circular parking orbit around the moon, E, established. When the proper point in the parking orbit is reached, the spacecraft is accelerated into the nominal transearth trajectory F by an impulse ΔV_2 at time t_E. The nominal transearth trajectory F allows a spacecraft to perform a re-entry at t_{R_2} along PMR (Pacific Missile Range). The detailed specifications for this mission are as follows:

Specification I. Nominal Translunar Trajectory (A)

(1) A circumlunar class translunar trajectory will be employed.

(2) The spacecraft is launched from Cape Canaveral with launch azimuth restrictions given by $70° \leq A \leq 110°$.

(3) Parking orbits are used prior to translunar injection.

(4) Injection may be either north or south relative to the MOP.

(5) Injection conditions are

$$h_0 = 250 \text{ km}$$

$$\gamma_0 = 5°$$

(6) The pericynthion altitude of the circumlunar trajectory is 185.2 km.

(7) At pericynthion, thrust is applied to decelerate the spacecraft into a circular lunar orbit.

Specification II. Abort Transearth Trajectory (B)

(8) The transearth portion of the circumlunar trajectory (if the lunar orbit is not established) must have an inclination of 31.5 ± 3° relative to the earth's equator and must return "direct."

(9) A maneuver (in order to adjust return inclination and/or time of (8)) on the transearth abort trajectory is permissible.

(10) Atmospheric re-entry at earth for the abort trajectory must be direct.

(11) Re-entry along PMR is specified and preferable, but it may occur along AMR (Atlantic Missile Range) if absolutely necessary.

(12) Vacuum perigee of the transearth trajectory is 46 km.

Specification III. Nominal Transearth Trajectory (F)

(13) The lunar landing is made in daylight and a stay time of approximately three days is desired on the lunar surface.

(14) For return, the spacecraft is launched from the lunar surface into a 185.2 km circular lunar parking orbit.

(15) The lunar launch and ascent to lunar orbit phase is to be conducted in daylight.

(16) There are no requirements for a lunar rendezvous.

(17) The spacecraft is injected from the lunar orbit into the nominal transearth trajectory that has the same specifications as given under headings 8 and 9, and the nominal re-entry occurs along the PMR.

(18) The nominal transearth injection conditions are

$$h_{PL} = 185.2 \text{ km}$$

$$\gamma_{☾0} = 0°$$

Step I: Choose a lunar landing site

The first step in analyzing this mission is to refer to lunar maps on which features of the lunar surface are shown. The maps that are available at present are listed and can be procured from the agencies listed in Chapter III, Subsection A-2g, and a typical lunar aeronautical chart has been reproduced in Chapter II. The objective at this time is to select the selenographic coordinates of the landing site. Although the detail required for the actual planning of missions cannot be obtained with the available maps, they nevertheless are adequate for preliminary analyses.

In the Sinus Medii region (from above referenced maps and from Fig. 1, Chapter III), it appears that a suitable landing site can be located approximately in the center of the region ($\phi_☾ = 0°$, $\lambda_☾ = 0°$). See Chapter III for additional explanation on selenographic coordinates $\phi_☾$, $\lambda_☾$.

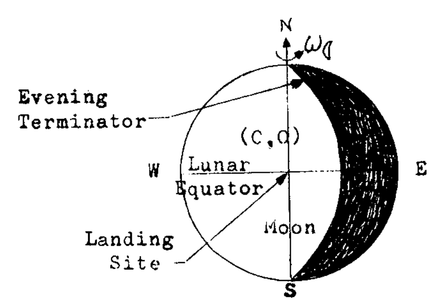
Step II: Choose an arrival date

The next step is to choose an approximate pericynthion arrival date. This can be determined from Section C, which presents the approximate lighting conditions of the moon throughout this decade. From the calendar in the appendix Table 1, showing the number of days in the year after January 0, is seen that for the month of October 1966, the day of the year varies from 274 to 304. Referring next to (Figs. 4a and 4b) showing the phases of the moon for 1966, it is evident that in October 1966 there are two periods in which a daylight lunar landing can be conducted.

(1) Days 274 to 281

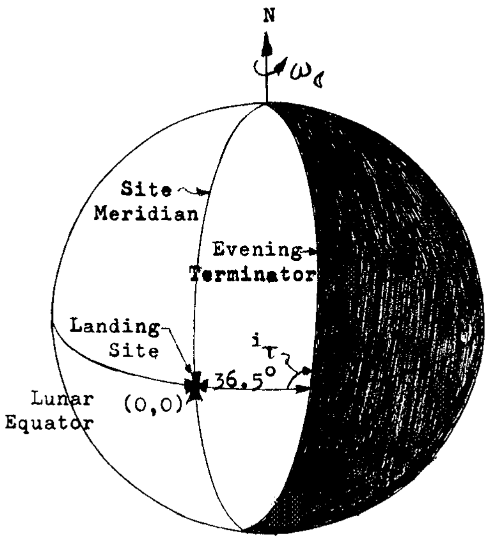
(2) Days 294 to 304.

The moon arrival time is arbitrarily chosen to be in the first period. Knowing that a stay time of about three days is desirable and that a launch from the lunar surface in daylight is specified (see item 15, above), an arrival time of $277^d.00$ is chosen. This time corresponds to 00:00 UT October 4, 1966. Also from Figs. 4a and 4b the declination of the moon δ_M is +20.7°, the inclination i_{em} of the MOP to the equator is 27°, and the earth-moon distance R_M is 61.9 ER. In addition, approximate lighting conditions on the arrival date can be estimated from the figures, as shown below:



The evening terminator is approximately 45° East of the landing site. The exact lighting conditions can also be determined from Section B, if an ephemeris of the sun is available. For comparison purposes, exact lighting is presented below for the $277^d.00$:

The evening terminator is 36.5° E of the landing site and its inclination to the lunar equator is $i_T = 91°$.



Step III: Obtain the translunar trajectory inclination limits

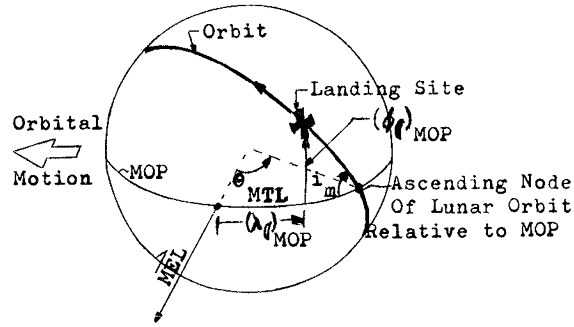
Referring again to Fig. 4a, $277^d_.00$ is between the ascending node and maximum northerly declination of the moon $\delta_M = +20.7°$. With this in mind, and turning to Section D, the central angle, $\beta_☾$, measured from the ascending node to the moon, can be determined. For a $\delta_M = +20.7°$ and $\beta_☾ = 51°$ from Fig. 10. From Fig. 11, the possible ranges of i_{VTL} are found for this (see the following table).

	i_{VE} (deg)	i_{VTL} (injection north) (deg)	i_{VTL} (injection south) (deg)
Minimum	28.5	38.2	3.8
Maximum	34.5	46.0	11.7

These permissible ranges (or limits) are now superimposed on Figs. 15, 18 and 29, the mission planning envelopes for circumlunar trajectories for $h_{PL} = 185.2$ km. Each figure corresponds to a given earth moon distance; i_{VTL} is plotted against θ_M; and i_m and i_{VTE} are parameters. (see Section F for chart explanation.) For purposes of clarification, one of the charts ($R_M = 60$) is repeated at this time in Fig. 41 with the above injection limits noted on it.

Step IV: Determine required i_m, θ_M

Since the data presented on the mission planning charts is relative to the MOP and since the EML does not generally intersect the MOP due to the lunar librations (Chapter III), the flight to the landing site must be carefully planned. For a given arrival date there is a specific θ_M - i_m relationship that allows a lunar orbit to pass over a particular site (see sketch below). This facet of the problem is fully discussed in Section A.



Utilizing the equations presented in Section A, the required θ_M - i_m relationship to ensure passing over the site, after a lunar orbit has been established, is given below in the table for the selenographic landing site coordinates $\lambda_☾ = 0°$, $\phi_☾ = 0°$ at $277^d_.00$. The landing site coordinates for this time relative to the $\overrightarrow{MEL}$ and MOP are $(\lambda_☾)_{MOP} = +5.69°$ and $(\phi_☾)_{MOP} = +1.94°$.

Required θ_M and i_m for Sample Mission I for Moon Arrival at $277^d_.0$

Injection	θ_M (deg)	i_m (deg)
Direct North	10	24.23
	20	7.79
	30	4.70
	40	3.43
	50	2.77
	60	2.39
	70	2.15
	80	2.01
	90	1.95
Direct South	100	1.94
	110	2.00
	120	2.13
	130	2.35
	140	2.71
	150	3.32
	160	4.46

Direct South Injection	θ_M (deg)	i_m (deg)
↓	170	7.13
	180	18.84

The required θ_M, i_m from the above table is now superimposed on the mission planning charts. A sample of this super position is also shown in the special included envelope (Fig. 41). For the allowable i_{VTL} - range, the resulting i_{VTE} of the circumlunar trajectory can be listed as shown below:

i_{VTL} (deg)	$R_{\oplus☾}$ (ER)	i_{VTE} (deg)	i_m (deg)	θ_M (deg)	$R_{\oplus☾}$ (ER)	i_{VTE} (deg)	i_m (deg)	θ_M (deg)	$R_{\oplus☾}$ (ER)	i_{VTE} (deg)	i_m (deg)	θ_M (deg)
Inject North												
38.2	56	-14	5.9	25	60	-11	5.6	26	64	-12	5.8	25.5
46	↓	-19	6.9	22	↓	-15	6.6	22	↓	-14.1	6.6	22.5
Inject South												
3.8	56	+21	3.0	144	60	-21	2.5	145	64	+19	3.0	147
11.7	↓	+26	4.3	158	↓	+27	4.2	158	↓	+27	4.5	160

However, only the mission parameters associated with the highest value of i_{VTL} for each case will be selected for the remainder of this sample mission. The actual parameters occurring at $R_{\oplus☾}$ = 61.9 ER are obtained by cross plotting i_{VTE}, i_m, θ_M versus $R_{\oplus☾}$ for i_{VTL} = 46° (for inject north) and 11.7° (for inject south) with the result for the mission parameters i_{VE} = 34.5°, $R_{\oplus☾}$ = 61.9 ER given below:

i_{VTL} (deg)	i_{VTE} (deg)	i_m (deg)	θ_M (deg)
46.0	-14.5	6.6	22 (inject north)
11.7	+27.0	4.3	159 (inject south)

Step V: Adjusted translunar trajectory characteristics

If the $\vec{x}_E$-axis (see Chapters III and VI) were directed toward the moon at the arrival time t_p, then the above i_{VTL} is correct. However, Section E gives an empirical equation that adjusts β for a specific mission, since such alignment of $\vec{x}_E$ with lunar position at t_p does not occur in practice. The equation for $\Delta\Phi_{TL}$ of Section E is repeated below:

$$\Delta\Phi_{TL} = \frac{\pm(0.22 + 0.000156\, h_{PL})}{C_0 \sin i_{VTE} + 1.0}$$

where

h_{PL} = pericynthion altitude, km

C_0 = constant (from Fig. 15)

use + for inject south cases
use - for inject north cases

For $R_{\oplus☾}$ = 61.9 ER, and h_{PL} 185.2 km, $\Delta\Phi_{TL}$ is +1.0° for the inject north case and +1.2° for the inject south case. Therefore, the $\beta_☾$ of 51.0° must be adjusted by the above increments in order to determine a more accurate value of i_{VTL}. Thus, Step III is repeated by re-entering Fig. 11 with $\beta_☾$ adjusted by approximately +1.1° or $(\beta_☾)_{adj}$ = 52.1°, to obtain a new value of i_{VTL} for inject north and inject south cases:

i_{VE} (deg)	i_{VTL} (inject north) (deg)	i_{VTL} (inject south) (deg)
34.5	45	12.0

Step IV is then repeated to refine the mission parameters of i_{VTE}, i_m and θ_M. The resulting adjusted parameters are:

Adjusted Parameters (i_{VE} = 34.50°, $R_{\oplus☾}$ = 61.9 ER)

i_{VTL} (deg)	i_{VTE} (deg)	i_m (deg)	θ_M (deg)	Case
45	-14.5	6.6	23	(inject north, return south direct)
12	+27.0	4.4	159	(inject south, return north direct)

Normally i_{VE} = 34.5° is used to evaluate the parameters. Step V can be omitted if the resulting values of i_{VTL} > 15° and the values obtained from Step IV can be used instead. However, for values of i_{VTL} < 15°, Step V must be carried through.

Step VI: Translunar flight time

To summarize steps (I) to (V), the following mission parameters have been found to acquire the landing site at $277^d.00$:

$$i_{VTL} = 45°,\ 12.0°$$

$$i_{VTE} = -14.5°,\ +27.0°$$

$$h_{PL} = 185.2 \text{ km}$$

$$R_{\oplus☾} = 61.9 \text{ ER}$$

$$i_m = 6.6°,\ 4.4°$$

$$\theta_M = 23.0°,\ 159°$$

$$h_0 = 250 \text{ km}$$

$$\gamma_0 = 5°.$$

It is now desired to find the transit time t_p from the earth injection point to pericynthion. From Chapter VI, Catalogue Figs. C-5, C-29 and C-64, t_p may be obtained as a function of $R_☾$ for the above parameters. For $R_☾ = 61.9$ ER $t_p = 73.0$ hr for the inject north case and 71.1 hr for the inject south case, and the time of injection onto the translunar trajectory, t_0 is given by

$$t_0 = 277^d.00 - t_p$$

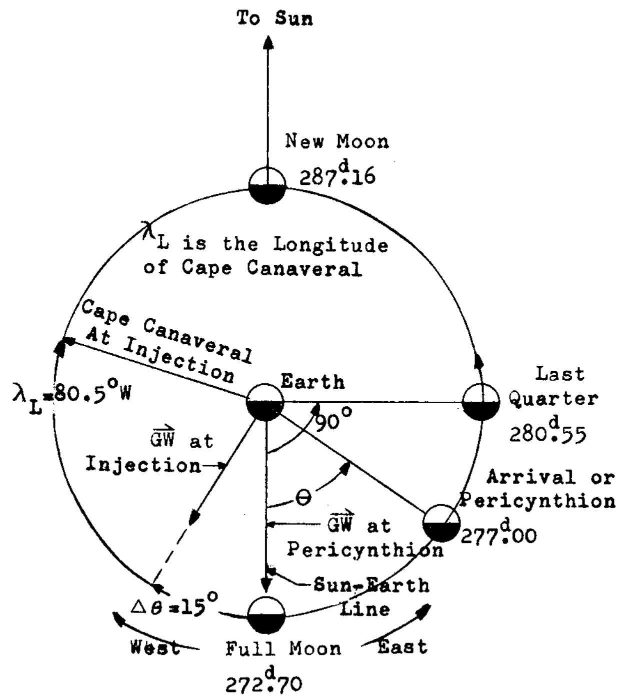
Step VII: Determine launch point position on injection date

The purpose of the following several steps is to determine the adjustment in arrival and injection time such that the launch can occur from Cape Canaveral. This must be done because the arrival time chosen in Step I does not, by any means, guarantee that the translunar trajectory and parking orbit will provide an earth ground trace which passes over Cape Canaveral at liftoff.

The procedure is as follows:

Referring to Table 2, full moon occurs on September 29, 1966, at 16:48 UT and the last quarter on October 7, 1966, at 13:09 UT. These phases bracket the moon arrival time and occur on $272^d.70$ and $280^d.55$, respectively. Also from Fig. 4a, the corresponding δ_M is -0.5° and +27.0°, respectively.

From now on, only the inject north case will be considered, since the steps involving the inject south case are identical. The following sketch illustrates the hour angles of the moon and sun and the location of the Greenwich meridian and of Cape Canaveral at injection,



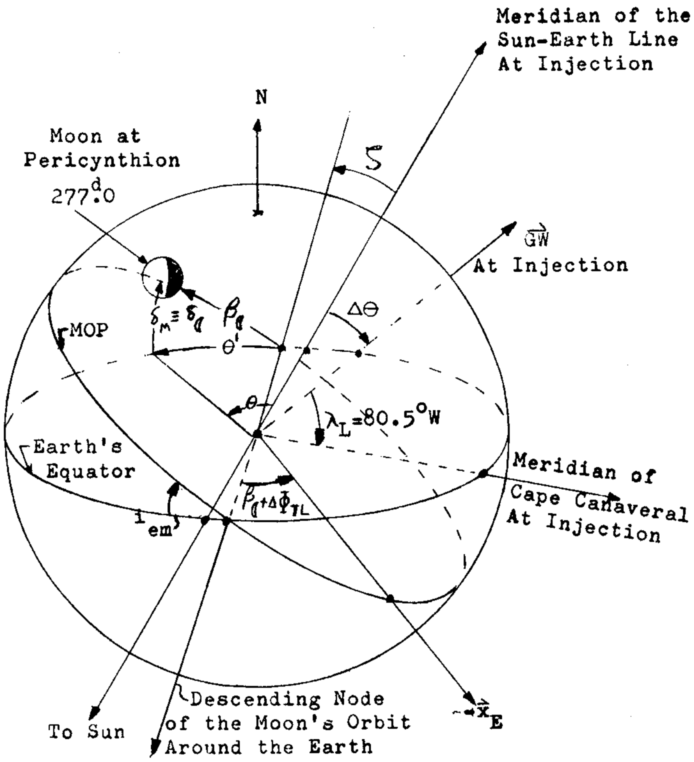
where the plane of the paper denotes the ecliptic plane. Consider the moon projected into the ecliptic plane. The average rotational rate of the moon relative to the sun-earth line in its orbit from full moon to the last quarter is given by:

$$\omega_{\oplus☾} = 90° / (280^d.55 - 272^d.70) = 11.47° / \text{day}$$

At arrival or pericynthion $277^d.00$, the moon is located θ degrees from the sun-earth line, where

$$\theta = (277^d.00 - 272^d.70)\, \omega_{\oplus☾} = 4^d.3 \times \frac{11.47°}{\text{day}} = 49.3°$$

Since pericynthion occurs at 00:00 UT, the Greenwich meridian at pericynthion, denoted by $\overrightarrow{GW}_p$ in the above sketch, is on the sun-earth line opposite to the direction to the sun. It has been determined in previous steps that injection for the inject north case takes place $73^h.0$ or $3^d.04$ earlier. Therefore, at injection the Greenwich meridian, now denoted by $\overrightarrow{GW}_0$, is located at $\Delta\theta = 15°$ where $\Delta\theta$ is positive to the west of the sun-earth line. For the inject south case $\Delta\theta = -13.5°$ (to the east of the sun-earth line) at the time of injection since the earth-moon transit time for the inject south case is $71^h.1 = 2^d.96$. The longitude of Cape Canaveral is 80.5° W; its location for the inject north case has been shown on the previous sketch. The next sketch illustrates the geometry in three dimensions for the inject north case by showing the celestial sphere centered at earth:



The hour angle ζ, which is measured from the midnight meridian eastward to the ascending node of the moon at injection, is given by

$$\zeta = \theta - \theta'$$

$$\theta = (273.96 - 272.7)\ 11.47 = 14.45°$$

where

$$\theta' = \theta'' \qquad 0 < \beta_{☾} < 180°$$

$$\theta' = 360 - \theta'' \qquad 180° < \beta_{☾} < 360°$$

and $\theta'' = \cos^{-1}\left(\dfrac{\cos \beta_{☾}}{\cos \delta_M}\right)$.

For the inject north case at injection $\beta_{☾}$ = 12° and δ_M = 5.5° so that

$$\theta'' = 10.3°$$

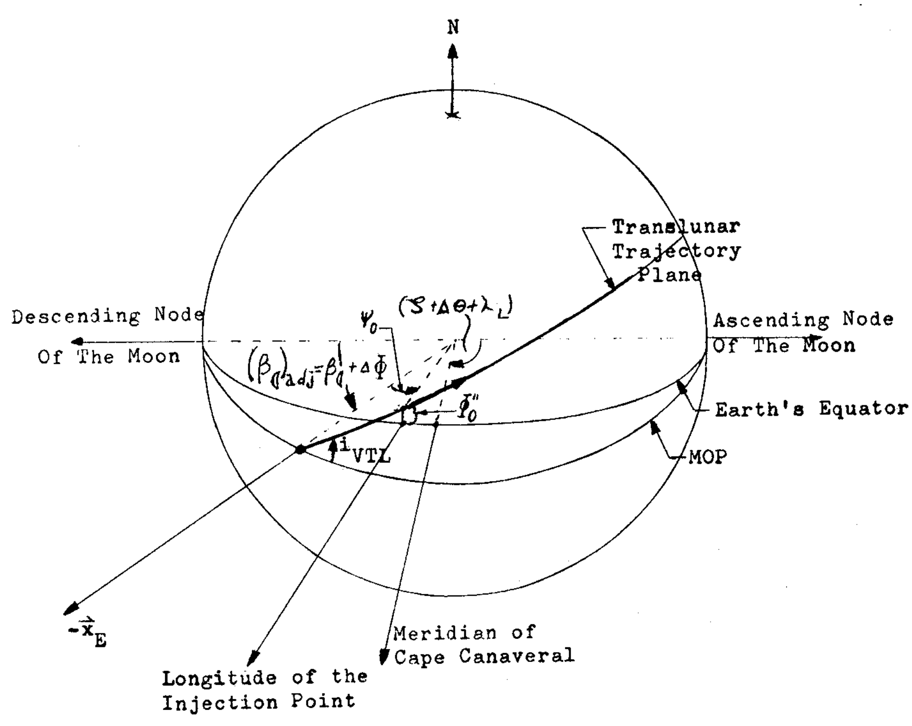
$$\theta' = 10.3°$$

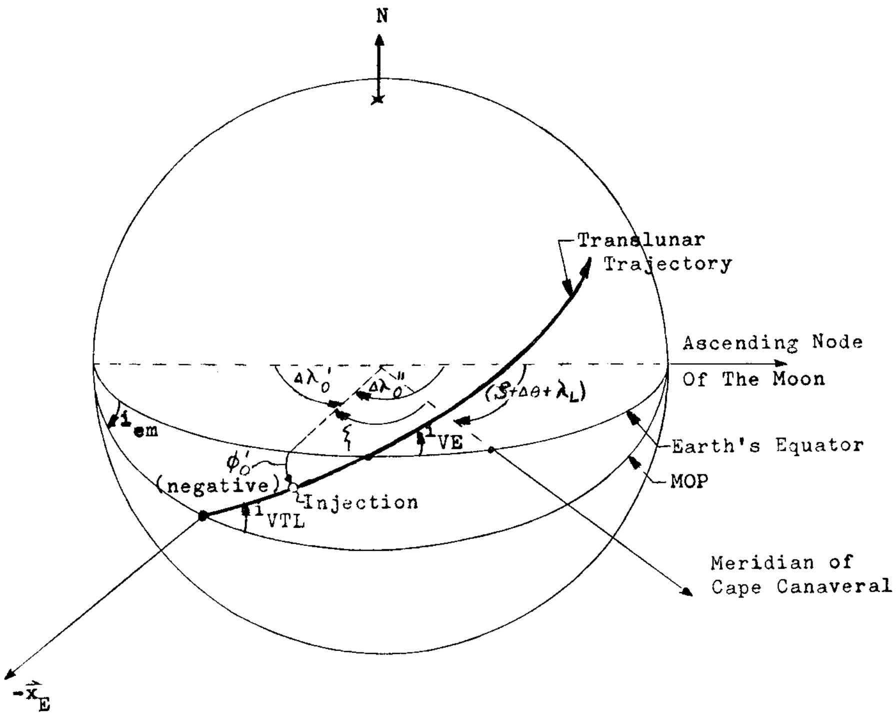
$$\zeta = 14.45 - 10.3° = 4.2°$$

Therefore, the meridian of Cape Canaveral is $\lambda_L + \Delta\theta + \zeta = 80.5° + 15° + 4.2° = 99.7°$ west of the ascending node of the moon at injection ($273^d.96$).

Step VIII: Determine injection point latitude and longitude

Next the latitude and longitude of the injection point at the time of injection for the inject north case are found. From the previous sketch in step VII, the $-\vec{x}_E$-axis is located in the MOP and the included angle measured eastward from the descending node is the adjusted value of $\beta_{☾}$, i.e., $(\beta_{☾})_{adj} = \beta_{☾} + \Delta\Phi_{TL} = 52.1°$.





The geocentric injection latitude ϕ'_0 is given by the following equation, where ψ_0 is found in Chapter VI, Figs. C-2, C-26, C-61 for $R_{\oplus\text{☾}} =$ 61.9 ER, $i_{VTL} = 45°$, and $\psi_0 = 17.8°$:

$$\phi'_0 + 90° = \cos^{-1}\Big[-\sin\psi_0 \sin i_{VTL} \cdot \cos i_m + \big(\cos\Psi_0 \sin(\beta_{\text{☾}} + \Delta\Phi_{TL}) - \sin\Psi_0 \cos i_{VTL} \cdot \cos(\beta_{\text{☾}} + \Delta\Phi_{TL})\big)\sin i_m\Big]$$

use $- i_{VTL}$ for inject south cases

use $+ i_{VTL}$ for inject north cases

where, for the present sample mission $\phi'_0 = -5°$.

The injection longitude λ_0 as illustrated in the sketch above, can be found by use of several steps:

Calculate first

$$\Delta\lambda'_0 = \cos^{-1}\left(\frac{L}{\left[L^2 + M^2\right]^{1/2}}\right)$$

where

$$L = \cos\psi_0 \cos(\beta_{\text{☾}} + \Delta\Phi_{TL}) - \sin\psi_0 \cos i_{VTL} \sin(\beta_{\text{☾}} + \Delta\Phi_{TL})$$

$$M = (\cos\psi_0 \sin(\beta_{\text{☾}} + \Delta\Phi_{TL}) + \sin\psi_0 \cos i_{VTL} \cos(\beta_{\text{☾}} + \Delta\Phi_{TL}) \cos i_m + \sin\psi_0 \sin(\pm i_{VTL}) \sin i_m$$

where the negative sign is used for inject north and the positive sign is to be used for inject south cases. If the quantity M is negative in the equation above, then $(360° - \Delta\lambda'_0)$ should be used instead of $\Delta\lambda'_0$ above. The quantity $\Delta\lambda'_0$ is the longitude increment of the injection point measured from the descending node eastward. As a second calculation, obtain $\Delta\lambda''_0$, the longitude increment of the injection point measured from the ascending node westward by use of

$$0 \lessapprox \Delta\lambda'_0 \lessapprox 180° \qquad \Delta\lambda''_0 = 180° - \Delta\lambda'_0$$

$$180° \lessapprox \Delta\lambda'_0 \lessapprox 360° \qquad \Delta\lambda''_0 = 540° - \Delta\lambda'_0.$$

For the inject north case under consideration

$$\Delta\lambda'_0 = 65°$$

$$\Delta\lambda''_0 = 115° .$$

Finally, from step VII the meridian of Cape Canaveral is 99.7° West of the ascending node of the moon at injection. Therefore the geographic injection longitude is $\lambda_0 = 80.5°$ W + ξ = 95.8° W, where $\xi = \Delta\lambda''_0 - 99.7° = 15.3°$ (measured positive westward as indicated on the sketch).

Step IX: Actual launch time and pericynthion time

The ascent trajectory will be neglected for the following calculations, i.e., the lunar space vehicle is assumed to "start out" or be launched at the parking orbit altitude h_{PL} = 185.2 km above Cape Canaveral, with the corresponding circular orbital velocity. This assumption is quite good for lunar spacecraft due to the short ascent time (on the order of 300 seconds). In any case, a further correction for launch and pericynthion time can be performed as soon as the booster for the lunar mission has been selected and ascent trajectory calculations for the booster have been performed.

With this assumption and the data generated in steps I to VIII, the next sketch can be drawn. The prime purpose of step IX is to obtain the longitude increment $\Delta\lambda$ of the parking orbit ground trace measured from the meridian of Cape Canaveral to the intersection of the latitude of Cape Canaveral by the parking orbit. Furthermore the orbital range angle Φ_{LO} from this intersection point to the translunar injection point must be determined.

Define an orbital central angle β_L from the ascending node of the parking orbit to its intersection with the launch site parallel of latitude. For i_{VE} = 34.5°, β_L becomes

$$\beta_L = \sin^{-1}\left(\frac{\sin \phi_L'}{\sin i_{VE}}\right) = \sin^{-1}\left(\frac{\sin 28.5°}{\sin 34.5°}\right) = 57.3°.$$

Define an orbital central angle from the injection point to the ascending node of the parking orbit by β_0', where β_0 = 360 - β_0', and

$$\beta_0' = \sin^{-1}\left(\frac{\sin \phi_0'}{\sin i_{VE}}\right) = 8.9° .$$

Since these two equations are spherical trigonometry relations, the quadrant of the angles must be determined from the previous sketch. The angles β_L, β_0' have been defined for a given hemisphere and node.

For the inject north case in question, the injection takes place in the southern hemisphere and the nearest node is the ascending node, as illustrated on the sketch.

The orbital central angle from injection to the intersection of the launch site latitude is $\beta_L + \beta_0'$ = 66.2° for this sample mission. Define the longitude increments corresponding to β_L as $\Delta\lambda_L$ and of β_0' as $\Delta\lambda_{PO}'$ where $\Delta\lambda_L$ and $\Delta\lambda_{PO}$ are given by

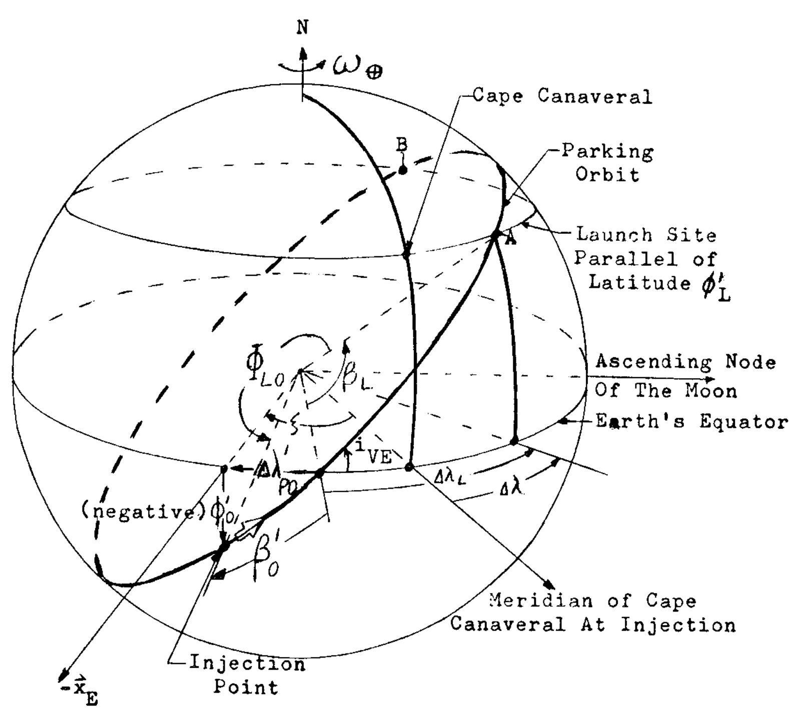
$$\Delta\lambda_L = \cos^{-1}\left(\frac{\cos \beta_L}{\cos 28.5°}\right) = 52° ,$$

$$\Delta\lambda_{PO} = \cos^{-1}\left(\frac{\cos \beta_0'}{\cos \phi}\right) = 6° .$$

Therefore the longitude increment from the location of Cape Canaveral to the intersection of the parking orbit with the launch parallel of latitude is

$$\Delta\lambda = \Delta\lambda_L + \Delta\lambda_{PO}' - \xi = 42.7° \text{ E}.$$

For the assumption of no ascent trajectory to the parking orbit, the spacecraft must be located either at point A or at point B at launch. Point A represents a launch azimuth A = 70°, while point B calls for A = 110°. The former point has been arbitrarily selected as the launch condition for the remainder of the sample mission. Thus, the



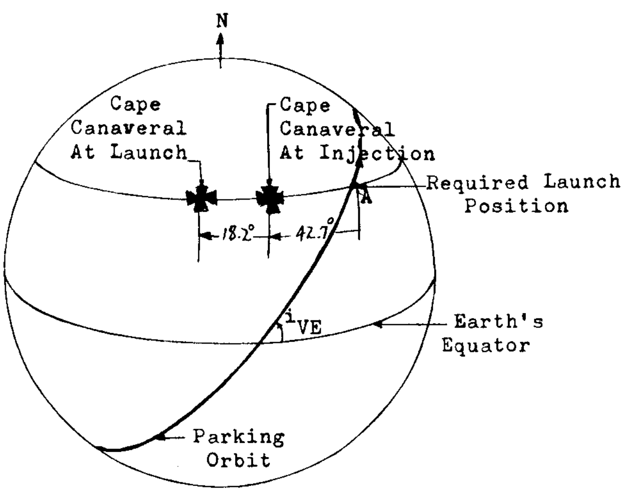
range angle Φ_{LO} from point A to the translunar injection point is

$$\Phi_{LO} = 360° - (\beta_L + \beta_0') = 297.8°.$$

Since the average angular velocity of the spacecraft in its parking orbit is 246.6 deg/hr, it must be launched from Cape Canaveral at a time equal to the injection time minus the time to travel the range angle Φ_{LO}, which is 297.8° / 246.6° /hr = 1.21 hr = 0.05^{d}. From step VII, translunar injection occurs on 273.96^{d}. Therefore the launch time is:

$$\text{launch time} = 273.96^{d} - 0.05^{d} = 273.91^{d}.$$

However, if the location of the spacecraft in the parking orbit is retraced back in time to the launch site latitude ϕ_L', Cape Canaveral itself rotates with the earth backward or westward during this interval. For an elapsed time of 1.21 hr Cape Canaveral has rotated westward by a longitude displacement equal to 1.21 hr · 15.04° / hr = 18.2°; where $\omega_{\oplus}$ = 15.04° /hr is the angular velocity of the earth about its axis. At launch, the actual position of Cape Canaveral is displaced from the required launch point (point A) by 42.7° + 18.2° = 60.9° toward the west as illustrated in the following sketch:



The parking orbit does not pass through Cape Canaveral at launch and the computed launch date is too early. Assuming that the flight time to the moon and all other parameters generated thus far remain unchanged by this change of launch time, then a first-order correction can be made to the launch date by conducting the launch at an interval of time Δt later, where Δt is the time required for Cape Canaveral to rotate with the earth to point A:

$$\Delta t = (18.2° + 42.7°)/15.04° /hr = 4.05 \text{ hr}.$$

Thus, the corrected launch time is:

$$\text{Launch time (corrected) } t_L = 273.96^{d} - 1.21^{h} + 4.05^{h}$$

$$= 273.96^{d} + 2.84^{h}$$

$$= 274.08^{d}$$

For the first-order correction, the range angle Φ_{LO} is the same for the new launch date, and the corrected injection date is

$$\text{Injection time (corrected)} = 273.96^{d} + 2.84^{h} + 1.21^{h} = 274.13^{d}$$

Similarly, the corrected arrival date is

$$\text{Moon arrival time (corrected)} = 273.96^{d} + 4.05^{h} + 73.00^{h}$$

$$= 277.17^{d}$$

Step X: Iteration with improved arrival time

Steps II to IX are now repeated with the corrected moon arrival (or pericynthion) time. This is essentially the first iteration, and the steps of the first iteration are denoted by an asterisk (*). For each step, only the important parameters and their values are listed.

Step II*. The lighting conditions for the new arrival date have changed only slightly. The evening terminator is now 34.6° E of the landing site, and its inclination i_T = 91°.

Step III*. Revised values for δ_M, R_M, $\beta_☾$ and maximum i_{VTL} are δ_M = +21°, $R_{\oplus ☾}$ = 61.8 ER, $\beta_☾$ = 52.2°, and the maximum i_{VTL} = 45° for the inject north case.

Step IV*. The required θ_M and i_m relationship for the direct inject north case are, by linear interpolation between the 277.00^{d} and 277.50^{d}:

θ_M (deg)	i_m (deg)
10	27.24
20	8.78
30	5.25
40	3.86
50	3.11
60	2.68
70	2.41
80	2.25
90	2.18

The trajectory parameters for $R_{\oplus ☾}$ = 61.8 ER are:

i_{VTL}	i_{VTE}	i_m	θ_m	(Inject north and return south direct)
45	-12.5°	6.2°	25.2°	

Step V*. Step V* is not required, since $i_{VTL} > 15°$. However, as $\Delta\Phi_{TL}$ is approximately the same, it will be used.

Step VI*. The revised transit time is $t_p = 72.8$ hr, with injection at

$$t_0 = 277.169^d - 72.8^h = 274.136^d$$

Step VII*. At $t_0 = 274.136^d$, the angular displacement of the moon from the sun-earth line is:

$$\theta = (277.17 - 272.7)(11.47) = 51.1°$$

The Greenwich meridian is $\Delta\theta = 45.8°$ east of the sun-earth line, and the hour angle ξ remains approximately the same, i.e.,

$$\zeta = 4.2°,$$

and the meridian of Cape Canaveral is 80.5° - 45.8° + 4.2° = 38.9° W of the ascending node of the moon.

Step VIII*. The revised values for the injection parameters of step VIII become

$$(\beta_\ell)_{adj} = 53.3$$

$$\psi_0 = 17.8°$$

$$\phi_0' = 4.4°$$

$$\Delta\lambda_0' = 63.5°$$

$$\Delta\lambda_0'' = 116.5°$$

$$\xi = 77.6°$$

$$\lambda_0 = 158.1° \text{ W}.$$

Step IX*. The revised values for the auxiliary parking orbit parameters are:

$$\beta_0' = 7.77°$$

$$\beta_L + \beta_0' = 65.07°$$

$$\Delta\lambda = 19.6° \text{ W}$$

$$\Phi_{LO} = 294.93° .$$

It takes 1.195^h to traverse the angle Φ_{LO} to yield

$$\text{Launch time} = 274.136^d - 1.195^h = 274.141^d$$

During this time, Cape Canaveral has rotated westward by a longitude increment of 18.0°.

At launch the actual longitude of Cape Canaveral is displaced from the parking orbit launch point A by 1.6° E. For this small discrepancy, the parking orbit trace, for all practical purposes, can be assumed to intersect Cape Canaveral at launch, thus satisfying the physical constraint of the mission, and eliminating the need for performing a second iteration.

However, if more accuracy is desired, the time of 0.1 hr may be subtracted from the revised launch, injection and moon arrival time to compensate for the 1.6° discrepancy mentioned above. This is also a logical point to include the ascent to parking orbit phase into the correction and to carry the calculations one decimal point further.

Step XI: Lighting conditions at earth launch time

The approximate lighting conditions may be found for the earth launch and earth injection phases with the aid of the next sketch, which shows a top view of the earth's equatorial plane.

From step VII*, $\zeta = 4.2°$, where ζ is the hour angle measured from the sun-earth line eastward to the ascending node of the moon and thus defines the location of the earth terminator as shown. Usually ζ is only accurate to within ±2°. From step VIII*, the injection meridian at the time of injection is 116.5° W of the moon's ascending node. Also from step VII*, the meridian of Cape Canaveral at injection is 38.9° W of the ascending node. Therefore the meridian of Cape Canaveral at launch is 38.9° W + 18.0° W or 56.9° W. Hence the spacecraft launch will definitely be in darkness and the injection in daylight. Launch or injection in darkness may or may not be acceptable, but for this sample mission there are no lighting restrictions for the earth-departure phase.

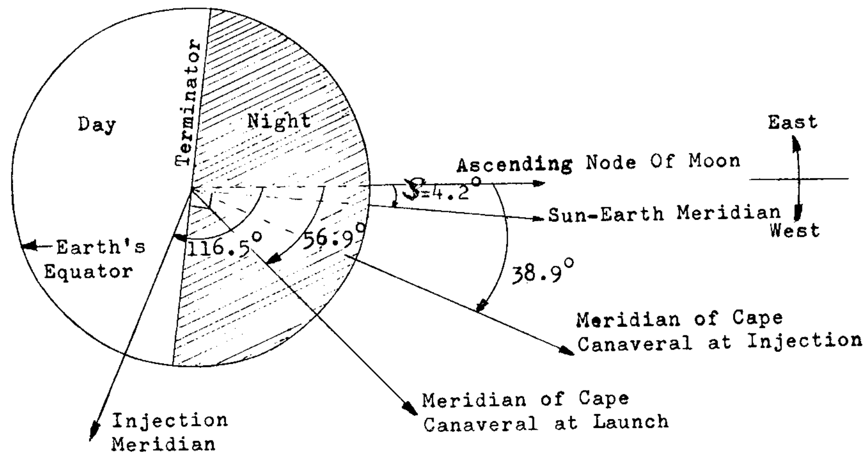
It should also be noted that the inject north case taken here represents the beginning of the launch tolerance for the variable translunar technique given in Chapter V. For inject north cases, launches can be made up to 4.9 hr beyond the above nominal launch time.

The same procedure can be used to determine trajectory characteristics and time of launch for the inject south case in step V.

Step XII: Aborted transearth trajectory

The next steps analyze the return-to-earth portion of the above circumlunar trajectory in the event entry into lunar orbit cannot be performed. From step IV* it was found that the return inclination i_{VTE} is -12.5° returning direct from the south. The total time of flight T (translunar injection to return vacuum perigee) is found in the same manner as the time to pericynthion t_p in steps VI and VI*. From Figs. C-6, C-30 and C-65 of the circumlunar catalogue in Chapter VI,

$$T = 143.4 \text{ hr.}$$



Mission specifications 8, 9 and 10 are directly related. From Chapter VI, Section A, a return inclination of $i_{VTEQ} \simeq 30°$ (relative to the equatorial plane of the earth) will place re-entry either along AMR or PMR. However, the re-entry direction must be specified relative to the equator. For re-entry along AMR, the return direction for the circumlunar trajectory is either direct from the north or retrograde from the south. Similarly, re-entry along PMR must be direct from the south or retrograde from the north. Two questions must now be resolved, namely:

(1) If the transearth trajectory does not return with the proper inclination, what is the propulsion requirement to correct the trajectory?

(2) If re-entry occurs geographically displaced from PMR, what propulsion requirement corrects the time of re-entry?

These questions will be answered in steps XIV and XV, respectively.

Step XIII: Transearth trajectory inclination (abort)

This step determines i_{VTEQ}. As was the case in steps V and V*, an empirical correction is applied to adjust the intersection of the transearth plane with the MOP. The correction factor is as follows and is discussed in Section E of this Chapter.

$$\Delta\Phi_{TE} = \pm(0.232 + 0.0001537\, h_{PL})\, D_0 \sin i_{VTE} - E_0$$

use + for inject south

use - for inject north

$$E_0 = 1.7 + 0.0375\,(R_{\oplus☾} - 56)$$

where $R_{\oplus☾}$ is measured in earth radii, h_{PL} in km, and D_0 is found in Fig. 16.

For the conditions:

$$h_{PL} = 185.2 \text{ km}$$

$$i_{VTE} = -12.5°$$

$$R_{\oplus☾} = 61.8 \text{ ER}$$

$$i_{VTL} = 45°$$

$$\Delta\Phi_{TE} = -1.9°$$

Therefore, $(\beta_☾)_{adj} = \beta_☾ - \Delta\Phi_{TE} = 52.2°$ (step III*) $- 1.9° = 50.3°$.

Figure 13 is now entered with the above value of $(\beta_☾)_{adj}$ and $i_{VTE} = -12.5°$, yielding $i_{VTEQ} = 21°$ (direct from the north).

Step XIV: ΔV required to correct i_{VTE} (abort)

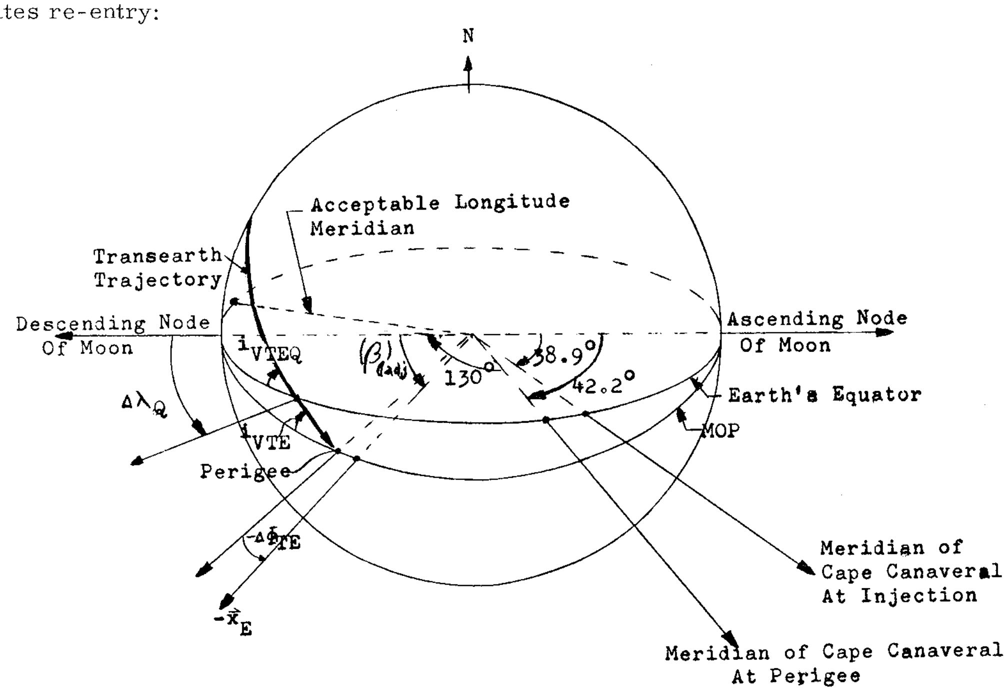
The previous step determined that the actual value of $i_{VTEQ} = 21°$. From mission specifications 8, 10 and 11, the minimum acceptable return inclination $i_{VTEQ} = -28.5°$, which is equivalent to $i_{VTEQ} = 331.5°$. (Note that this inclination is not inclination as normally defined, since it is measured from the west point on the horizon at the descending node to the branch of the trajectory in which the vehicle is approaching the node. Thus this inclination varies from 0 to 360° and carries with it the sense of vehicle motion.) Therefore, to correct the actual return inclination to the desired value, an i_{VTE} = change Δi_{VTE} of $-27.5°$ is required, an estimate obtained also from Fig. 13. Assuming that the correction, in the form of a velocity impulse ΔV_i, is applied at a $R_{\oplus\Delta}$ of 40 ER, a first approximation of the ΔV required can be found through the use of Figs. 18 and 21. For the above conditions, $\Delta V_i = 134$ m/sec $\Delta\Phi_{TE} = +8.5°$. Re-enter Fig. 13 with $i_{VTE} = -12.5° - 27.5° = -40°$ and $(\beta'_☾)_{adj} = 53.3° + 8.5° = 61.8°$ and note that $i_{VTEQ} = 325°$. This manipulation is repeated by altering Δi_{VTE} until the desired value of i_{VTEQ} has been obtained. For this sample mission, the final values are: ΔV_i =

110 m/sec, $(\beta'_{\mathbb{C}})_{adj} = 61.3°$, $\Delta\Phi_{TE} = 8°$, $i_{VTEQ} = 330°$ and $i_{VTE} = -35°$. The total time of the circumlunar trajectory remains T = 143.4 hr.

Step XV: Re-entry point position (abort)

It is assumed that the re-entry point is the vacuum perigee of the transearth trajectory, and that this point lies in the MOP. These assumptions are valid because the spacecraft's angular velocity at re-entry is much greater than the earth's rotational rate and because the actual re-entry point is but a few degrees (less than 10°) from the MOP for earth return. The following sketch illustrates re-entry:

If the ground trace of the re-entering spacecraft passes at some time through the point ($\phi' = 0°$, $\lambda = 149.5°$ E), re-entry will be along PMR (see Chapter VI, Subsection B-2). Define λ = 149.5° E as the "acceptable longitude" meridian; it is displaced westward from the meridian of Cape Canaveral ($\lambda_L = 80.5°$ W) by 130°, and in order to satisfy the mission specifications the "acceptable longitude" meridian must pass through the ascending node of the transearth trajectory (point Q on the previous sketch). Redrawing the area of interest for the actual situation, the angle $\Delta\lambda_{PE}$ is found by spherical trigonometry.



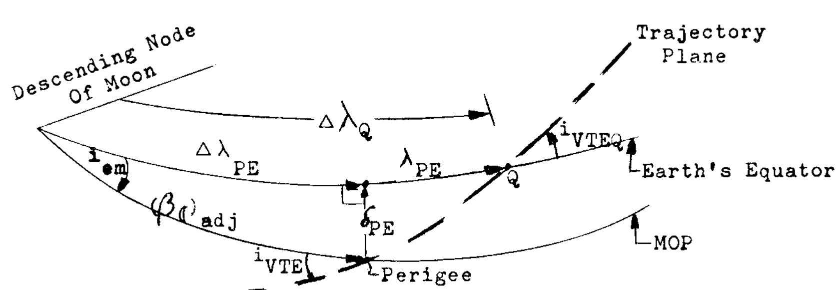
From step VII*, the meridian of Cape Canaveral is located 38.9° W of the lunar ascending node at translunar injection. During the interval from translunar injection to vacuum perigee, T = 143.4 hr, the meridian rotates with the earth at a rate of approximately 15.04 deg/hr. Therefore, at the time of perigee the position of this meridian is 42.2° W of the lunar ascending node.

where:

$$\Delta\lambda_{PE} = \tan^{-1}\left[\cos i_m \tan\left((\beta_{\mathbb{C}})_{adj} + \Delta\Phi_{TE}\right)\right]$$

$$\delta_{PE} = \sin^{-1}\left[\sin i_m \sin\left((\beta_{\mathbb{C}})_{adj} + \Delta\Phi_{TE}\right)\right]$$

$$\pm\lambda_{PE} = \sin^{-1}\left[\frac{\tan \delta_{PE}}{\tan i_{VTEQ}}\right]$$



use + for + i_{VTE} (direct south return)

use - for - i_{VTE} (direct north return)

$\Delta\lambda_{PE}$ = 58.6°

δ_{PE} = 23.4°

λ_{PE} = 30°

$\Delta\lambda_Q = \Delta\lambda_{PE} + \lambda_{PE} = 88.6°$

Again, great care must be taken in determining the quadrants of the angles for each particular mission geometry, since spherical trigonometry has been used.

The longitude increment between Q and the meridian of Cape Canaveral at perigee is $\Delta\lambda_{LQ}$ = 180° - 88.6° - 42.2° = 47.2°. Therefore, the "acceptable longitude" meridian for PMR is located 82.8° W of Q. In order for Q to be coincident with the acceptable meridian, it becomes necessary to delay the arrival at perigee by 82.8° /15.04°/hr = 5.5 hr. From Fig. 17, if a time-delaying maneuver is conducted at $R_{\oplus\Delta}$ = 40 ER, the ΔV_t required to achieve the acceptable re-entry is 345 m/sec.

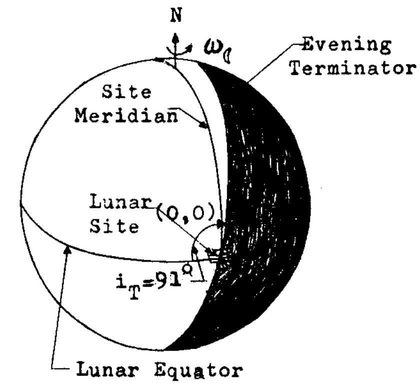
Step XVI: ΔV required to enter lunar orbit (also injection velocity)

The previous steps have analyzed the mission in the event that a circular orbit about the moon is not established, i.e., the mission is aborted. However, in the nominal case, a circular orbit is established (mission specification 7) and the energy required to accomplish this also may be found. From Chapter VI, Figs. C-4, C-28 and C-63 of the catalogue are entered with the data of step IV*, namely, i_{VTL} = 45°, i_{VTE} = -12.5°. Then for $R_{\oplus☾}$ = 61.8 ER, the energy required is ΔV_1 = 905 m/sec.

The injection velocity V_0 is found in a similar manner from the catalogue (Figs. C-1, C-25 and C-60), namely, V_0 = 10900.8 m/sec.

Step XVII: Lighting conditions at lunar launch

Mission specification 13 on page XI-11 requires an approximate stay time on the lunar surface of 3 days. Since the arrival time at the moon is $277^d.16$, injection into a transearth trajectory from lunar orbit is arbitrarily assumed at $280^d.00$. Allowing time for landing, takeoff and orbit, the actual stay time on the lunar surface is approximately 2.5 days. The lighting conditions for lunar takeoff are found in the same manner as in Step II. For $280^d.00$ the estimated (see Section C) and exact lighting agree closely; the exact conditions are shown below. The evening terminator passes through the site, and therefore, the lunar launch will take place at dusk. Launch must be westward and must occur prior to $280^d.00$ if daylight is desired during this mission phase.



Step XVIII: Requirements for establishing the transearth trajectory

Referring to Figs. 4a and 4b the following lunar information may be obtained for $280^d.00$:

i_m = 27.2°

δ_M = 27.2°

$R_{\oplus☾}$ = 59.8 ER

$\beta_☾$ = 90°

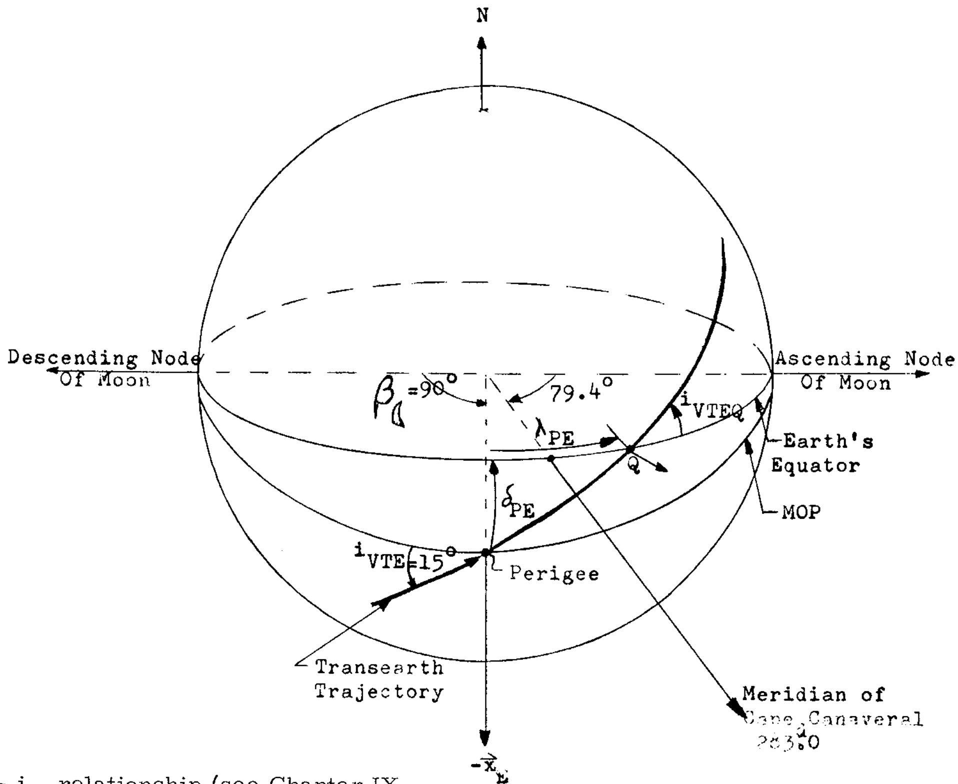
(see step II for definitions).

The object of this step is to determine the required transearth trajectory inclination and time of flight from the transearth injection point to vacuum perigee on earth arrival.

At translunar injection, $t_0 = 274^d.136$ (step VI*), the meridian of Cape Canaveral is 38.9° W of the lunar ascending node. Now the "acceptable longitude" meridian for re-entry along AMR is displaced eastward from Cape Canaveral by 55.5° (step XV), and the corresponding meridian for re-entry along the PMR is 130° W of Cape Canaveral. At $280^d.00$, the meridian of Cape Canaveral is 81.9° W of the lunar ascending node. A first estimate for the return trip duration is exactly 3 days, and the meridian of Cape Canaveral at re-entry is 82.4° - 15.04 · (72) = 79.4° W of the lunar ascending node. The following sketch illustrates re-entry.

Returning to Fig. 10, it is noted that for $\beta_☾$ = 90°, i_{VTE} must equal - 15° to satisfy mission specification 17, i.e., i_{VTEQ} = 30° and direct re-entry along PMR.

Before proceeding further, step IV is repeated here for the transearth injection time $280^d.00$ and the lunar site with $\phi_☾ = 0°$, $\lambda_☾ = 0°$. The re-



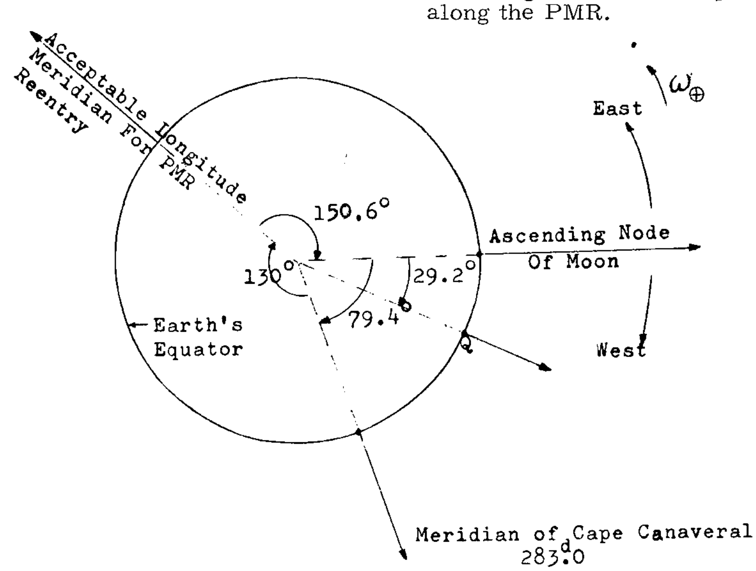
quired θ_{MTE} - i_m relationship (see Chapter IX for an explanation) is as follows:

θ_{MTE} (deg)	i_m (deg)
10	59.28
20	23.02
30	13.9
40	10.11
50	8.13
60	6.97
70	6.26
80	5.84
90	5.64
100	5.61
110	5.75
120	6.09

Since $R_{\oplus\mathbb{C}} \approx 60$ ER, the above data are superimposed on mission planning envelopes in Section F (Figs. 35, 36 and 37), a sample of which is presented in Fig. 42. These particular envelopes satisfy mission specifications 14, 17 and 18. It is confirmed that an i_{VTE} of ± 15° can be obtained for return flight times from 70 to 110 hr, and re-entry may occur direct either along AMR or PMR. For this mission, nominally a direct re-entry along the PMR is specified (specification 17), implying that "direct from the south" return be made or i_{VTE} be -15°, as shown in the above sketch. Note that $\delta_M = \delta_{PE}$ for this case, and from spherical trigonometry

$$\lambda_{PE} = \sin^{-1}\left(\frac{\tan \delta_{PE}}{\tan i_{VTEQ}}\right) = 60.8°$$

The next sketch shows more clearly the geometry of timing the arrival at perigee so that it is direct along the PMR.



Thus, the re-entry should be timed to occur

$$\frac{130° + 53.2°}{15.04°/hr} = 12.17 \text{ hr later, or } \left(\frac{147.6° + 29.2°}{15.04°/hr}\right) =$$

11.73 hr earlier. The later return flight time of 72 + 12.17 = 84.17 hr will be chosen. The corresponding i_m and θ_{MTE} needed to achieve this time is obtained by cross-plotting the superimposed data of Figs. 35, 36 and 37 for an i_{VTE} = - 15.0, and they turn out to be $i_m = 9.3°$ and $\theta_{MTE} = 43.0°$.

Step XIX: The transearth trajectory

Figures 26 to 50 in the catalogue of Chapter IX can be entered with these values of i_m, θ_{MTE} and the velocity impulse required to eject from lunar orbit ΔV_2, and the lunar orbit ejection position β_{M0} can be cross-plotted against i_m. They are

$$\Delta V_2 = 840 \text{ m/sec}$$

$$\beta_{M0} = -27°$$

An approximate idea of the lighting conditions at the time of transearth injection from lunar orbit is obtained by means of the following sketch, which neglects the lunar librations (see step XVII):

Transearth injection will take place in daylight, as shown, but subsequent flights will be eclipsed by the moon for a few hours.

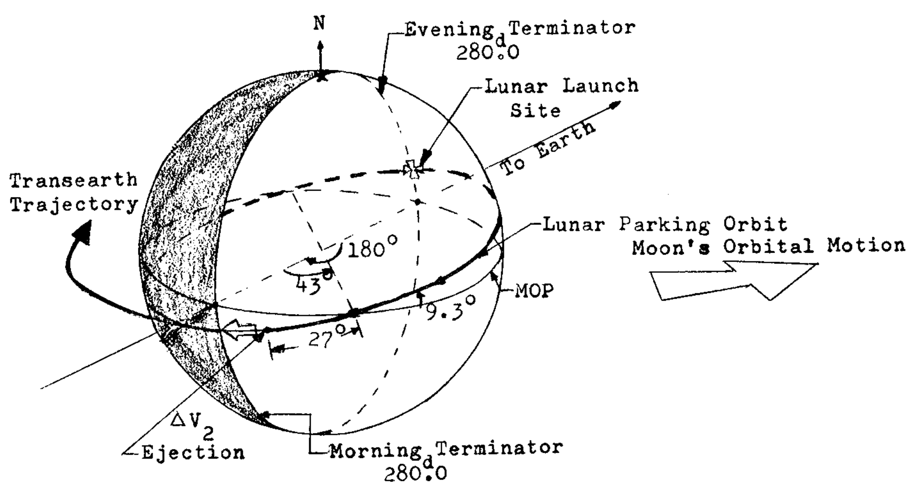
From step XVIII, the earth arrival date is $280.00^d + 89.17^h = 283.51^d$. Lighting conditions at earth for re-entry may be estimated in the following manner. First, the second sketch in the previous step is partially redrawn. Secondly, repeat step VII in determining the hour angle ζ which is measured from the midnight meridian eastward to the ascending node for positive angles. Since the average rotational rate of the moon around the earth from the last quarter to new moon is $\omega_{\oplus☾} = 13.6$ deg/day,

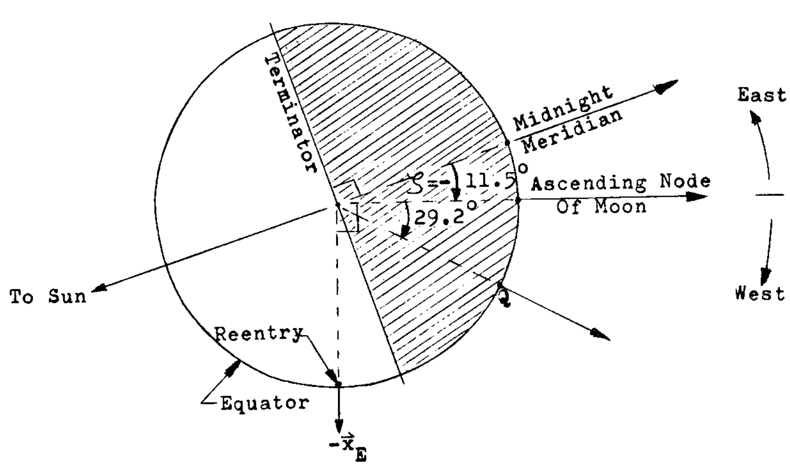
$$\zeta = 130.1° - 141.6° = -11.5°$$

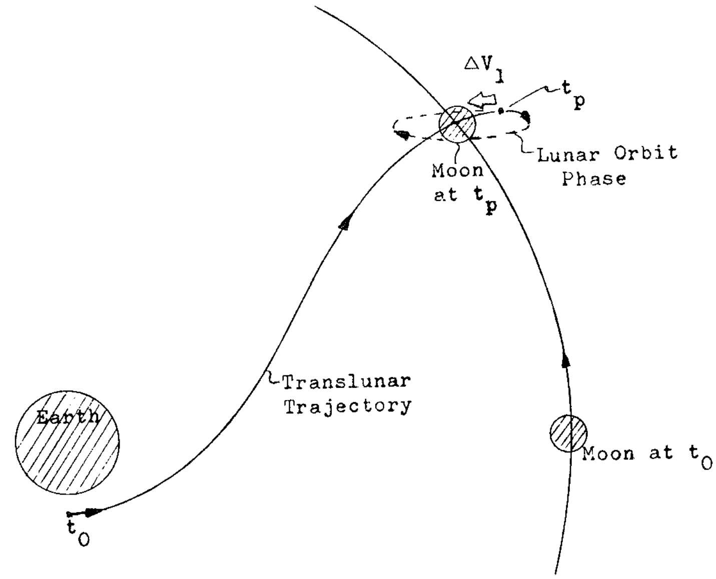
Note that the $-x_E$-axis lies in daylight, but close to the earth's terminator. Therefore, the re-entry will occur in daylight, but if the re-entry range to the required landing site is large, part of the re-entry phase and the landing will be in darkness.

2. Sample Mission II

Mission: Establish a photo-reconnaissance satellite (unmanned) around the moon to map the moon. Launch date: October 1966.







A mission schematic is shown above for clarity.

Injection into the translunar trajectory takes place at time t_0. The spacecraft then flies ballistically along the nominal trajectory until pericynthion at time t_p. At t_p, a velocity impulse ΔV_1 is applied against the direction of motion to reduce the speed of the spacecraft to the circular lunar orbit speed, from which the mission is carried out. The translunar trajectory does not necessarily have to be a portion of a circumlunar trajectory, nor is there any requirement to recover the spacecraft either nominally or in the event of a malfunction.

The detailed specifications for this mission are as follows:

Specification I. Nominal Translunar Trajectory

(1) A translunar trajectory of the approach class (Chapter IV) will be employed.

(2) The spacecraft is launched from Cape Canaveral with launch azimuth restrictions of $70° \leq A \leq 110°$.

(3) Parking orbits are used prior to translunar injection.

(4) Injection is over AMR, which implies an inject south case.

(5) Injection conditions are

$$h_0 = 250 \text{ km}$$

$$\gamma_0 = 5°$$

(6) The pericynthion altitude of the circumlunar trajectory is 185.2 km.

(7) At pericynthion, thrust is applied to reduce the speed of the spacecraft to the circular lunar orbit speed.

Specification II. Lunar Satellite Orbit

(8) The orbit is to be circular with an altitude of 185.2 km above the moon.

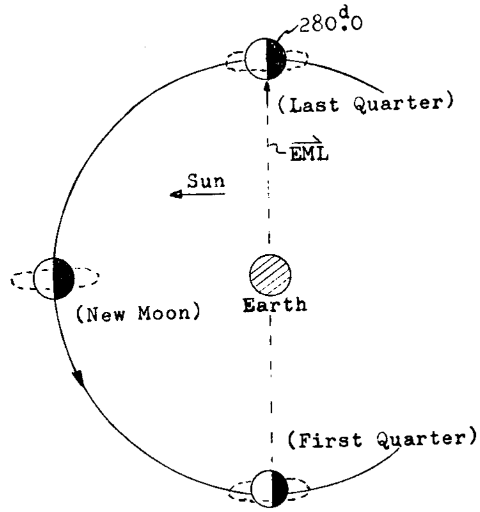
(9) The orbit is highly inclined to the MOP to obtain nearly maximum coverage.

(10) The foremost requirement of the mission is data acquisition from the unseen side of the moon.

Step I: Determine required i_m, θ_{MTL}, and arrival time

Since there is no specific small coverage area, a particular i_m, θ_{MTL} relationship is not required. However, from mission specification 9 , the lunar orbit must be highly inclined, and thus, for this mission, a polar orbit ($i_m = 90°$) is chosen. The value of θ_{MTL}, which is dependent on lighting conditions and specification 10 , must still be determined. From step XVII of sample mission I, the lighting at $280.^d00$, 1966 is as shown in the sketch below.

If the orientation of the lunar orbit, $\theta_{MTL} \simeq \pm 90°$ on $280.^d00$, then its plane is nearly perpendicular to the terminator (also shown in the sketch). As the moon rotates about the earth to its first quarter position, it is observed from the previous sketch that the unseen face of the moon is adequately lighted and will be completely mapped in 0.5 lunar months. Thus, the foremost requirement of the mission (specification 10) is carried out as soon as possible, the entire lunar surface can be mapped in one lunar month, and any value of $\theta_{MTL} \simeq \pm 90°$ is acceptable for this mission.



Step II: Obtain the translunar trajectory inclination

Referring to Figs. 4a and 4b, the following lunar orbit information is obtained for the $280^d.00$ day:

$$i_m = 27.2°$$
$$\delta_M \equiv \delta_☾ = 27.2°$$
$$R_M \equiv R_{\oplus ☾} = 59.8 \text{ ER}$$
$$\beta_☾ = 90°$$

From Fig. 11, for an $i_{VE} = 34.5°$, the i_{VTL} for the inject south case is $i_{VTL} = 23.5°$. Since $i_{VTL} > 10°$, there is no adjustment required, and, therefore, Eqs (21) and (22) of Section E are not needed.

Step III: Translunar flight time

The following required mission parameters are now known

$$i_{VTL} = 23.5° \text{ (inject south)}$$
$$h_0 = 250 \text{ km}$$
$$\gamma_0 = 5°$$
$$R_{\oplus ☾} = 59.8 \text{ ER} \approx 60 \text{ ER}$$
$$i_m = 90°$$
$$t_p = 280.00$$

Referring to the transearth mission envelopes (Figs. 35, 36 and 37) for $R_{\oplus ☾} = 60$ ER, the translunar flight time that most nearly satisfies the required i_m and θ_{MTL} is $t_p = 110$ hr. This flight time gives, for the above trajectory parameters:

$$i_m = 90°$$
$$\theta_{MTL} = 95° \text{ or } -85°$$

Only $\theta_{MTL} = 95°$ will be considered in the remaining discussion. Thus, the injection into the translunar trajectory occurs on

$$t_0 = 280^d.00 - t_p = 275^d.416.$$

Step IV: Determine launch point position at injection

This step is essentially identical to step VII of sample mission I, so only the pertinent data is given here.

Relative to the sun-earth line, the average rotation rate of the moon about the earth during the lunar phase at return is $\omega_{\oplus ☾} = 11.47°$/day.

Thus, at t_0 the moon is located

$$\theta = (275.416 - 272.7)\ 11.47 = 31.0°$$

from the sun-earth line. Since the Greenwich meridian $\overline{GW}$ is coincident with the sun-earth line at $276^d.00$, at injection, $\overline{GW}$ is at $\Delta\theta = 210°$ W of the sun-earth line.

The hour angle ζ, measured from the midnight meridian eastward to the ascending node of the MOP at t_0, is:

$$\zeta = \theta - \theta',$$

which yields $\beta_☾ = 31°$, $\delta_☾ = 13.5°$, $\theta' = 28.5°$ and $\zeta = 2.5°$. Therefore, the meridian of Cape Canaveral is $(80.5° + 210° + 2.5°) = 293.0°$ W of the ascending node of the moon at t_0.

Step V: Determine injection latitude and longitude

From the last sketch in step VII of sample mission I, the $-x_E$-axis is located in the MOP, and the included angle measured eastward from the descending node is the adjusted value $(\beta_{\text{☾}})_{adj} = \beta_{\text{☾}} + \Delta\Phi_{TE}$. However, since $i_{VTL} > 10°$, $\Delta\Phi_{TE} = 0$ can be assumed, and $(\beta_{\text{☾}})_{adj} = 90°$ at t_p.

From Fig. 50 in the catalogue of Chapter IX, $\psi_0 = -2°$ for $i_{VTL} = 23.5°$ (inject south), $R_{\oplus\text{☾}} = 60$ ER, $\theta_{MTL} = 95°$, $i_m = 90°$, and $\gamma_0 = 0°$. However, mission specification 5 requires that $\gamma_0 = +5°$. Therefore, for $\gamma'_0 = 5°$ from empirical Eq (19), $\psi_0' = -2° + 10° = 8°$.

The injection latitude ϕ_0' may now be found by the latitude equation of step VIII in sample mission I, and it is $\phi_0' = -30°$.

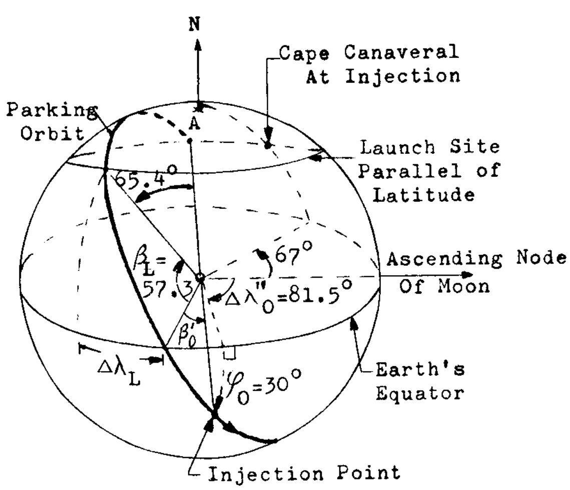
The injection longitude increments $\Delta\lambda_0'$ and $\Delta\lambda_0''$ can also be found from step VIII of sample mission I, and they are $\Delta\lambda_0' = 98.5°$, $\Delta\lambda_0'' = 81.5°$.

From step IV, the meridian of Cape Canaveral is 293° W of the ascending node at injection. Hence, $\lambda_0 = 148.5°$ W.

Step VI: Determine parking orbit parameters

A sketch of the actual situation at injection is shown below. The prime purpose in this step is to determine the longitude increment of the parking orbit ground trace measured from the meridian of Cape Canaveral to the intersection of the parallel of latitude of Cape Canaveral by the parking orbit. Furthermore, the central angle from this intersection to the injection point must be determined.

The same procedure as in step IX of sample mission I is followed.



For $i_{VE} = 34.5°$, $\beta_L = 57.3°$ and $\beta'_0 = 62°$ the injection takes place in the southern hemisphere and the nearest node of the parking orbit is the descending node.

The central angle from injection to the intersection of the launch site latitude at point A (requiring a launch azimuth of A = 70°) is

$$\beta_L + \beta_0' + 65.4° = 184.7° .$$

The longitude increment of β_L is $\Delta\lambda_L$ and of β'_0 is $\Delta\lambda'_{PO}$, where $\Delta\lambda_L = 52°$ and $\Delta\lambda'_{PO} = 57.3°$. Therefore the longitude increment $\Delta\lambda$ of the intersection point A of the parking orbit with the launch site latitude from Cape Canaveral is at t_0:

$$\Delta\lambda = 180° - \Delta\lambda_L + \Delta\lambda'_{PO} + 180° - 52.0°$$

$$+ 57.3° + (67° + 81.5°) = 333.8° \text{ W}$$

$$= 26.2° \text{ E}$$

The range angle that the spacecraft must traverse prior to reaching the injection point is $\Phi_{LO} = 184.7°$.

With the average angular velocity of the spacecraft in the parking orbit being 246.6 deg/hr, launch takes place 184.7 deg/246.6 deg/hr = 0.749 hr prior to t_0 at $t_L = 275^d.416 - 0.749$ hr $= 275^d.385$. However, at launch the actual location of Cape Canaveral is 37.5° W of point A, and, consequently, the parking orbit does not pass over the launch site. Assuming that the flight time to the moon and all other parameters generated so far are constant, a first-order correction to the launch date can be made by conducting the launch at an interval of time Δt later, which is the time required for the launch site to rotate beneath the parking orbit plane. In sample mission II

$$\Delta t = \frac{[26.2° + 15.04 \text{ deg/hr } (0.749 \text{ hr})]}{15.04 \text{ deg/hr}} =$$

$$2.49 \text{ hr}$$

and the corrected launch time is:

Launch time (corrected):

$$t_L = 275^d.416 - 0^h.749 + 2^h.49$$

$$= 275^d.489$$

Translunar injection time (corrected):

$$t_0 = 275^d.416 + 2^h.49$$

Lunar arrival time (corrected) =

$$280^d.00 + 2^h.49 = 280^d.104.$$

Step VII: Iteration with corrected arrival time

Steps II to VI are now repeated with the corrected arrival date. Each repeated step is denoted by an asterisk (*), and only the important parameters and their values are listed.

Step II*

For $t_p = 280.^d104$:

$$i_m \simeq 27.2°$$

$$\delta_M \equiv \delta_{☾} \simeq 27.2°$$

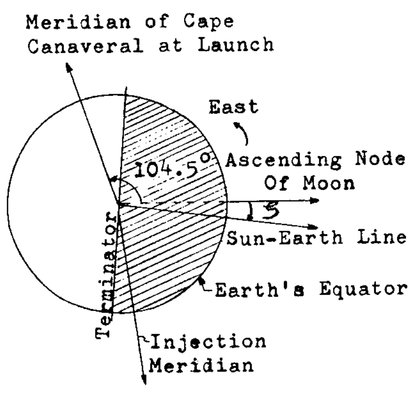
$$R_M \equiv R_{\oplus ☾} \simeq 59.8 \text{ ER}$$

$$\beta_{☾} \simeq 90°.$$

However, for this mission, from step II* it is immediately observed that insignificant changes take place in the lunar data for the corrected arrival date. In fact, very little change is detected in the following iterated steps. Therefore, it can be assumed that the translunar trajectory characteristics do not change, and, consequently, the determination of the timing and of other flight mechanics preliminary aspects is complete.

Step VIII: Lighting conditions at earth launch and injection

From step IV, $\zeta = 2.5°$, and the meridian of Cape Canaveral at launch (step VI) is 104.5° E of the ascending node of the moon. Also from step VI, the injection meridian is 81.5° W of the ascending node. The following sketch illustrates the lighting conditions.



Launch takes place in the morning and in daylight. Injection takes place in the evening, mostly in darkness.

For the inject south case taken here, the launch time represents the beginning of the launch tolerance for the variable translunar technique of Chapter V. Therefore, launch is possible throughout most of the morning in daylight.

Step IX: Required injection velocity

The required injection velocity, V_0, can be obtained from Fig. 49 of the catalogue in Chapter IX for the following parameters:

$$i_{VTL} = 23.5° \text{ (inject south)}$$

$$\theta_{MTL} = 95°$$

$$i_m = 90°$$

$$R_{\oplus ☾} = 60 \text{ ER}$$

$$\gamma_0 = 5°$$

$$h_0 = 182.9 \text{ km}$$

and it turns out to be

$$V_0 = 10929 \text{ m/sec.}$$

But mission specification 5 requires that $h_0 = 250$ km; therefore, Eq (20) of Section E is used to find V_0'

$$V_0' = \left[V_0^2 - V_p^2 \left(\frac{\Delta h_0}{r_0}\right) \right]^{1/2}$$

$$V_0' = 10861 \text{ m/sec}$$

Step X: ΔV_1 required to enter lunar orbit

The velocity impulse ΔV_1 required to reduce the vehicle speed to the value of circular lunar orbital speed at t_p is obtained from Fig. 47 of the catalogue in Chapter IX for the conditions listed in the previous step, and it is

$$\Delta V_1 = 780 \text{ m/sec.}$$

3. Concluding Remarks

The two sample missions illustrated the application of the translunar and circumlunar trajectory catalogue, and the material in Chapters V to X as well as of Sections A to F of Chapter XI to the design of nominal lunar missions. The next step would be to lift the two major assumptions underlying the calculations for the sample missions, namely:

(1) Use a constant angular velocity $\omega_{\oplus ☾}$ through a quarter of one revolution of the moon around the earth

(2) Neglect the ascent trajectory

and to recalculate the trajectory and lighting parameters for the nominal mission.

The second step is to calculate dispersed trajectories, i.e., to find trajectory and lighting parameters if earth launch or translunar injection or some other phase of the mission cannot occur at the nominal times and estimate navigation and guidance requirements. This is about as far as the Voice-calculated trajectories and catalogues can assist the mission designer. However, it should be noted with what ease all mission parameters can be obtained: a trajectory analyst can obtain all necessary data for one mission in a matter of hours as compared to the days of labor required to obtain these preliminary trajectory parameters by use of numerical integration of the equations of motion. The slow and tedious preliminary design of lunar missions by numerical integration is entirely due to the high sensitivity of translunar injection parameters to lunar mission constraints and requirements as was stated in Chapters III and IV.

The third, and final step for the preliminary design of a lunar mission is the use of the parameters and initial conditions established in Step I and II for numerically integrated trajectories; the accuracy of the resulting mission timing and profile depends on the force model and the numerical integration technique used for each mission phase, as was discussed in detail in Chapter IV.

The actual mechanization of a lunar mission involves the calculation of precision trajectories as opposed to the feasibility trajectories of the three steps discussed above in the preliminary design of lunar missions. Navigation charts would have to be prepared for this, and trajectory correction points and energy requirements established for the actual mission. This would typically occur several months before the actual mission is flown to allow ample time for checking and preparation.

Throughout the mission there will be requirements for trajectory calculations in the vehicle, as well as on earth to assimilate actual position data,to predict the trajectory, and to decide on the course of action to be followed if the actual trajectory does not conform to the nominal one.

Postflight analysis will provide inputs for future improvements in procedures and trajectory simulation and calculation.

H. REFERENCES

1. Woolston, O. S., "Declination, Radial Distance, and Phases of the Moon for the Years 1961 to 1971 for Use in Trajectory Considerations," NASA TN D-911, August 1961.

TABLES AND ILLUSTRATIONS

LIST OF TABLES AND ILLUSTRATIONS

LIST OF TABLES AND ILLUSTRATIONS (continued)

LIST OF TABLES AND ILLUSTRATIONS (continued)

LIST OF TABLES AND ILLUSTRATIONS (continued)

TABLE 1.- CALENDAR SHOWING NUMBER OF DAY IN YEAR

AFTER JANUARY 0

(a) Nonleap years: 1961, 1962, 1963, 1965, 1966, 1967, 1969, 1970, and 1971

Day of month	Day of year											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

TABLE 1.- CALENDAR SHOWING NUMBER OF DAY IN YEAR

AFTER JANUARY 0 - Concluded

(b) Leap years: 1964 and 1968

Day of month	Day of year											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	61	92	122	153	183	214	245	275	306	336
2	2	33	62	93	123	154	184	215	246	276	307	337
3	3	34	63	94	124	155	185	216	247	277	308	338
4	4	35	64	95	125	156	186	217	248	278	309	339
5	5	36	65	96	126	157	187	218	249	279	310	340
6	6	37	66	97	127	158	188	219	250	280	311	341
7	7	38	67	98	128	159	189	220	251	281	312	342
8	8	39	68	99	129	160	190	221	252	282	313	343
9	9	40	69	100	130	161	191	222	253	283	314	344
10	10	41	70	101	131	162	192	223	254	284	315	345
11	11	42	71	102	132	163	193	224	255	285	316	346
12	12	43	72	103	133	164	194	225	256	286	317	347
13	13	44	73	104	134	165	195	226	257	287	318	348
14	14	45	74	105	135	166	196	227	258	288	319	349
15	15	46	75	106	136	167	197	228	259	289	320	350
16	16	47	76	107	137	168	198	229	260	290	321	351
17	17	48	77	108	138	169	199	230	261	291	322	352
18	18	49	78	109	139	170	200	231	262	292	323	353
19	19	50	79	110	140	171	201	232	263	293	324	354
20	20	51	80	111	141	172	202	233	264	294	325	355
21	21	52	81	112	142	173	203	234	265	295	326	356
22	22	53	82	113	143	174	204	235	266	296	327	357
23	23	54	83	114	144	175	205	236	267	297	328	358
24	24	55	84	115	145	176	206	237	268	298	329	359
25	25	56	85	116	146	177	207	238	269	299	330	360
26	26	57	86	117	147	178	208	239	270	300	331	361
27	27	58	87	118	148	179	209	240	271	301	332	362
28	28	59	88	119	149	180	210	241	272	302	333	363
29	29	60	89	120	150	181	211	242	273	303	334	364
30	30		90	121	151	182	212	243	274	304	335	365
31	31		91		152		213	244		305		366

TABLE 2. - PHASES OF THE MOON

[Universal time]

(a) 1963

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
				Jan.	3	1	3	Jan.	9	23	9	Jan.	17	20	35
Jan.	25	13	43	Feb.	1	8	51	Feb.	8	14	53	Feb.	16	17	40
Feb.	24	2	7	Mar.	2	17	18	Mar.	10	7	50	Mar.	18	12	9
Mar.	25	12	11	Apr.	1	3	16	Apr.	9	0	58	Apr.	17	2	54
Apr.	23	20	30	Apr.	30	15	9	May	8	17	25	May	16	13	37
May	23	4	1	May	30	4	57	June	7	8	32	June	14	20	54
June	21	11	47	June	28	20	25	July	6	21	57	July	14	1	58
July	20	20	44	July	28	13	14	Aug.	5	9	32	Aug.	12	6	22
Aug.	19	7	36	Aug.	27	6	55	Sept.	3	19	35	Sept.	10	11	43
Sept.	17	20	52	Sept.	26	0	40	Oct.	3	4	45	Oct.	9	19	28
Oct.	17	12	44	Oct.	25	17	21	Nov.	1	13	56	Nov.	8	6	38
Nov.	16	6	52	Nov.	24	7	57	Nov.	30	23	55	Dec.	7	21	35
Dec.	16	2	8	Dec.	23	19	56	Dec.	30	11	5				

TABLE 2.- PHASES OF THE MOON - Continued

[Universal time]

(b.) 1964

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
												Jan.	6	15	59
Jan.	14	20	45	Jan.	22	5	30	Jan.	28	23	24	Feb.	5	12	43
Feb.	13	13	2	Feb.	20	13	25	Feb.	27	12	40	Mar.	6	10	1
Mar.	14	2	15	Mar.	20	20	40	Mar.	28	2	50	Apr.	5	5	46
Apr.	12	12	39	Apr.	19	4	10	Apr.	26	17	51	May	4	22	21
May	11	21	3	May	18	12	43	May	26	9	30	June	3	11	9
June	10	4	23	June	16	23	3	June	25	1	9	July	2	20	32
July	9	11	32	July	16	11	48	July	24	15	59	Aug.	1	3	30
Aug.	7	19	18	Aug.	15	3	21	Aug.	23	5	26	Aug.	30	9	16
Sept.	6	4	35	Sept.	13	21	25	Sept.	21	17	32	Sept.	28	15	3
Oct.	5	16	21	Oct.	13	16	58	Oct.	21	4	46	Oct.	27	22	0
Nov.	4	7	18	Nov.	12	12	21	Nov.	19	15	44	Nov.	26	7	12
Dec.	4	1	20	Dec.	12	6	3	Dec.	19	2	42	Dec.	25	19	28

TABLE 2.- PHASES OF THE MOON - Continued

[Universal time]

(c) 1965

New moon	d	h	m	First quarter	d	h	m	Full moon	d	h	m	Last quarter	d	h	m
Jan.	2	21	8	Jan.	10	21	1	Jan.	17	13	38	Jan.	24	11	8
Feb.	1	16	37	Feb.	9	8	54	Feb.	16	0	28	Feb.	23	5	40
Mar.	3	9	57	Mar.	10	17	53	Mar.	17	11	25	Mar.	25	1	38
Apr.	2	0	22	Apr.	9	0	41	Apr.	15	23	3	Apr.	23	21	8
May	1	11	57	May	8	6	21	May	15	11	53	May	23	14	41
May	30	21	14	June	6	12	12	June	14	2	1	June	22	5	37
June	29	4	53	July	5	19	37	July	13	17	3	July	21	17	54
July	28	11	46	Aug.	4	5	48	Aug.	12	8	23	Aug.	20	3	51
Aug.	26	18	51	Sept.	2	19	29	Sept.	10	23	33	Sept.	18	11	59
Sept.	25	3	19	Oct.	2	12	39	Oct.	10	14	15	Oct.	17	19	1
Oct.	24	14	12	Nov.	1	8	27	Nov.	9	4	16	Nov.	16	1	55
Nov.	23	4	11	Dec.	1	5	26	Dec.	8	17	22	Dec.	15	9	53
Dec.	22	21	4	Dec.	31	1	47								

TABLE 2. - PHASES OF THE MOON - Continued

[Universal time]

(d) 1966

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
								Jan.	7	5	17	Jan.	13	20	1
Jan.	21	15	47	Jan.	29	19	49	Feb.	5	15	59	Feb.	12	8	54
Feb.	20	10	50	Feb.	28	10	16	Mar.	7	1	46	Mar.	14	0	20
Mar.	22	4	47	Mar.	29	20	44	Apr.	5	11	14	Apr.	12	17	29
Apr.	20	20	36	Apr.	28	3	50	May	4	21	2	May	12	11	20
May	20	9	43	May	27	8	51	June	3	7	41	June	11	4	59
June	18	20	10	June	25	13	24	July	2	19	37	July	10	21	44
July	18	4	31	July	24	19	1	Aug.	1	9	6	Aug.	9	12	57
Aug.	16	11	49	Aug.	23	3	3	Aug.	31	0	15	Sept.	8	2	8
Sept.	14	19	14	Sept.	21	14	26	Sept.	29	16	48	Oct.	7	13	9
Oct.	14	3	53	Oct.	21	5	36	Oct.	29	10	1	Nov.	5	22	19
Nov.	12	14	27	Nov.	20	0	21	Nov.	28	2	41	Dec.	5	6	23
Dec.	12	3	14	Dec.	19	21	42	Dec.	27	17	44				

TABLE 2.- PHASES OF THE MOON - Continued

[Universal time]

(e) 1967

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
												Jan.	3	14	20
Jan.	10	18	7	Jan.	18	19	42	Jan.	26	6	41	Feb.	1	23	4
Feb.	9	10	45	Feb.	17	15	57	Feb.	24	17	44	Mar.	3	9	11
Mar.	11	4	31	Mar.	19	8	32	Mar.	26	3	22	Apr.	1	20	59
Apr.	9	22	21	Apr.	17	20	49	Apr.	24	12	4	May	1	10	34
May	9	14	56	May	17	5	19	May	23	20	23	May	31	1	53
June	8	5	15	June	15	11	13	June	22	4	58	June	29	18	40
July	7	17	1	July	14	15	54	July	21	14	40	July	29	12	15
Aug.	6	2	49	Aug.	12	20	45	Aug.	20	2	28	Aug.	28	5	36
Sept.	4	11	38	Sept.	11	3	6	Sept.	18	17	0	Sept.	26	21	45
Oct.	3	20	25	Oct.	10	12	12	Oct.	18	10	12	Oct.	26	12	5
Nov.	2	5	49	Nov.	9	1	1	Nov.	17	4	54	Nov.	25	0	24
Dec.	1	16	11	Dec.	8	17	58	Dec.	16	23	22	Dec.	24	10	49
Dec.	31	3	39												

TABLE 2.- PHASES OF THE MOON - Continued

[Universal time]

(f) 1968

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
				Jan.	7	14	24	Jan.	15	16	12	Jan.	22	19	39
Jan.	29	16	30	Feb.	6	12	21	Feb.	14	6	44	Feb.	21	3	29
Feb.	28	6	56	Mar.	7	9	21	Mar.	14	18	53	Mar.	21	11	8
Mar.	28	22	49	Apr.	6	3	28	Apr.	13	4	52	Apr.	19	19	36
Apr.	27	15	22	May	5	17	55	May	12	13	6	May	19	5	45
May	27	7	31	June	4	4	47	June	10	20	14	June	17	18	14
June	25	22	25	July	3	12	42	July	10	3	18	July	17	9	12
July	25	11	50	Aug.	1	18	35	Aug.	8	11	33	Aug.	16	2	14
Aug.	23	23	57	Aug.	30	23	35	Sept.	6	22	8	Sept.	14	20	32
Sept.	22	11	9	Sept.	29	5	7	Oct.	6	11	47	Oct.	14	15	6
Oct.	21	21	45	Oct.	28	12	40	Nov.	5	4	26	Nov.	13	8	54
Nov.	20	8	2	Nov.	26	23	31	Dec.	4	23	8	Dec.	13	0	50
Dec.	19	18	19	Dec.	26	14	15								

TABLE 2.- PHASES OF THE MOON - Continued

[Universal time]

(g) 1969

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
								Jan.	3	18	28	Jan.	11	14	1
Jan.	18	4	59	Jan.	25	8	24	Feb.	2	12	57	Feb.	10	0	9
Feb.	16	16	26	Feb.	24	4	31	Mar.	4	5	18	Mar.	11	7	45
Mar.	18	4	52	Mar.	26	0	49	Apr.	2	18	46	Apr.	9	13	59
Apr.	16	18	16	Apr.	24	19	45	May	2	5	14	May	8	20	12
May	16	8	27	May	24	12	16	May	31	13	19	June	7	3	40
June	14	23	9	June	23	1	45	June	29	20	4	July	6	13	18
July	14	14	12	July	22	12	10	July	29	2	46	Aug.	5	1	39
Aug.	13	5	17	Aug.	20	20	4	Aug.	27	10	33	Sept.	3	16	58
Sept.	11	19	56	Sept.	19	2	25	Sept.	25	20	21	Oct.	3	11	6
Oct.	11	9	40	Oct.	18	8	32	Oct.	25	8	45	Nov.	2	7	14
Nov.	9	22	12	Nov.	16	15	46	Nov.	23	23	54	Dec.	2	3	51
Dec.	9	9	42	Dec.	16	1	10	Dec.	23	17	36	Dec.	31	22	53

TABLE 2.- PHASES OF THE MOON - Continued

[Universal time]

(h) 1970

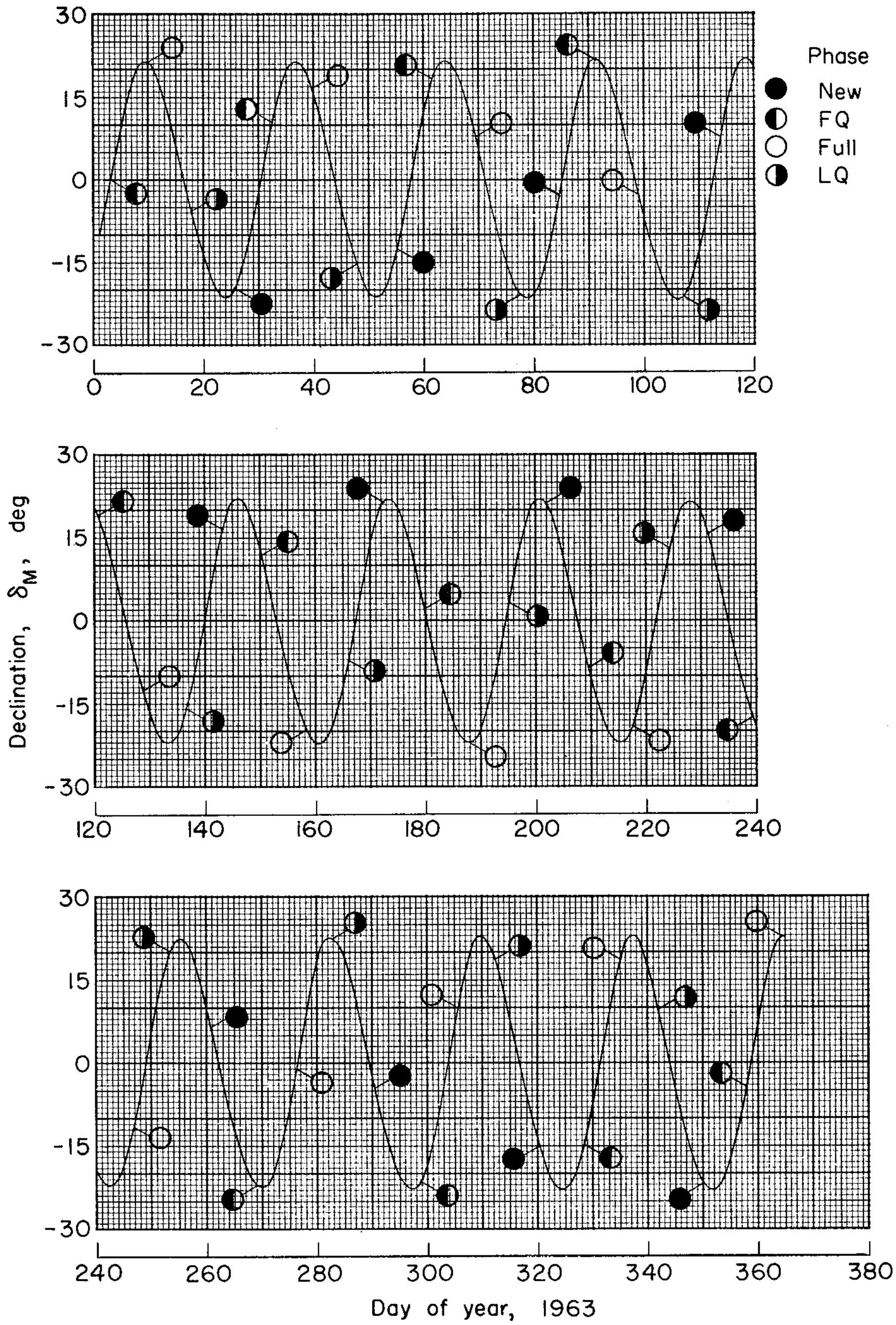
New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	7	20	36	Jan.	14	13	19	Jan.	22	12	56	Jan.	30	14	39
Feb.	6	7	13	Feb.	13	4	11	Feb.	21	8	19	Mar.	1	2	34
Mar.	7	17	43	Mar.	14	21	16	Mar.	23	1	53	Mar.	30	11	5
Apr.	6	4	10	Apr.	13	15	44	Apr.	21	16	22	Apr.	28	17	19
May	5	14	51	May	13	10	27	May	21	3	38	May	27	22	32
June	4	2	22	June	12	4	7	June	19	12	28	June	26	4	2
July	3	15	18	July	11	19	43	July	18	19	59	July	25	11	0
Aug.	2	5	59	Aug.	10	8	50	Aug.	17	3	16	Aug.	23	20	35
Aug.	31	22	2	Sept.	8	19	39	Sept.	15	11	10	Sept.	22	9	43
Sept.	30	14	32	Oct.	8	4	43	Oct.	14	20	22	Oct.	22	2	48
Oct.	30	6	28	Nov.	6	12	47	Nov.	13	7	28	Nov.	20	23	14
Nov.	28	21	15	Dec.	5	20	36	Dec.	12	21	4	Dec.	20	21	9
Dec.	28	10	43												

TABLE 2.- PHASES OF THE MOON - Concluded

[Universal time]

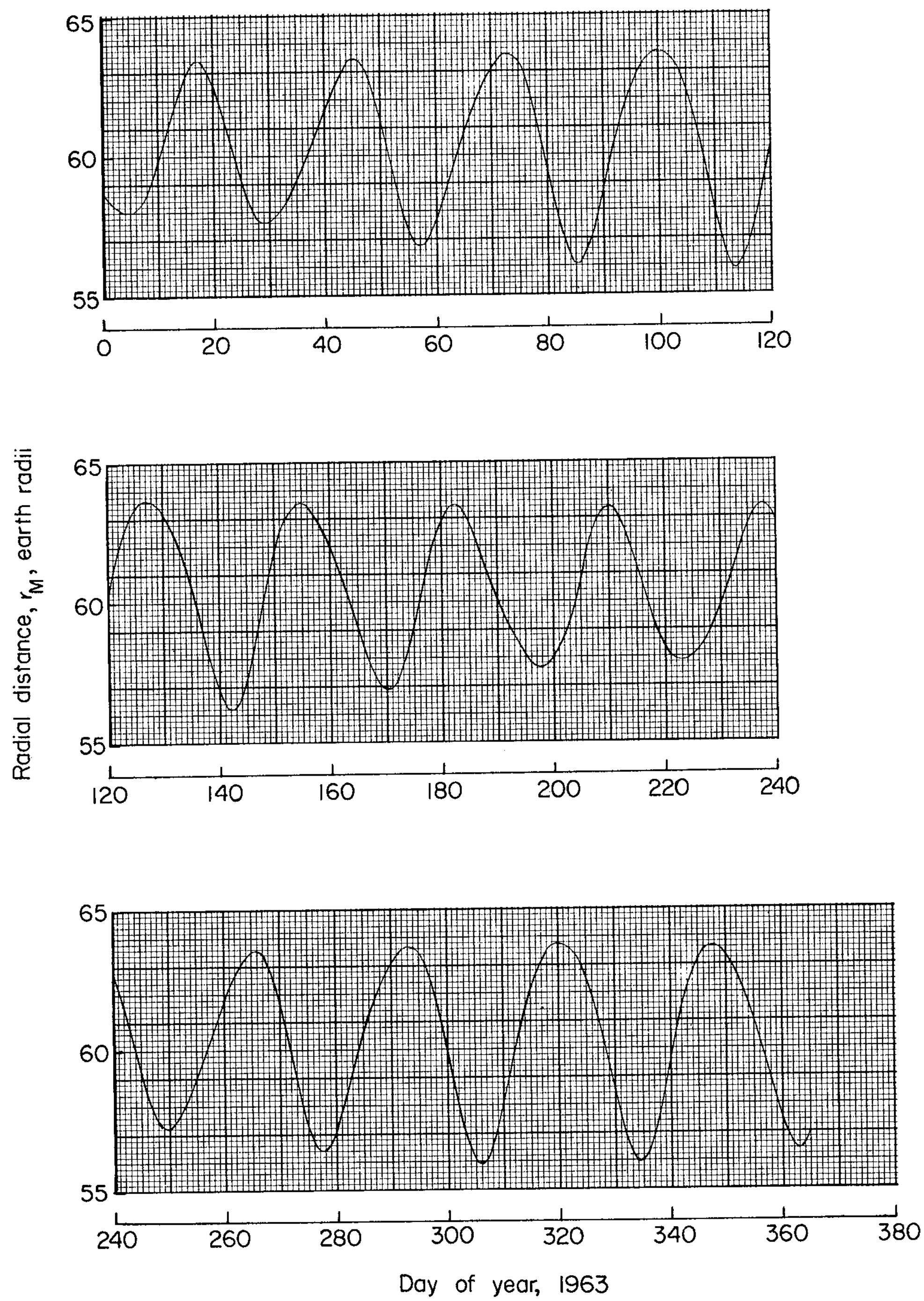
(i) 1971

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
				Jan.	4	4	55	Jan.	11	13	21	Jan.	19	18	9
Jan.	26	22	55	Feb.	2	14	31	Feb.	10	7	42	Feb.	18	12	14
Feb.	25	9	49	Mar.	4	2	1	Mar.	12	2	33	Mar.	20	2	30
Mar.	26	19	24	Apr.	2	15	46	Apr.	10	20	10	Apr.	18	12	58
Apr.	25	4	2	May	2	7	34	May	10	11	24	May	17	20	15
May	24	12	32	June	1	0	43	June	9	0	4	June	16	1	25
June	22	21	58	June	30	18	11	July	8	10	37	July	15	5	47
July	22	9	15	July	30	11	7	Aug.	6	19	43	Aug.	13	10	55
Aug.	20	22	54	Aug.	29	2	56	Sept.	5	4	3	Sept.	11	18	23
Sept.	19	14	43	Sept.	27	17	18	Oct.	4	12	20	Oct.	11	5	30
Oct.	19	7	59	Oct.	27	5	55	Nov.	2	21	20	Nov.	9	20	52
Nov.	18	1	46	Nov.	25	16	37	Dec.	2	7	48	Dec.	9	16	3
Dec.	17	19	3												



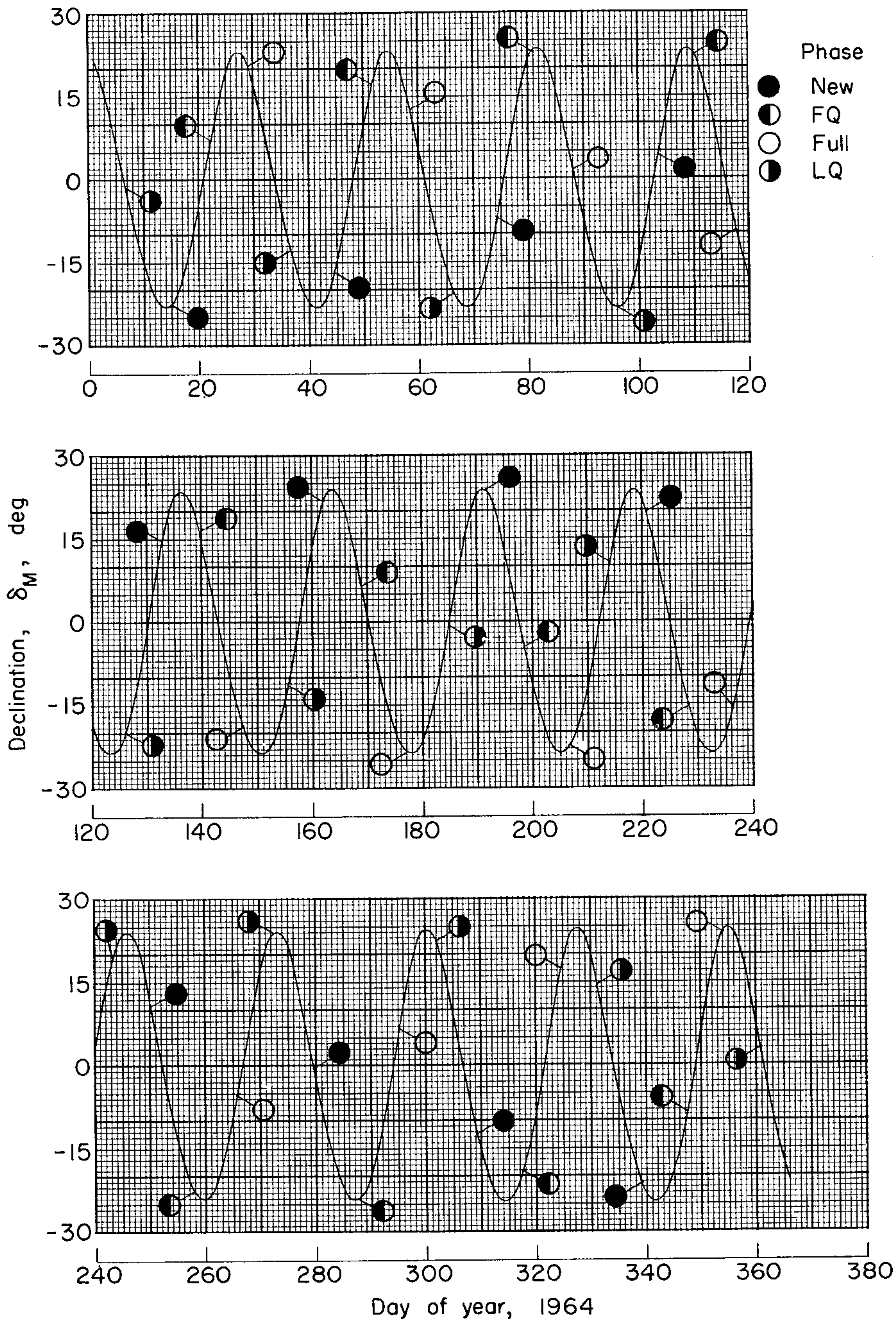
(a) Declination and phases.

Figure 1.- Declination and phases and radial distance of moon for year 1963.



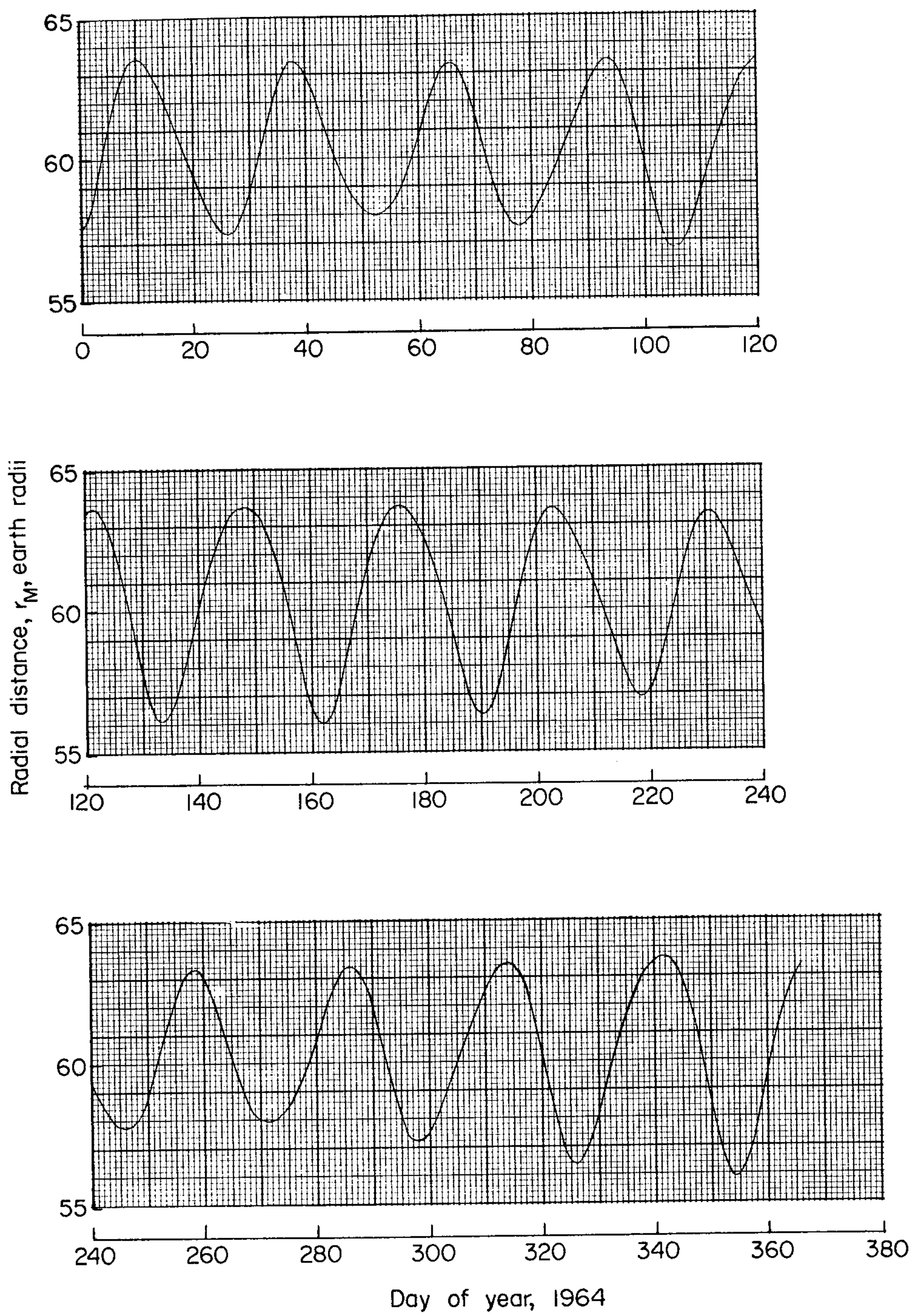
(b) Radial distance.

Figure 1. - Concluded.



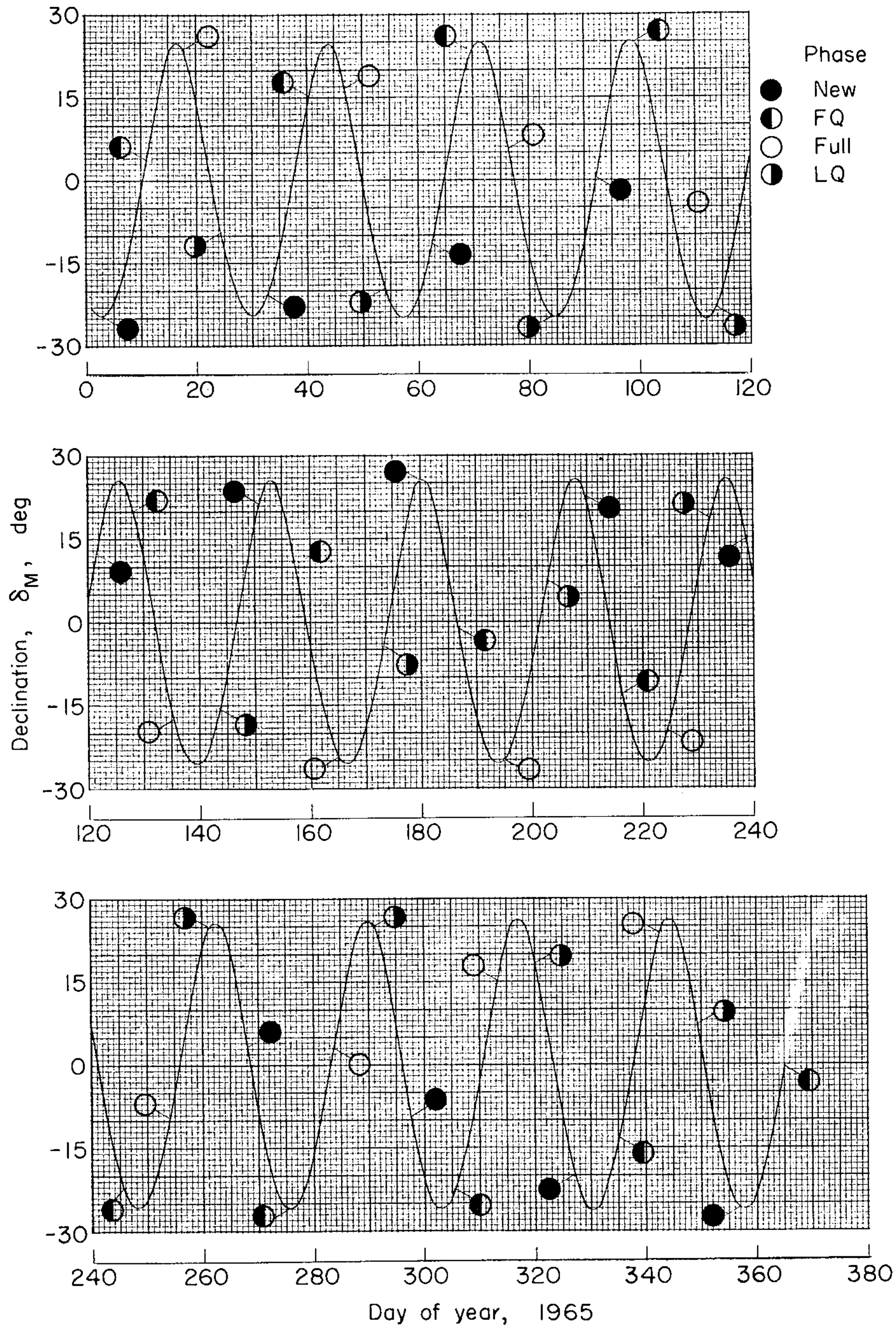
(a) Declination and phases.

Figure 2.- Declination and phases and radial distance of moon for year 1964.



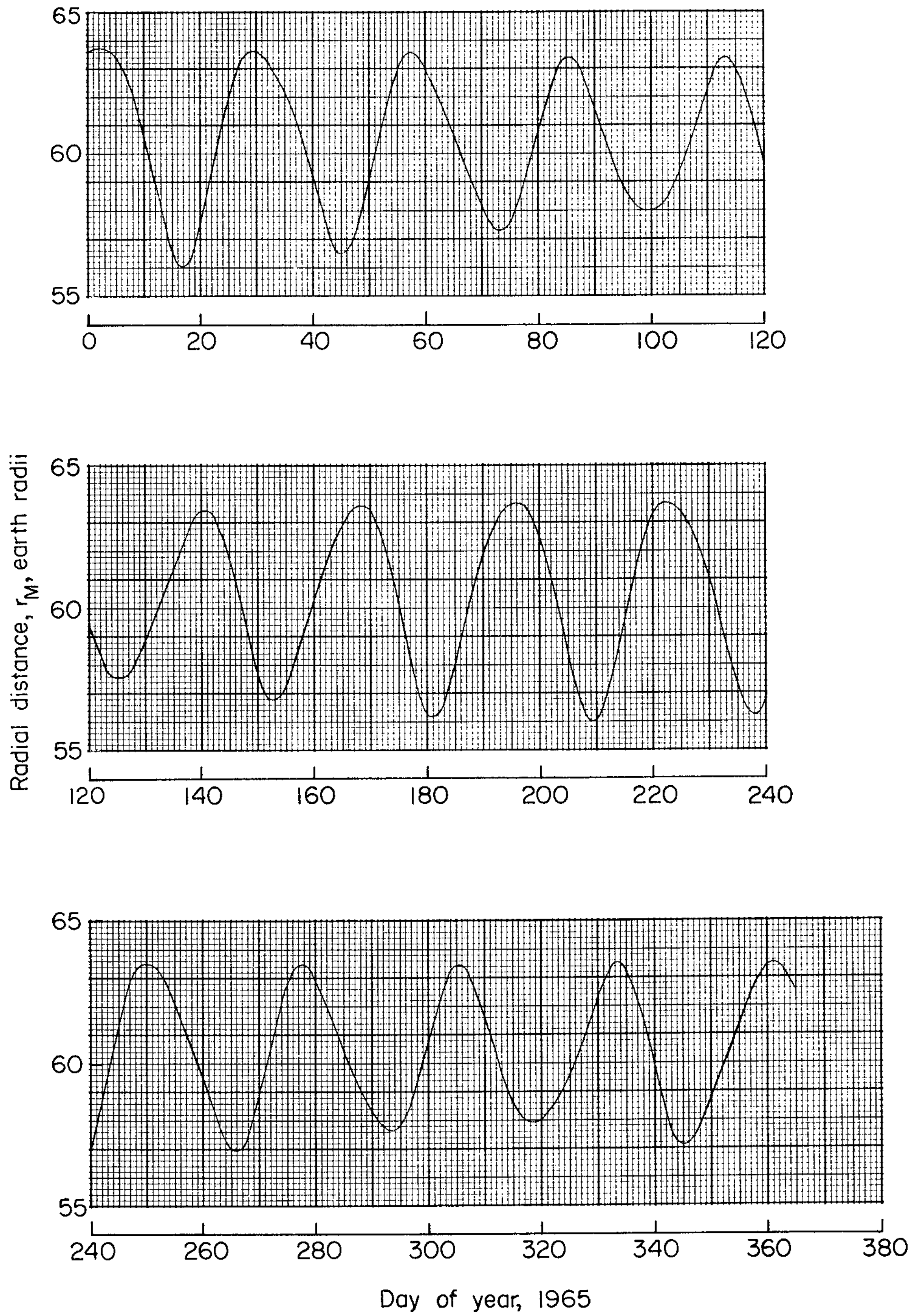
(b) Radial distance.

Figure 2.- Concluded.



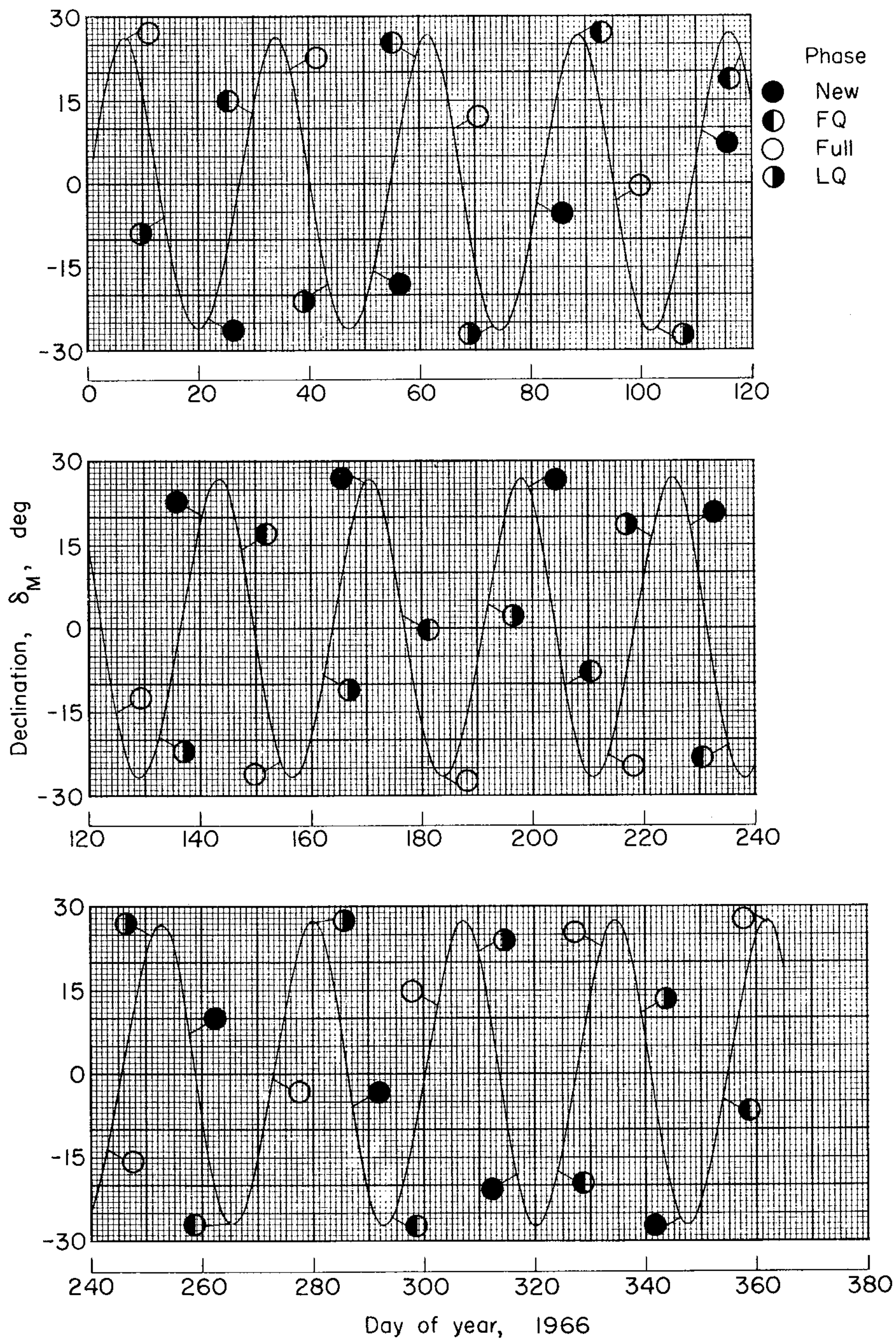
(a) Declination and phases.

Figure 3.- Declination and phases and radial distance of moon for year 1965.



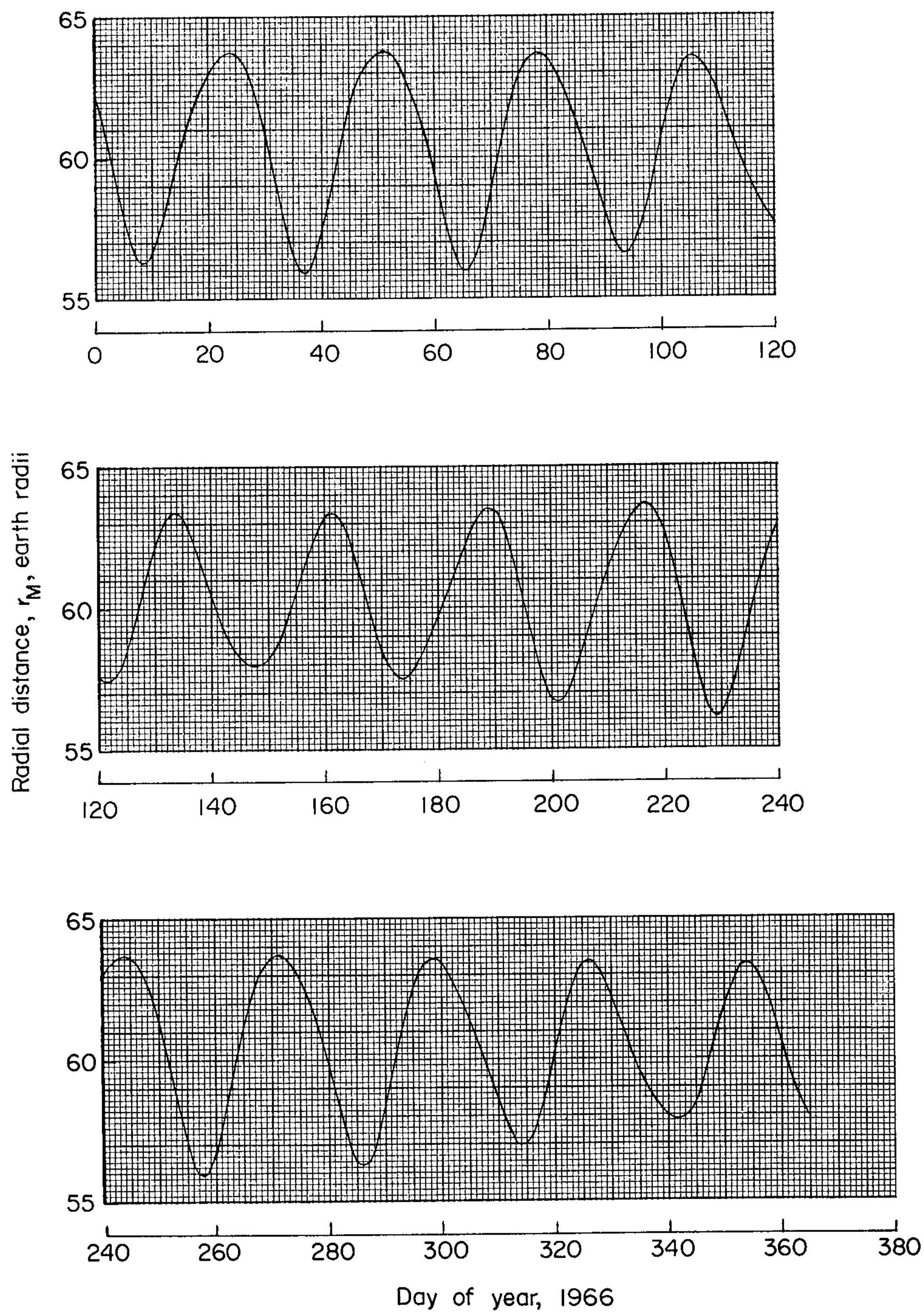
(b) Radial distance.

Figure 3.- Concluded.



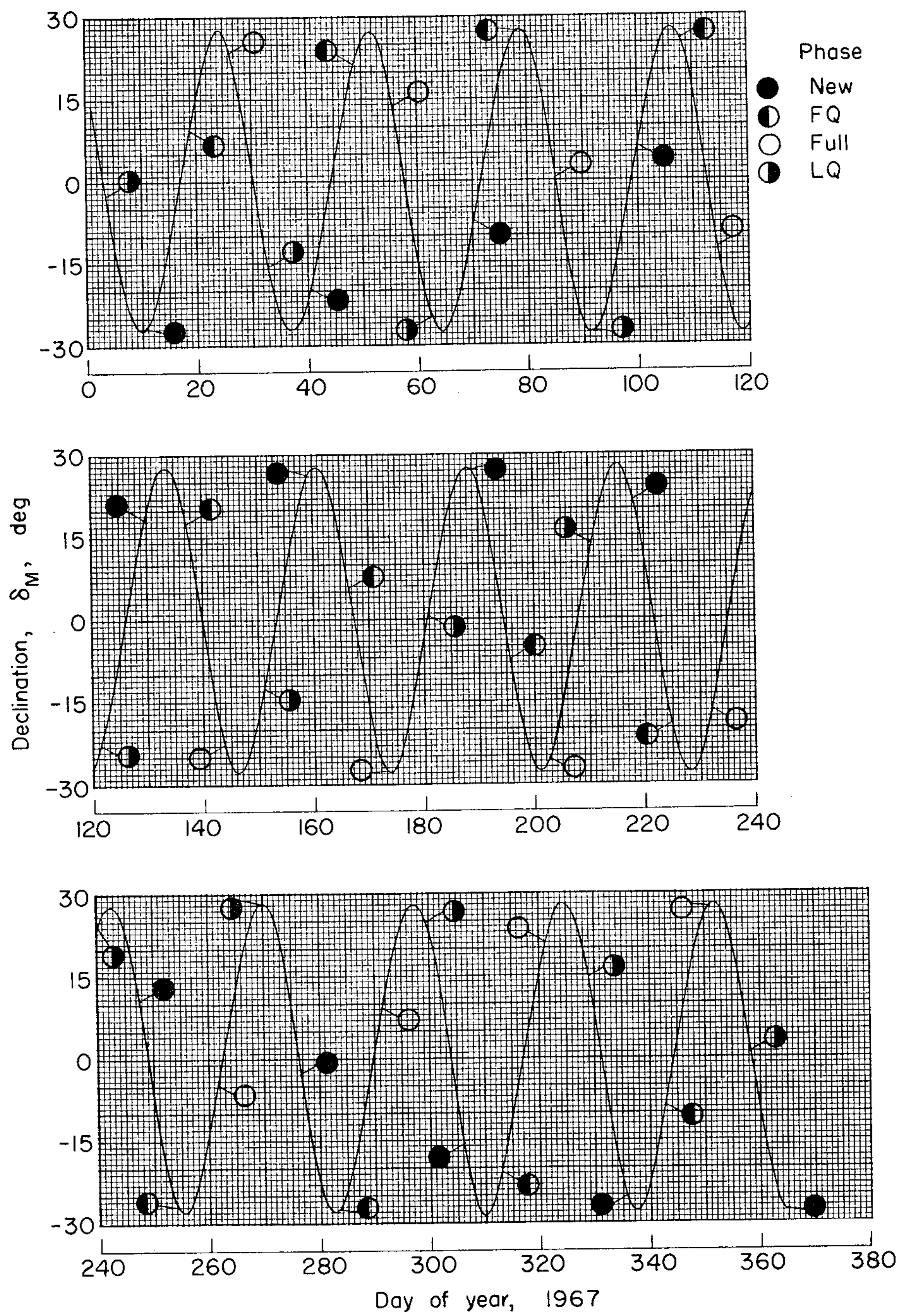
(a) Declination and phases.

Figure 4.- Declination and phases and radial distance of moon for year 1966.



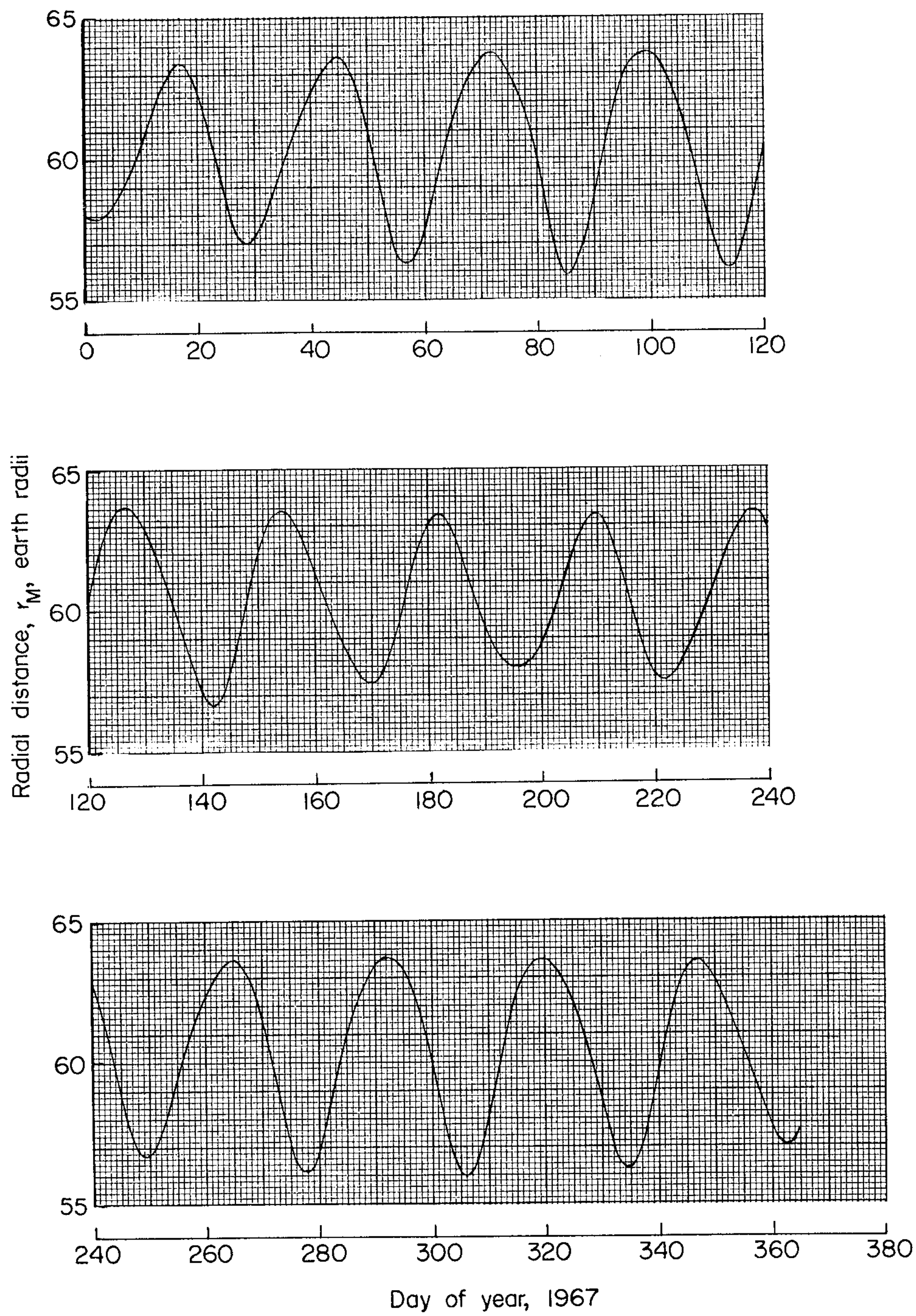
(b) Radial distance.

Figure 4. - Concluded.



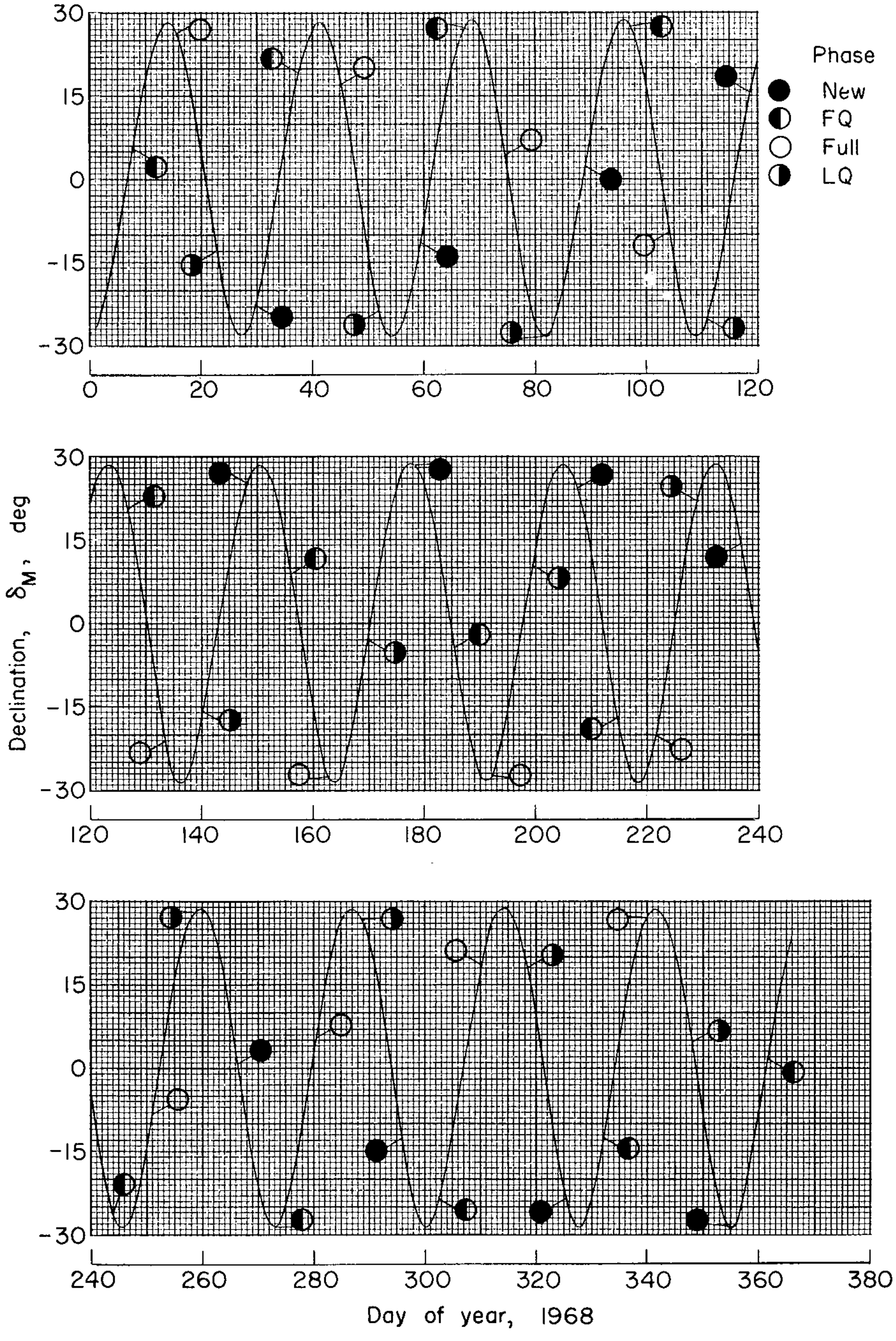
(a) Declination and phases.

Figure 5.- Declination and phases and radial distance of moon for year 1967.



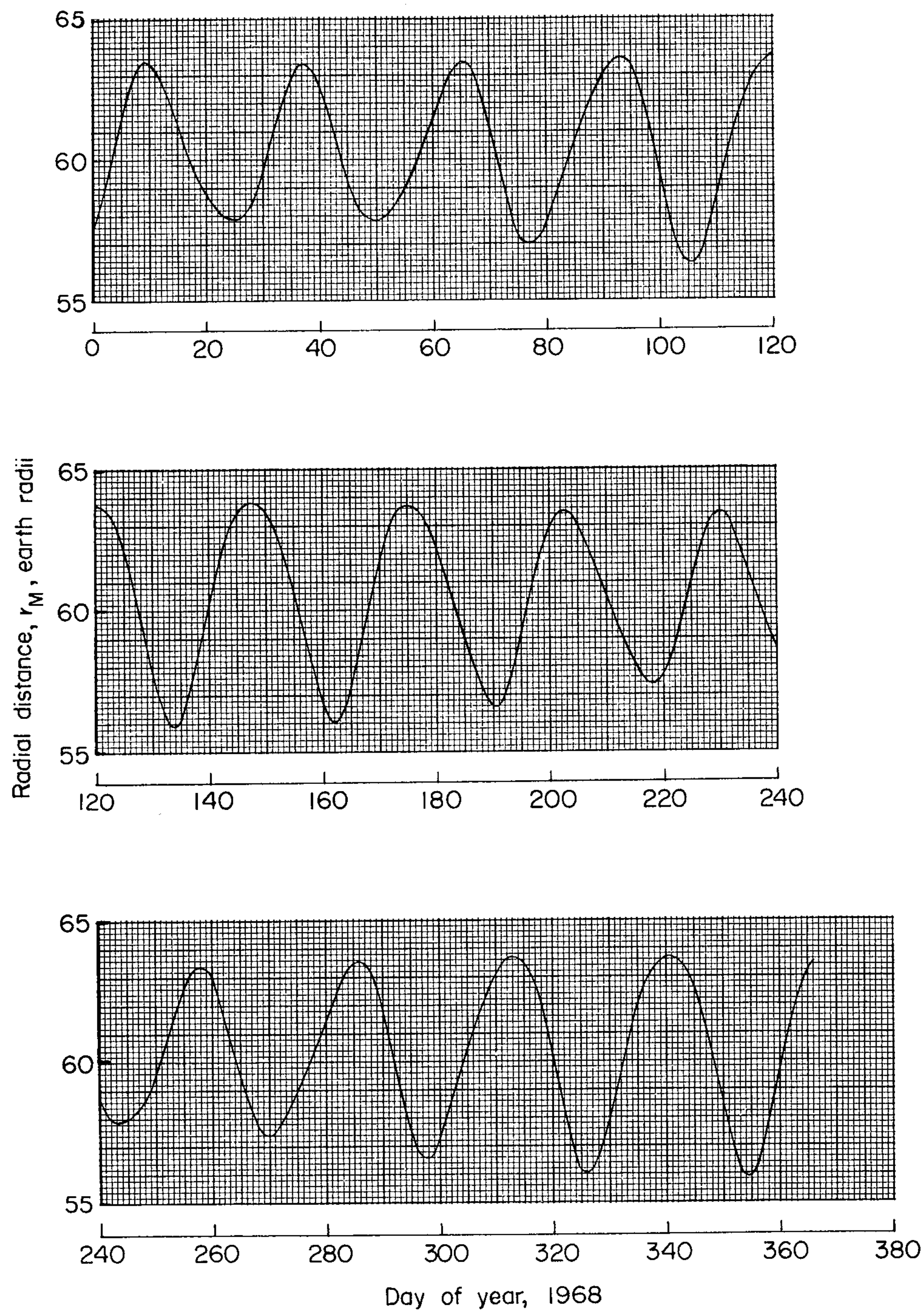
(b) Radial distance.

Figure 5.- Concluded.



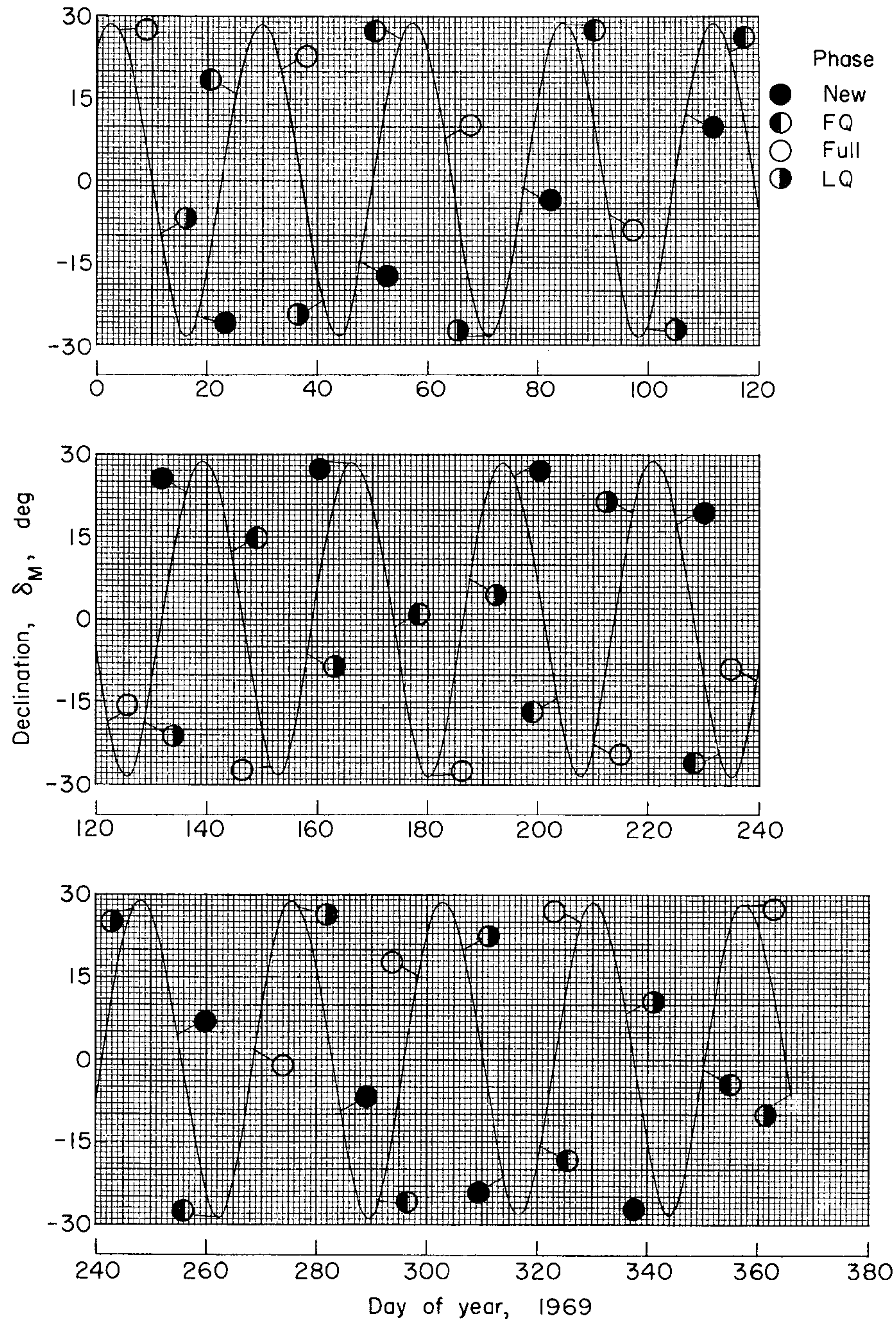
(a) Declination and phases.

Figure 6. - Declination and phases and radial distance of moon for year 1968.



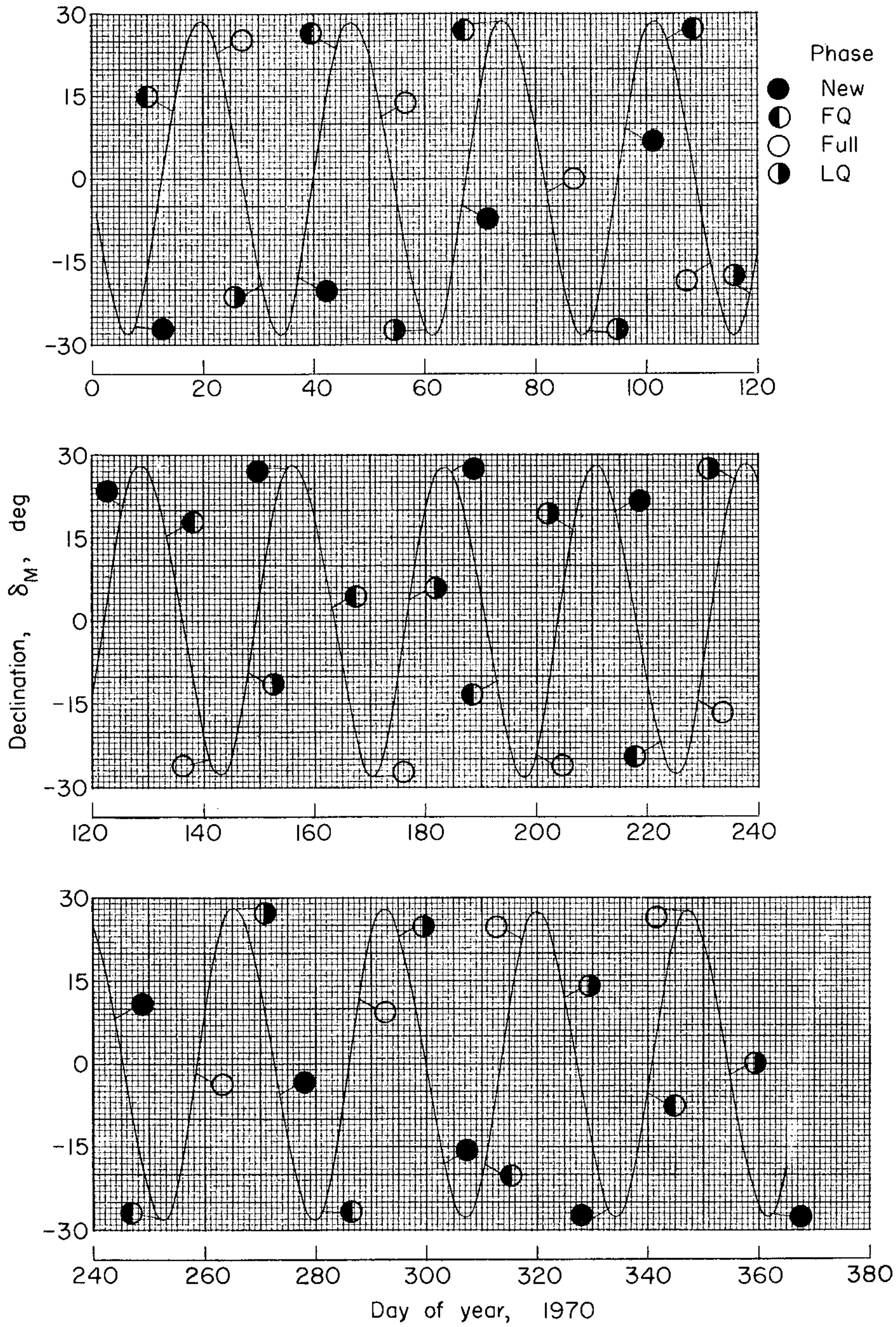
(b) Radial distance.

Figure 6.- Concluded.



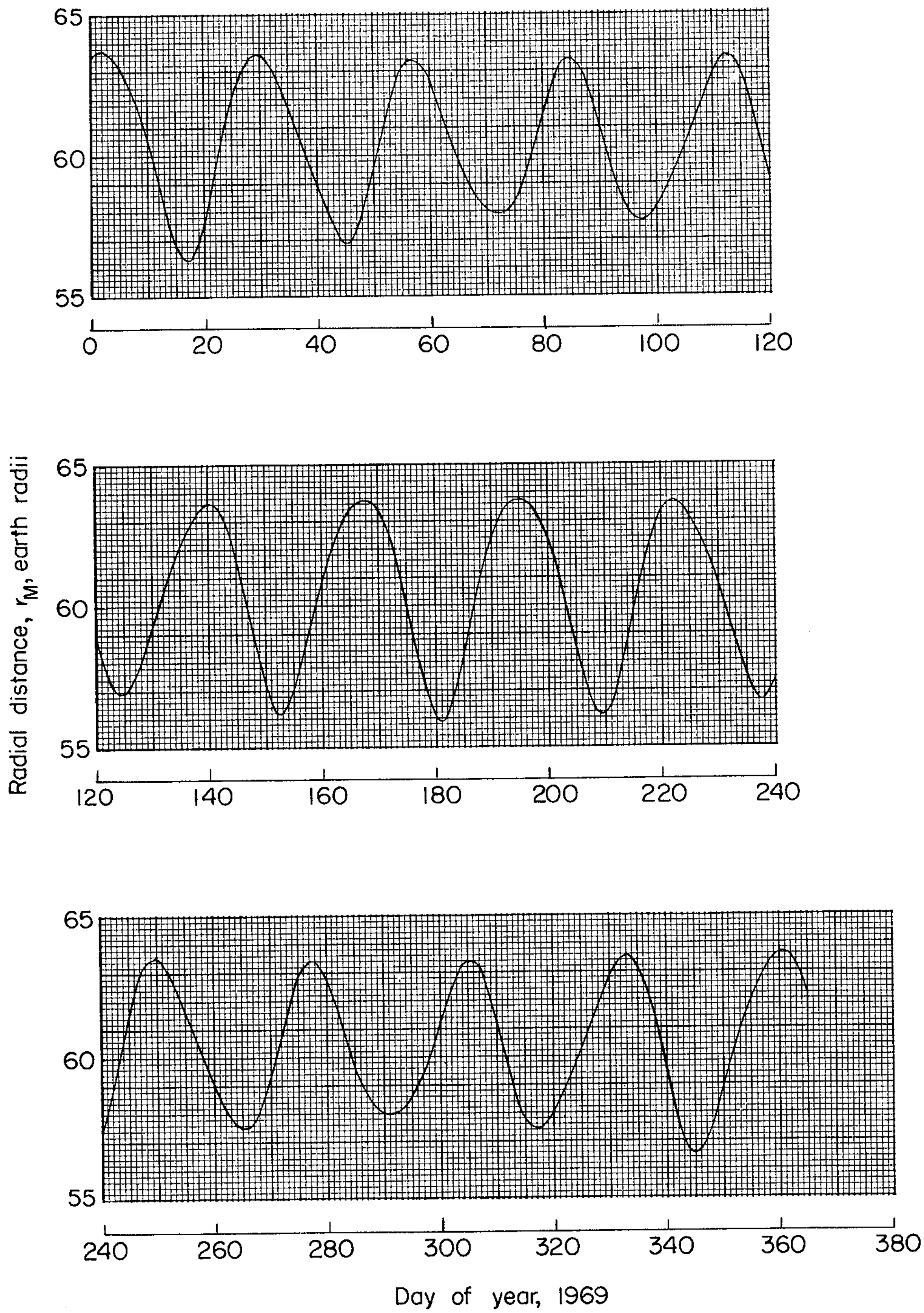
(a) Declination and phases.

Figure 7. - Declination and phases and radial distance of moon for year 1969.



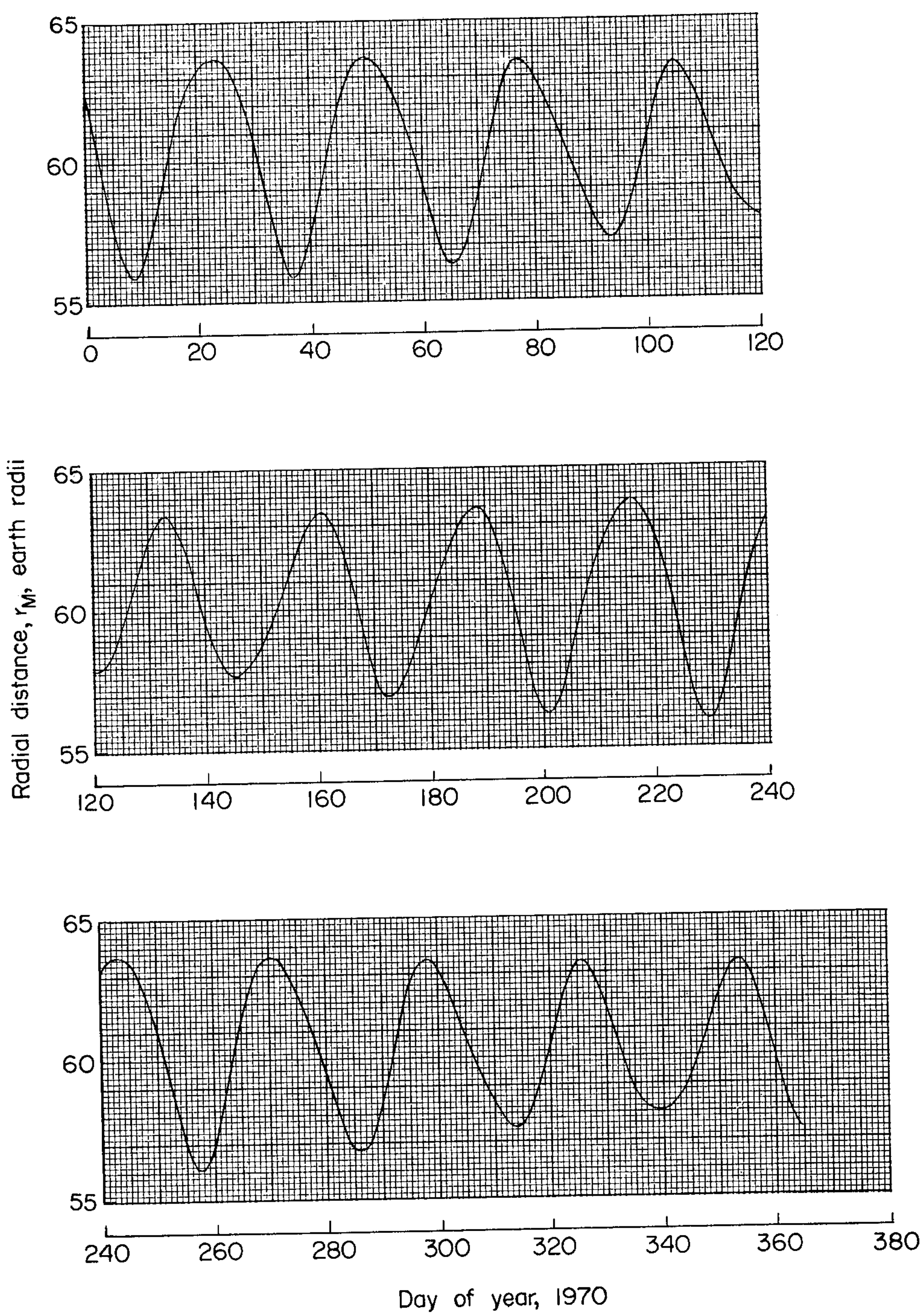
(a) Declination and phases.

Figure 8.- Declination and phases and radial distance of moon for year 1970.



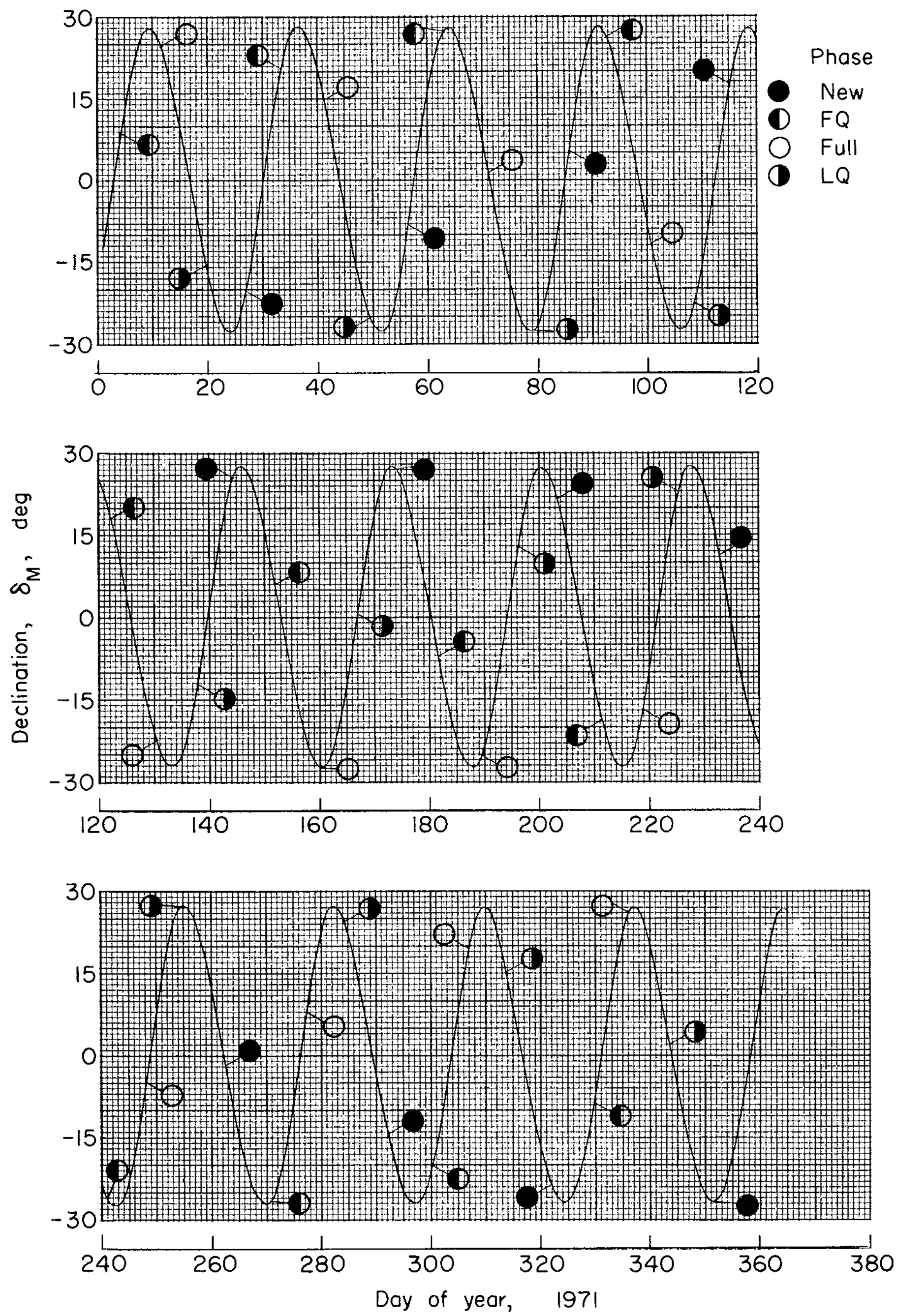
(b) Radial distance.

Figure 7. - Concluded.



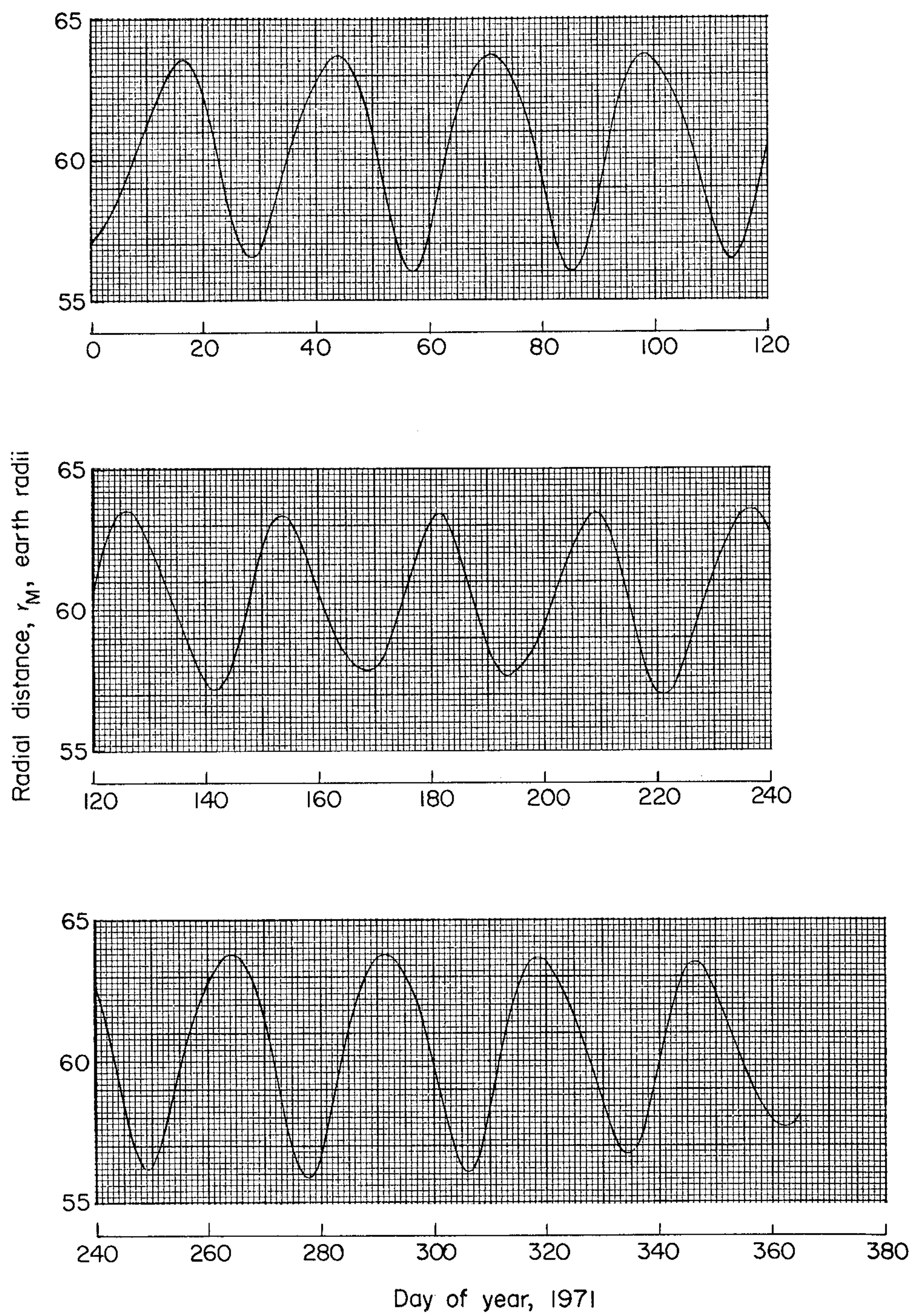
(b) Radial distance.

Figure 8. - Concluded.



(a) Declination and phases.

Figure 9.- Declination and phases and radial distance of moon for year 1971.



(b) Radial distance.

Figure 9. - Concluded.

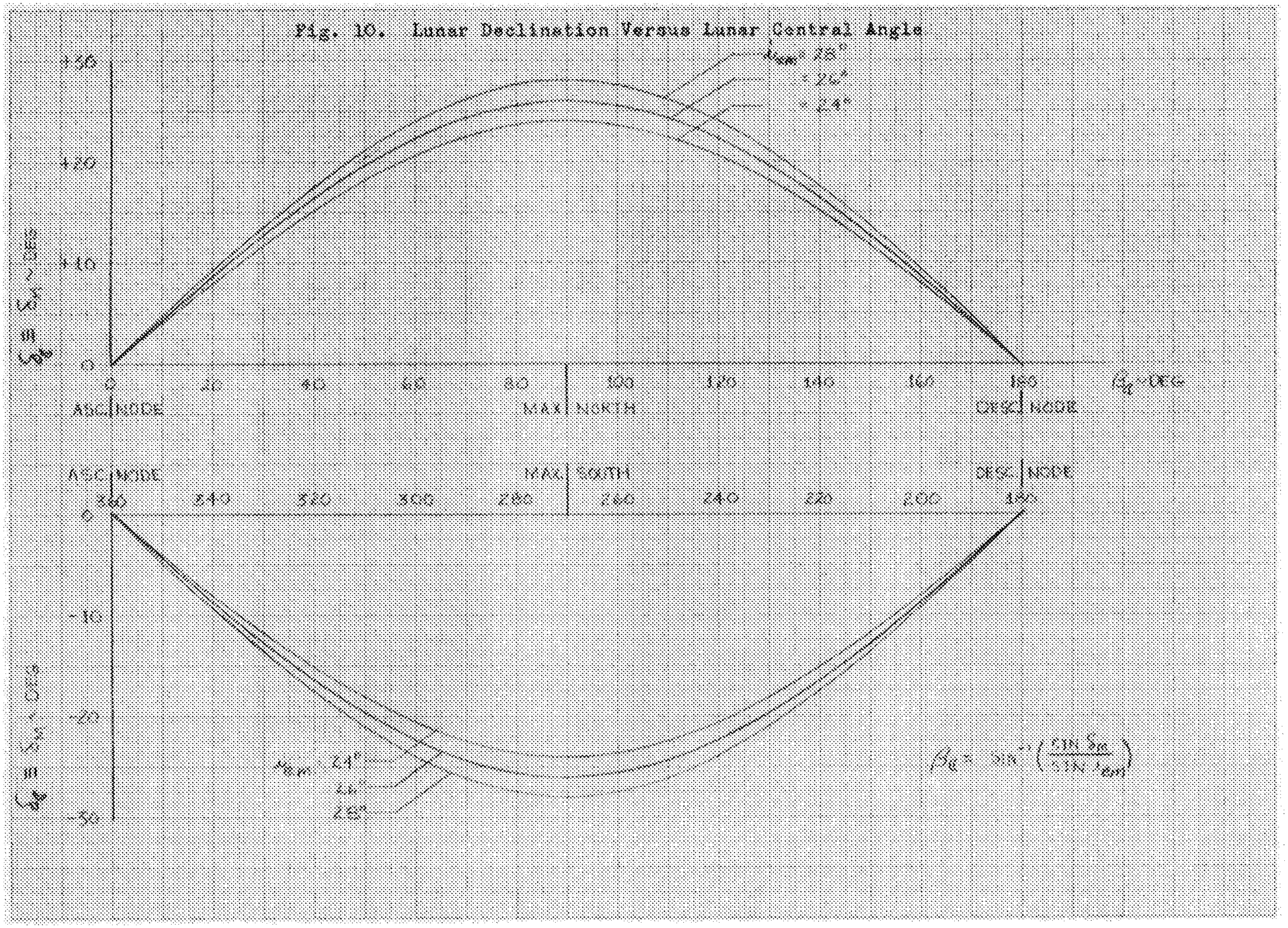


Fig. 10. Lunar Declination Versus Lunar Central Angle

Fig. 11. Required Translunar Plane Inclination (i_{VTL})
As A Function Of Lunar Central Angle ($\beta_{☾}$)

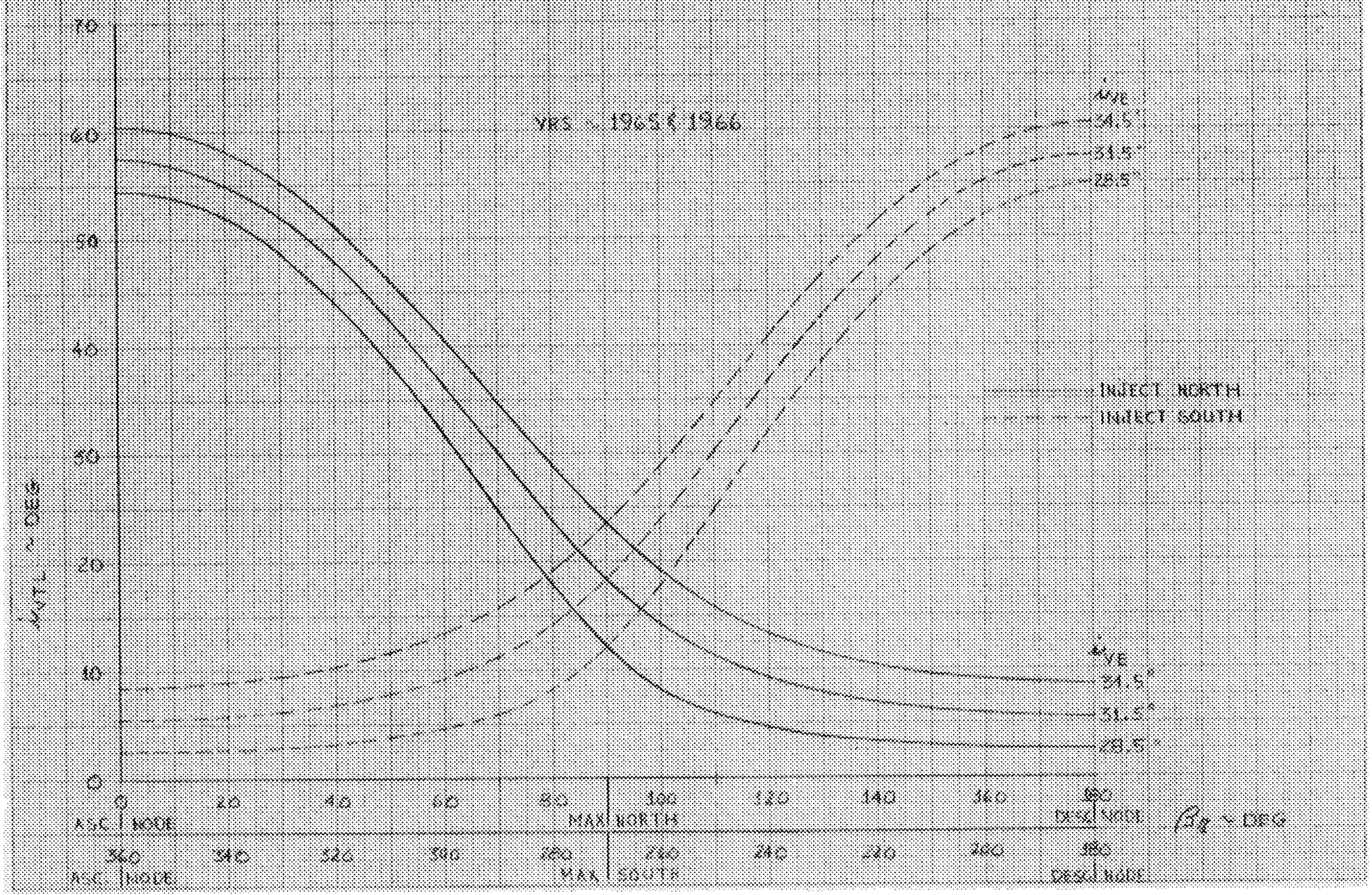


Fig. 12. Required Translunar Plane Inclination (i_{VTL})
As A Function Of Lunar Central Angle ($\beta_{☾}$)

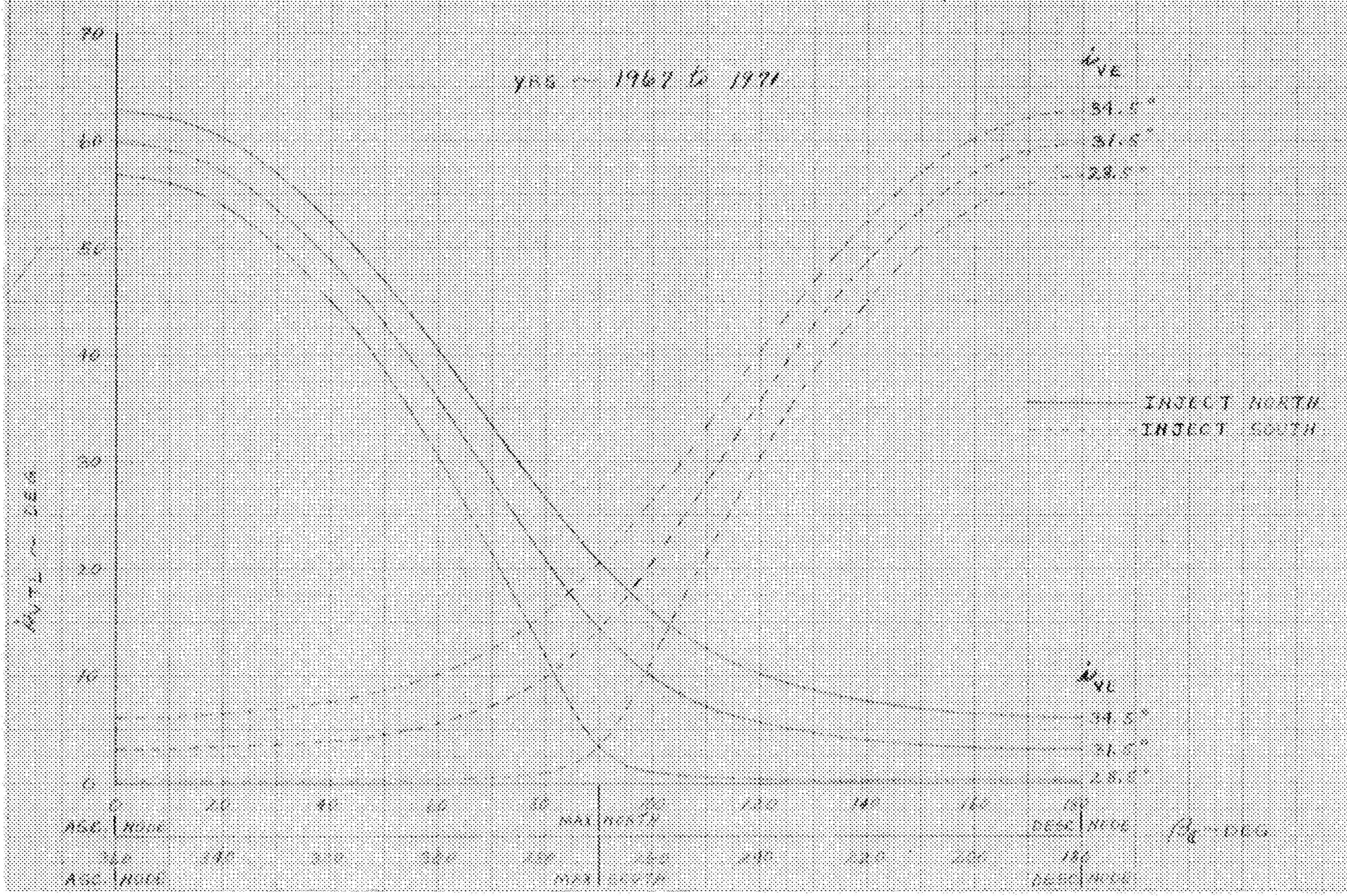
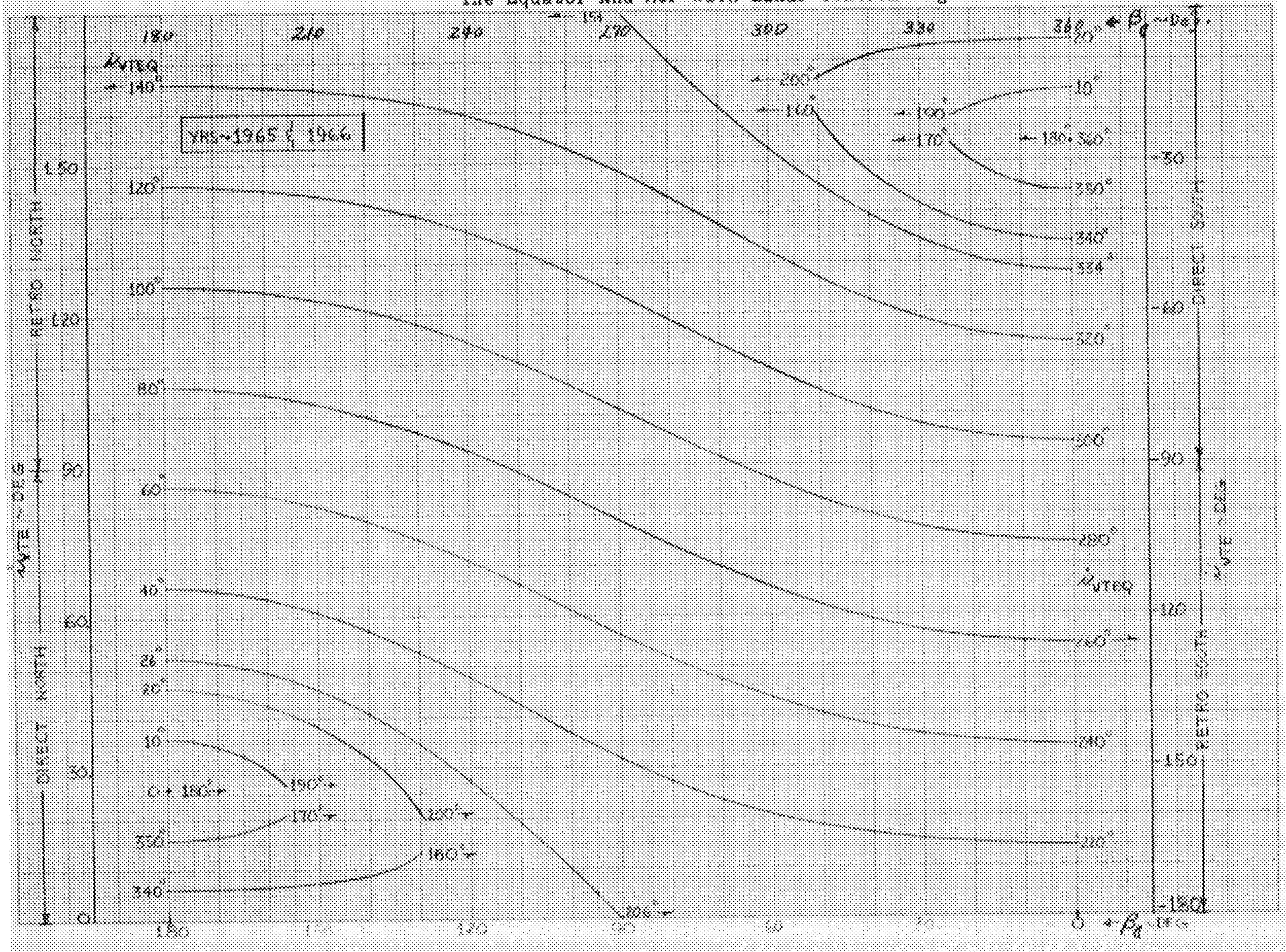


Fig. 13. Relationship Of Transearth Inclination To The Equator And MOP With Lunar Central Angle



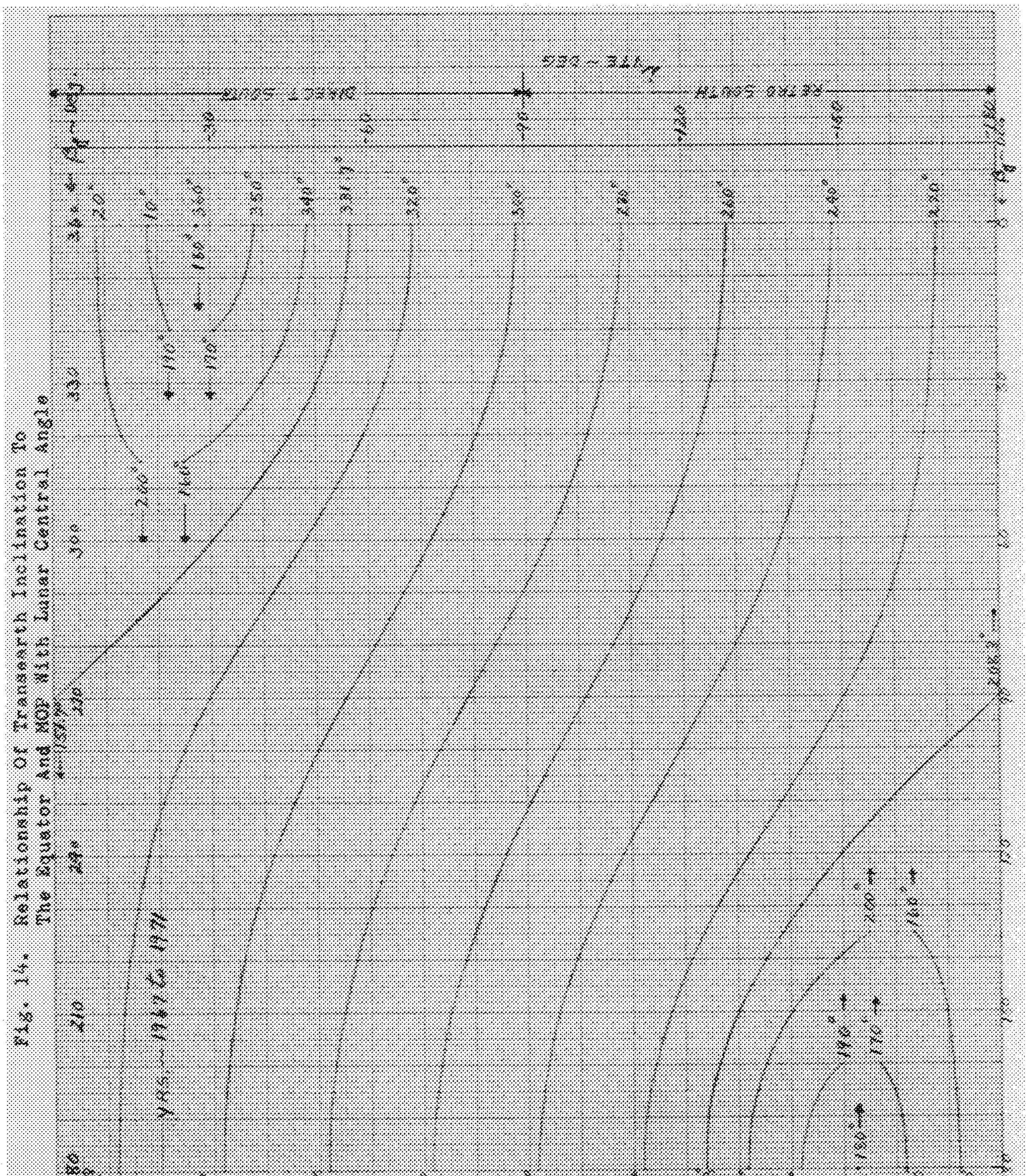
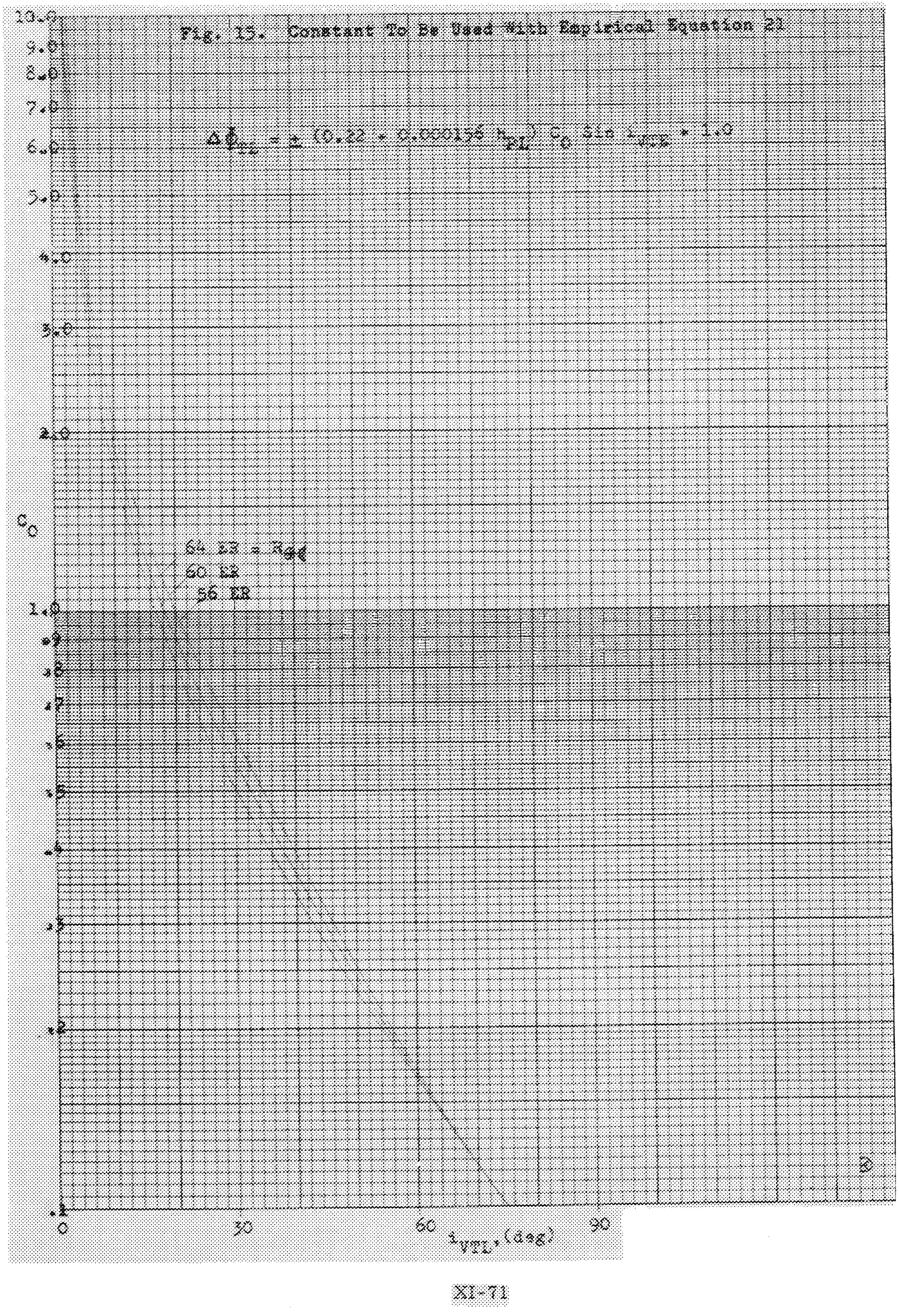


Fig. 14. Relationship Of Transearth Inclination To The Equator And MOP With Lunar Central Angle

Fig. 15. Constant To Be Used With Empirical Equation 21



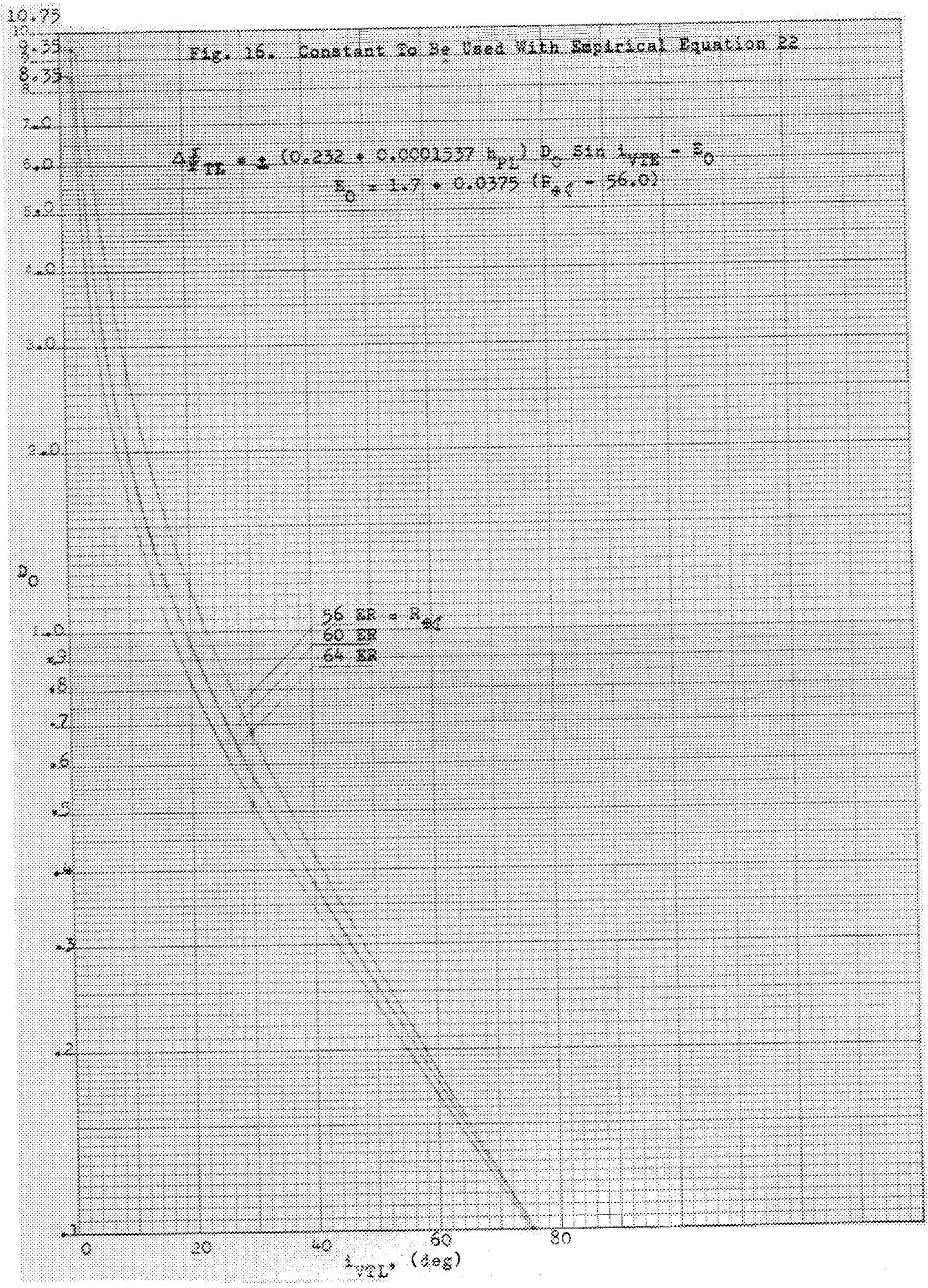


Fig. 16. Constant To Be Used With Empirical Equation 22

Fig. 17. ΔV_{TE} Required To Control Transearth Perigee Time

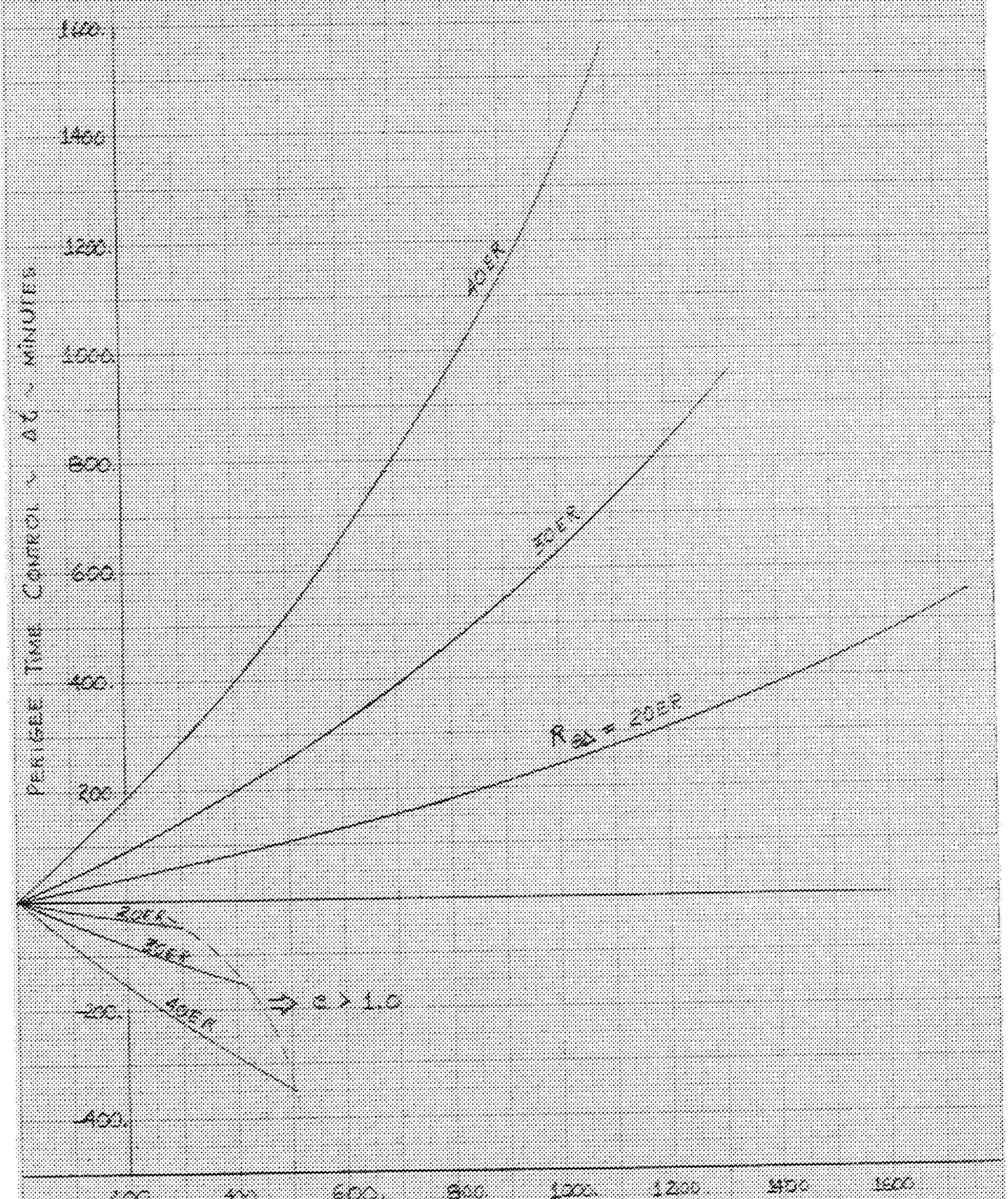


Fig. 18. ΔV_i Required To Change Transearth Plane Inclination

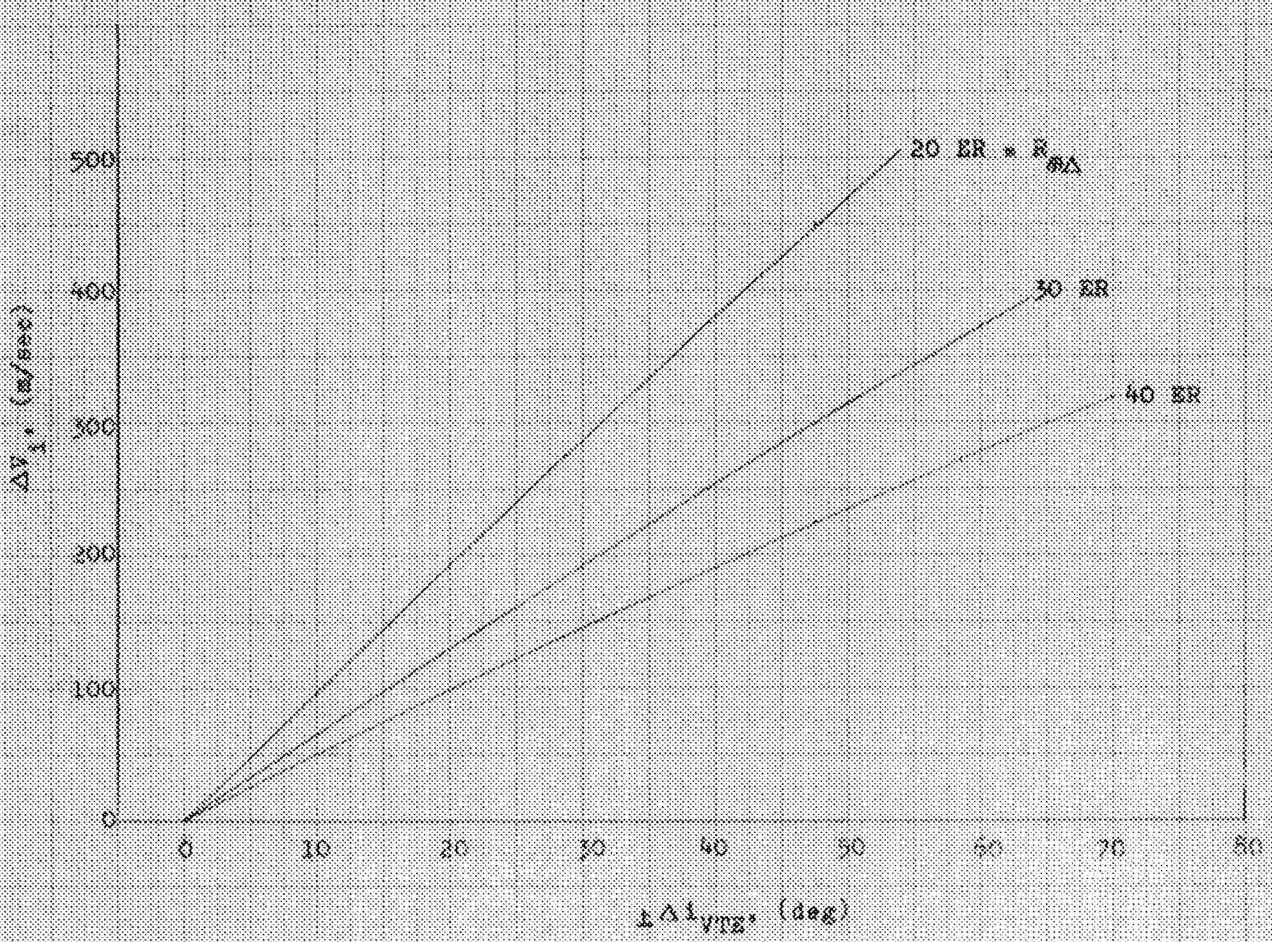


Fig. 19. Angular Change ($\Delta\oint_{TE}'$) Of The Transearth Plane Intersection With The MOP Due To An Inclination Change ($\pm\Delta i_{VTE}$) Maneuver

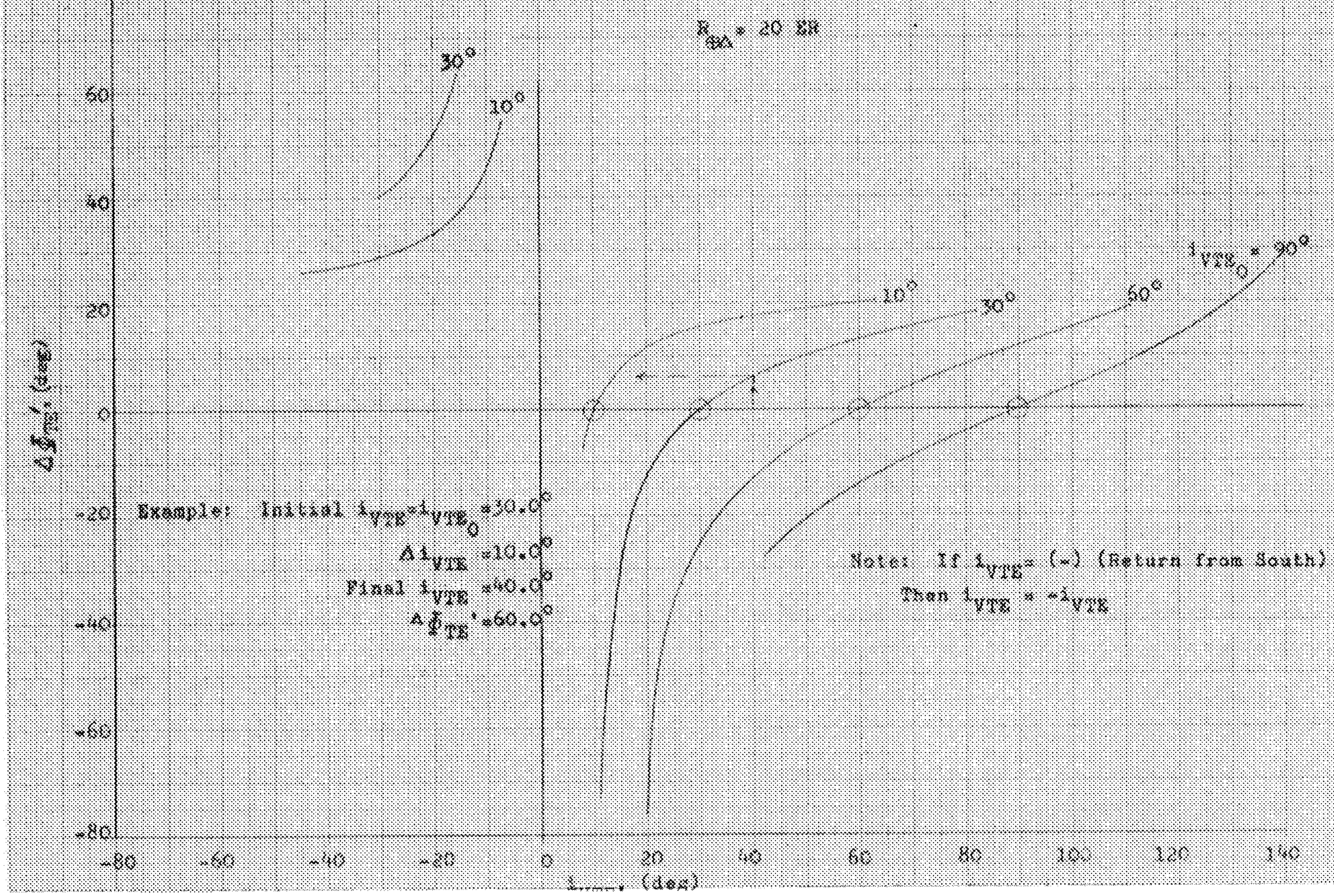


Fig. 20. Angular Change ($\Delta \mathcal{J}_{TE}'$) Of The Transearth Plane Intersection With The MOP Due To An Inclination Change ($\pm \Delta i_{VTE}$) Maneuver

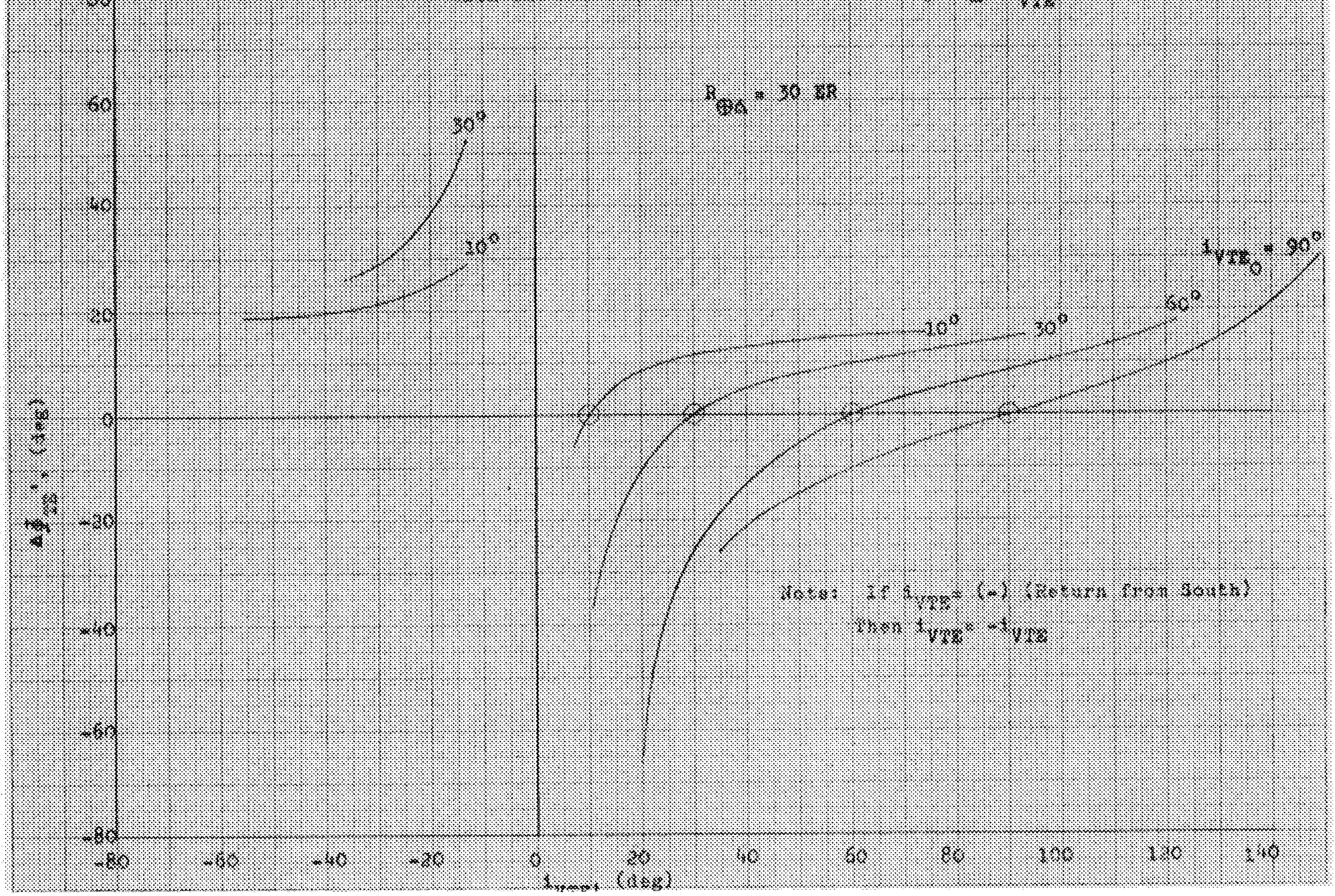
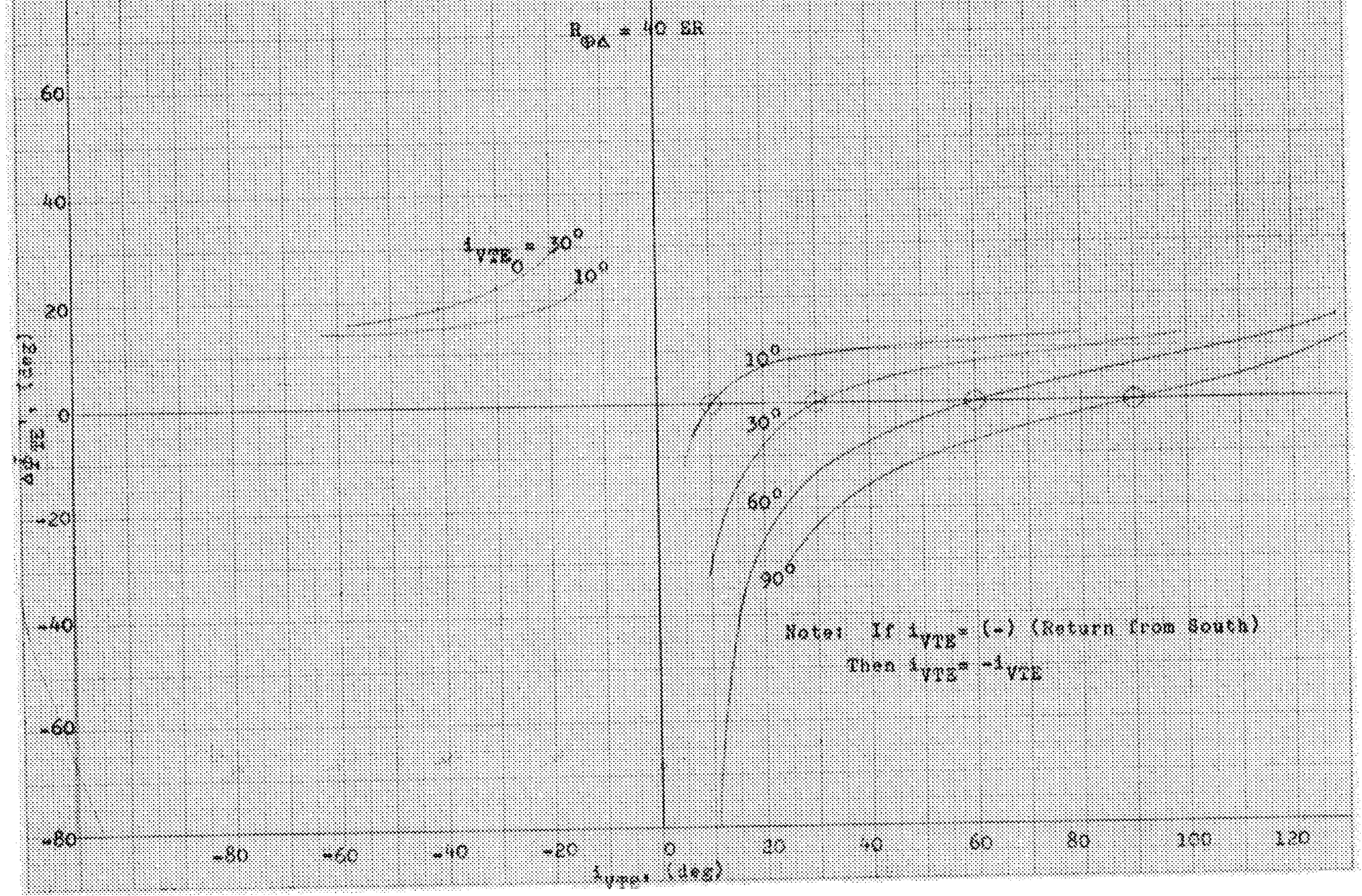


Fig. 21. Angular Change ($\Delta \xi_{TE}$*) Of The Transearth Plane Intersection With The MOP Due To An Inclination Change ($\pm \Delta i_{VTE}$) Maneuver



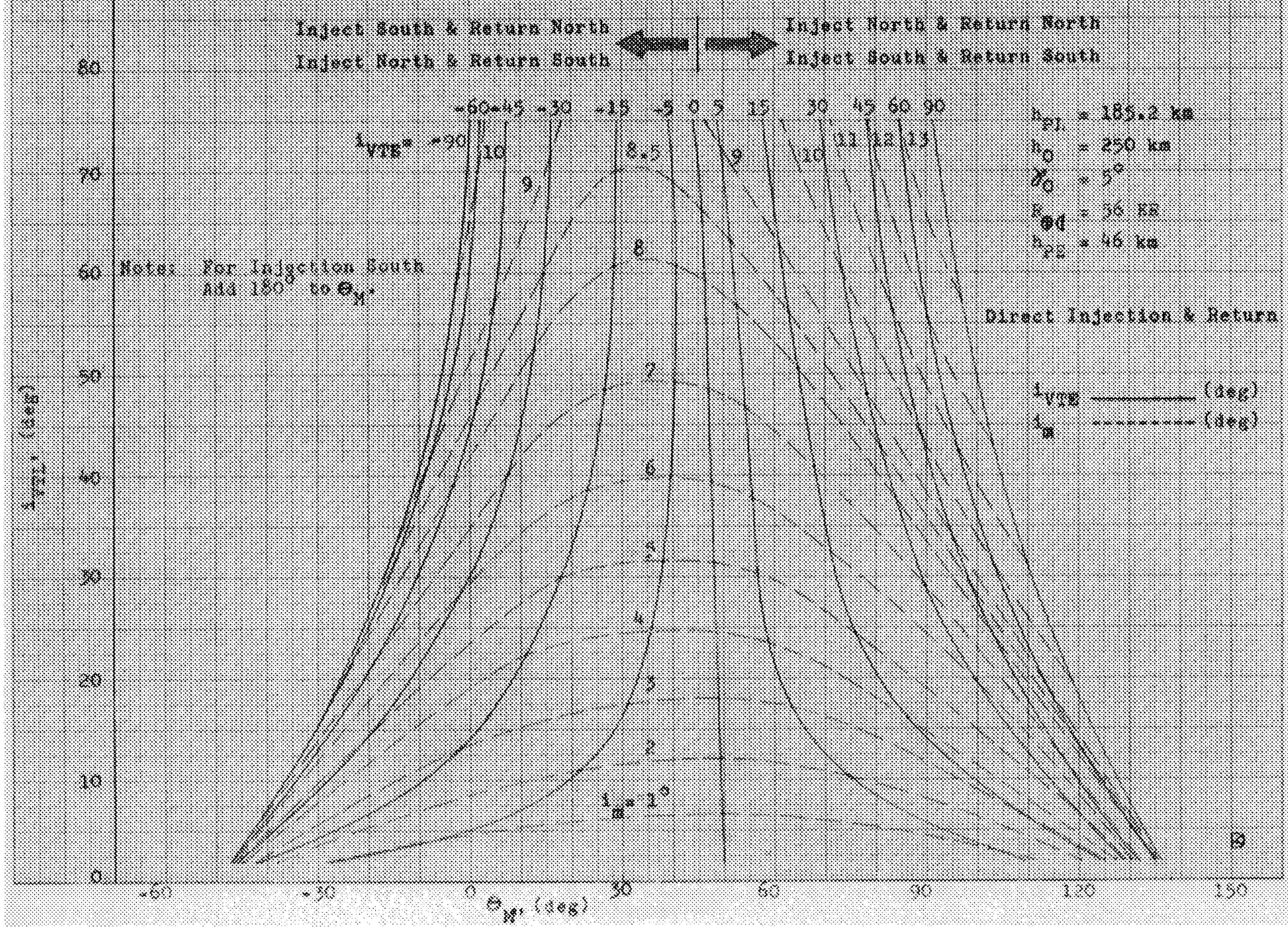


Fig. 22. Mission Planning Envelope (Circumlunar Trajectory Class)

Fig. 23. Mission Planning Envelope (Circumlunar Trajectory Class)

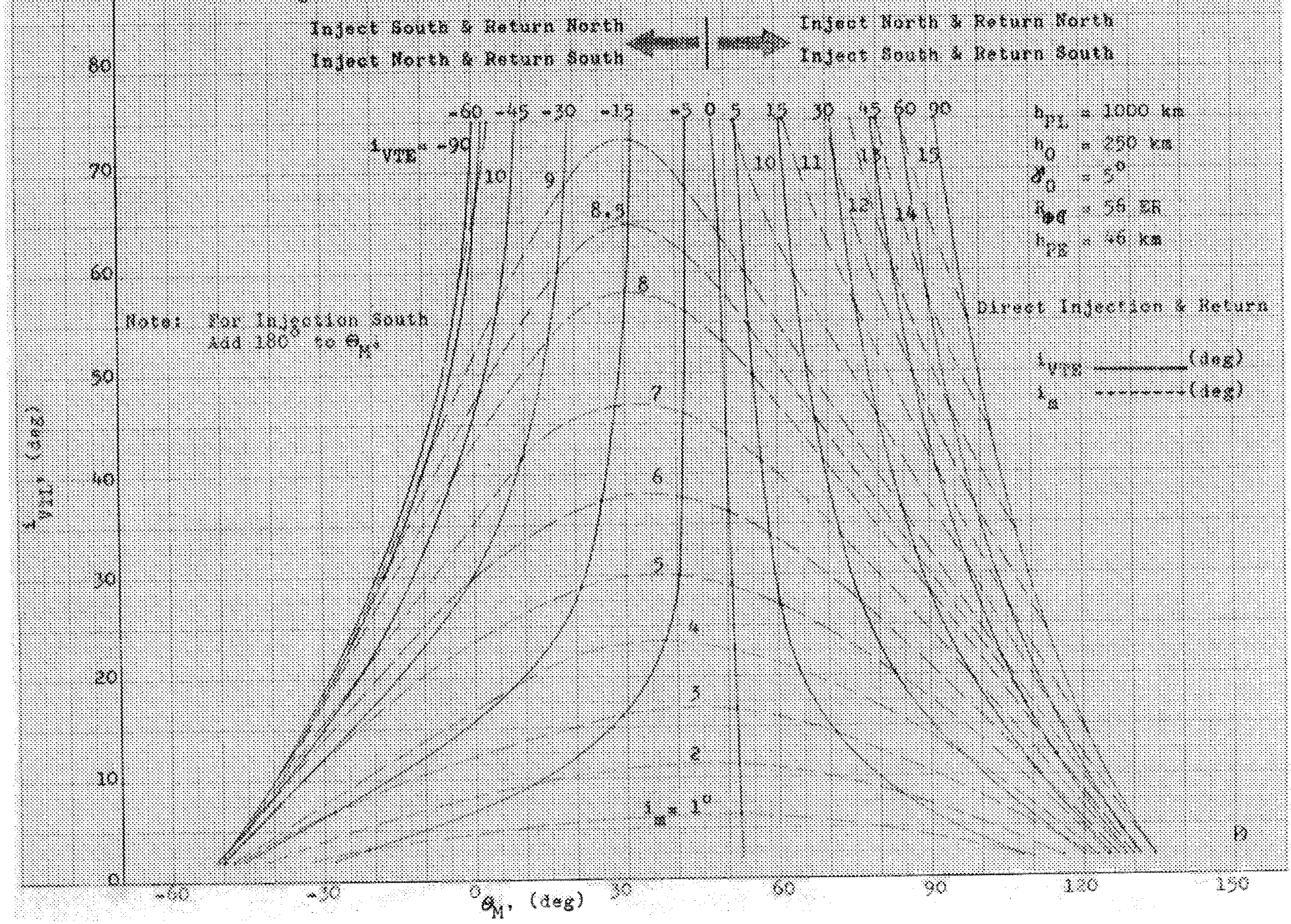


Fig. 24. Mission Planning Envelope (Circumlunar Trajectory Class)

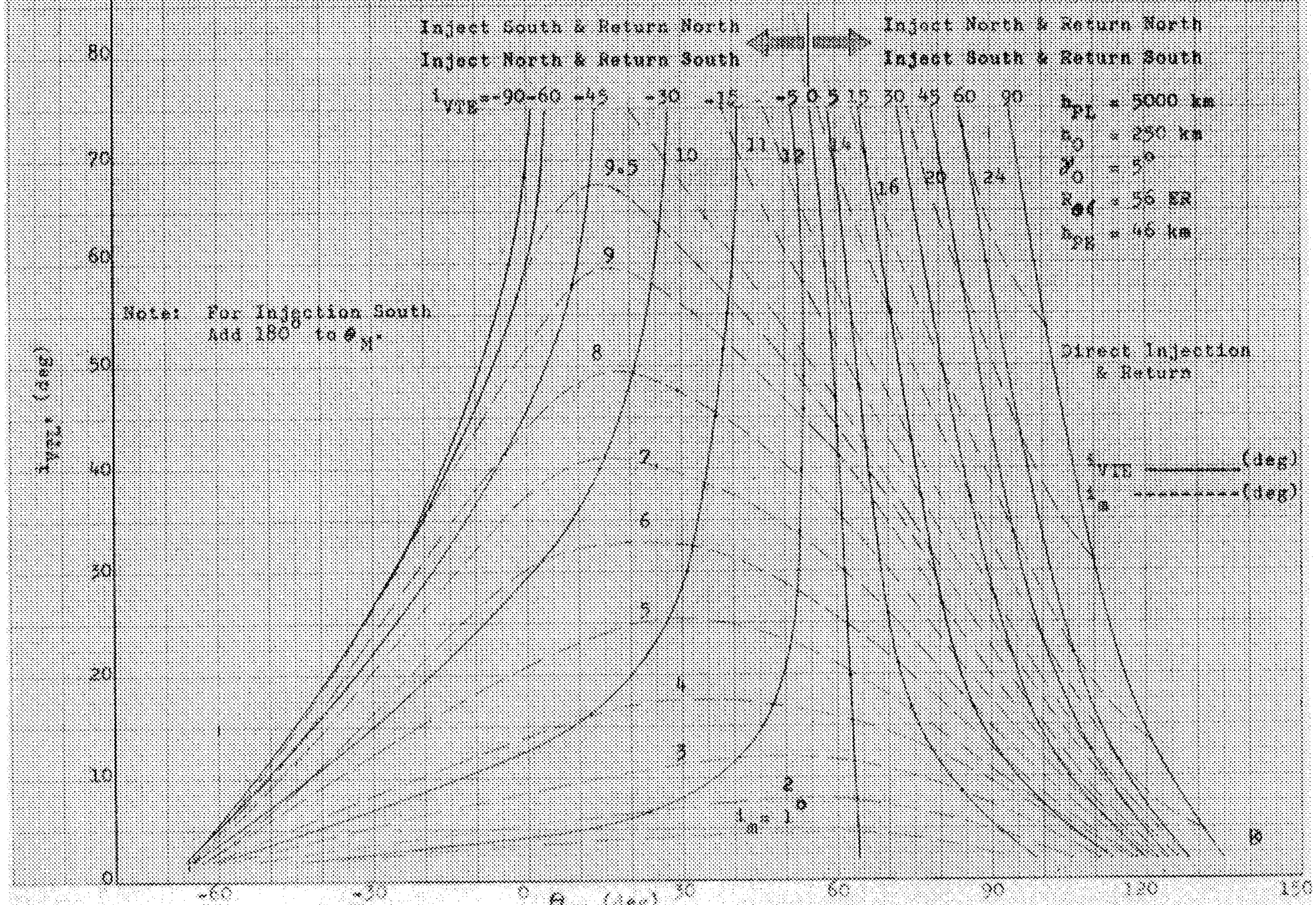


Fig. 25. Mission Planning Envelope (Circumlunar Trajectory Class)

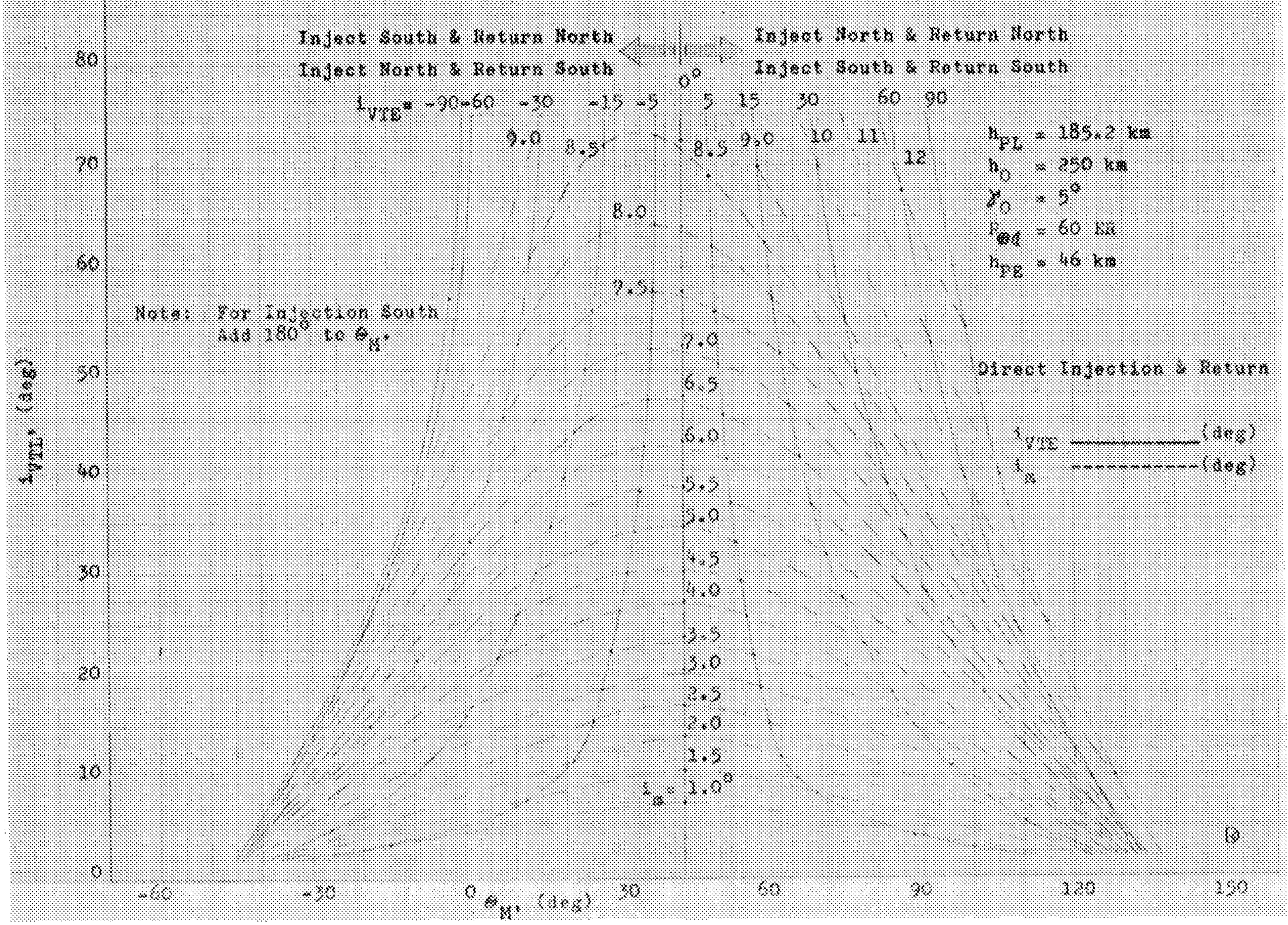
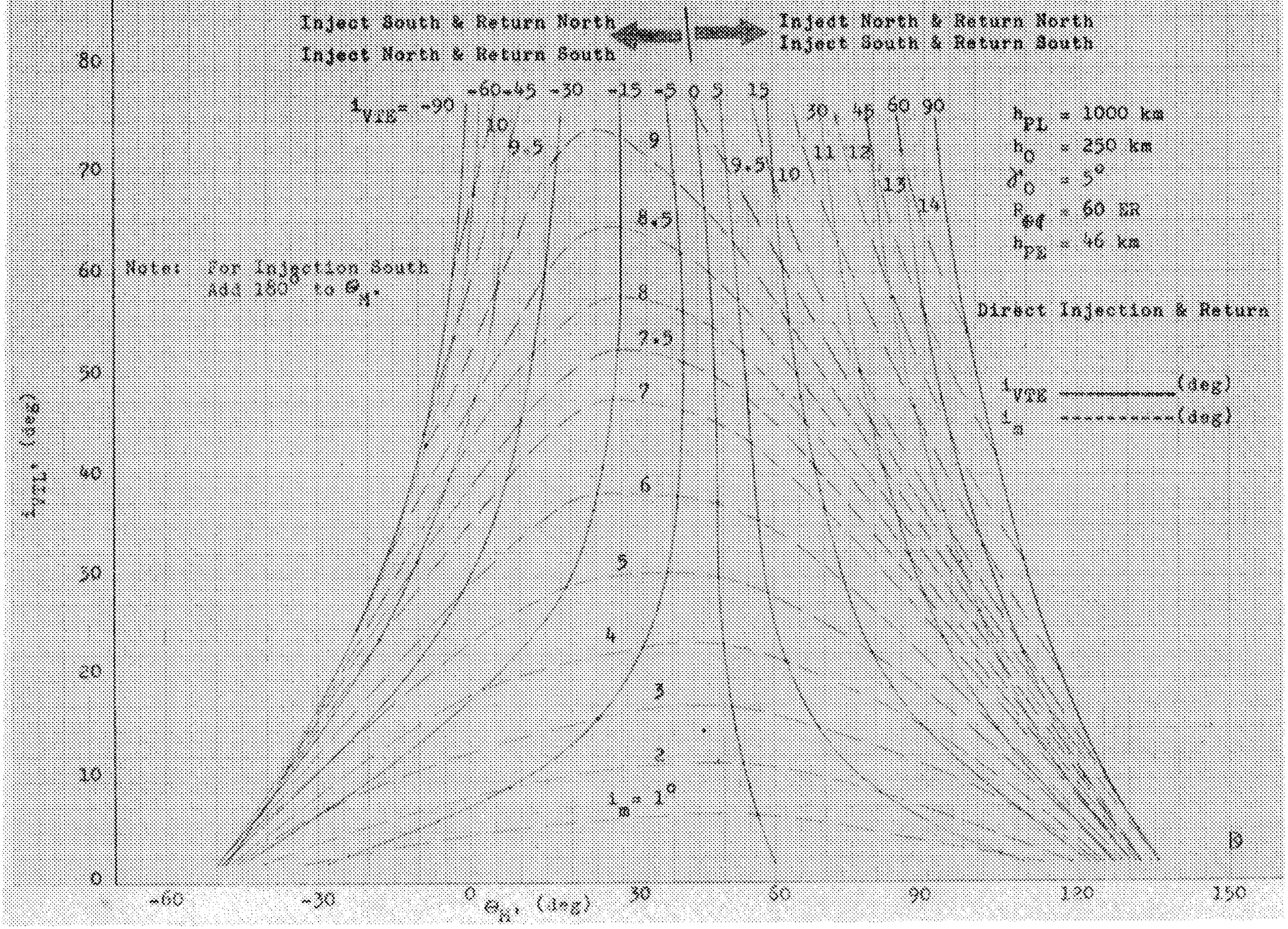


Fig. 26. Mission Planning Envelope (Circumlunar Trajectory Class)



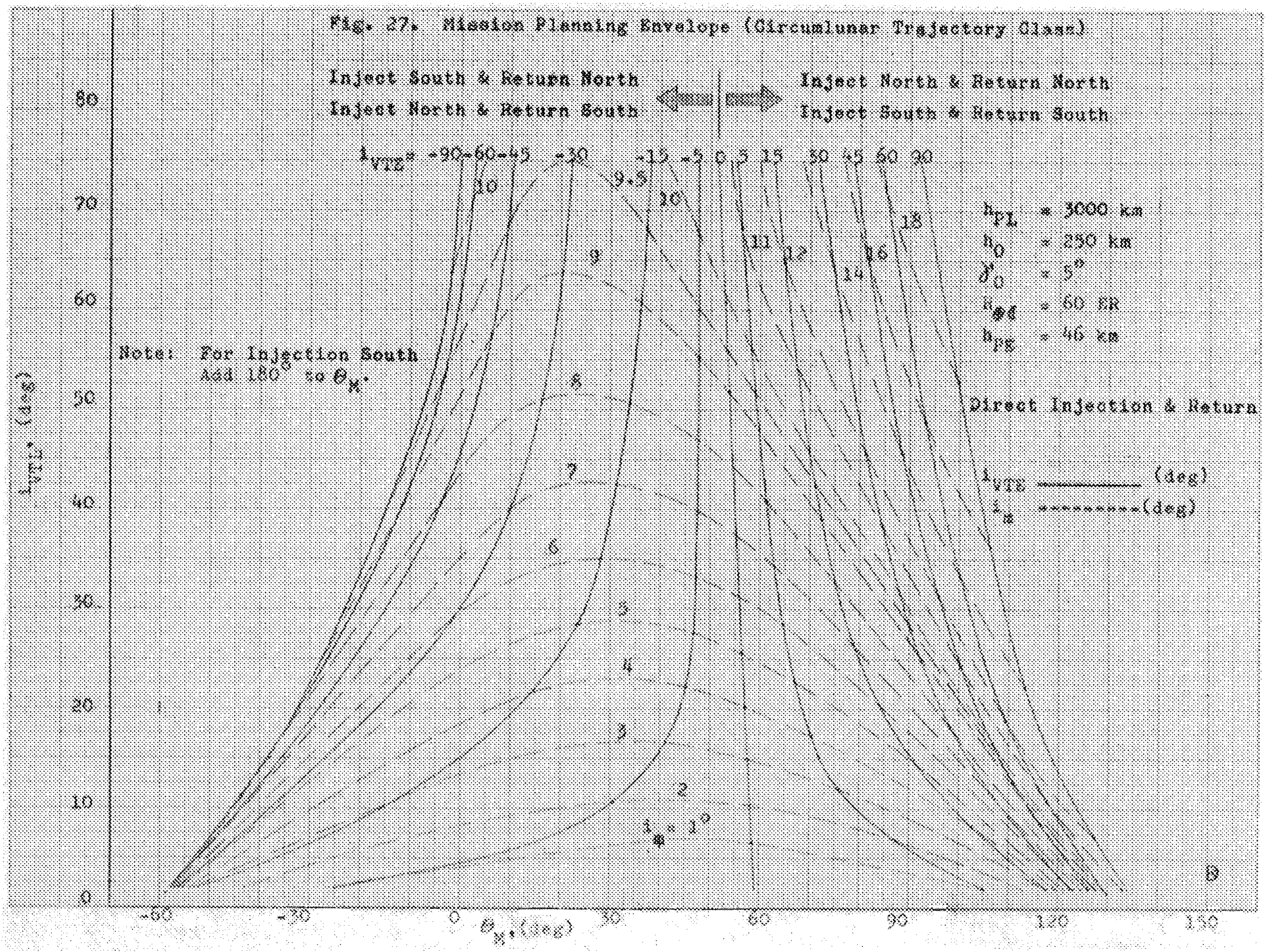


Fig. 27. Mission Planning Envelope (Circumlunar Trajectory Class)
Inject South & Return North
Inject North & Return South
Inject North & Return North
Inject South & Return South
i_{VTE} = -90 -60 -45 -30 -15 -5 0 5 15 30 45 60 90
h_{PL} = 3000 km
h_0 = 250 km
γ_0 = 5°
$R_{\oplus\mathbb{C}}$ = 60 ER
h_{PE} = 46 km
Direct Injection & Return
i_{VTE} ———— (deg)
$i_{\oplus}$ --------- (deg)
Note: For Injection South Add 180° to θ_M.
$i_{\oplus}$ = 1°
i_{VTL}, (deg)
θ_M, (deg)

Fig. 28. Mission Planning Envelope (Circumlunar Trajectory Class)

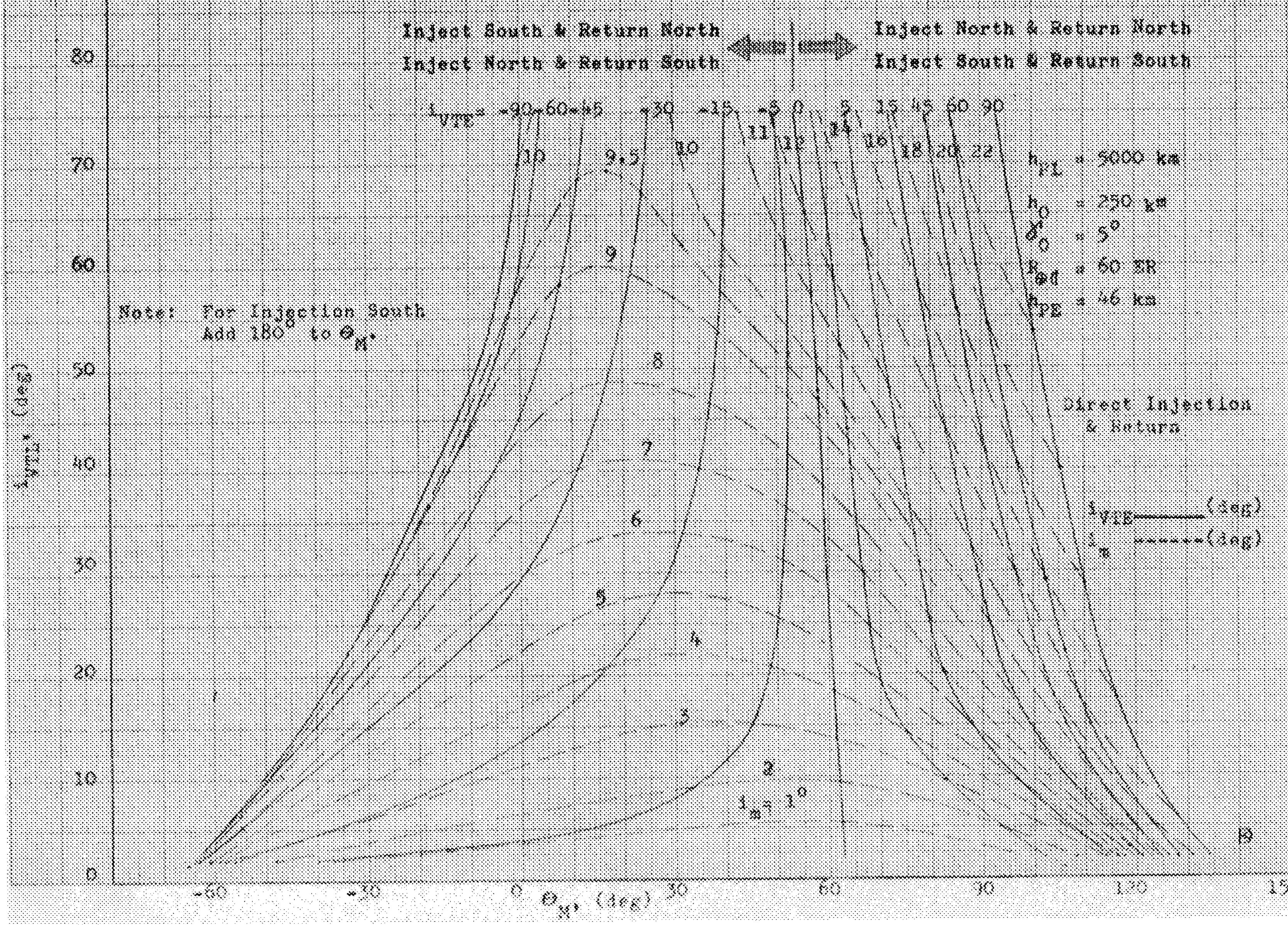
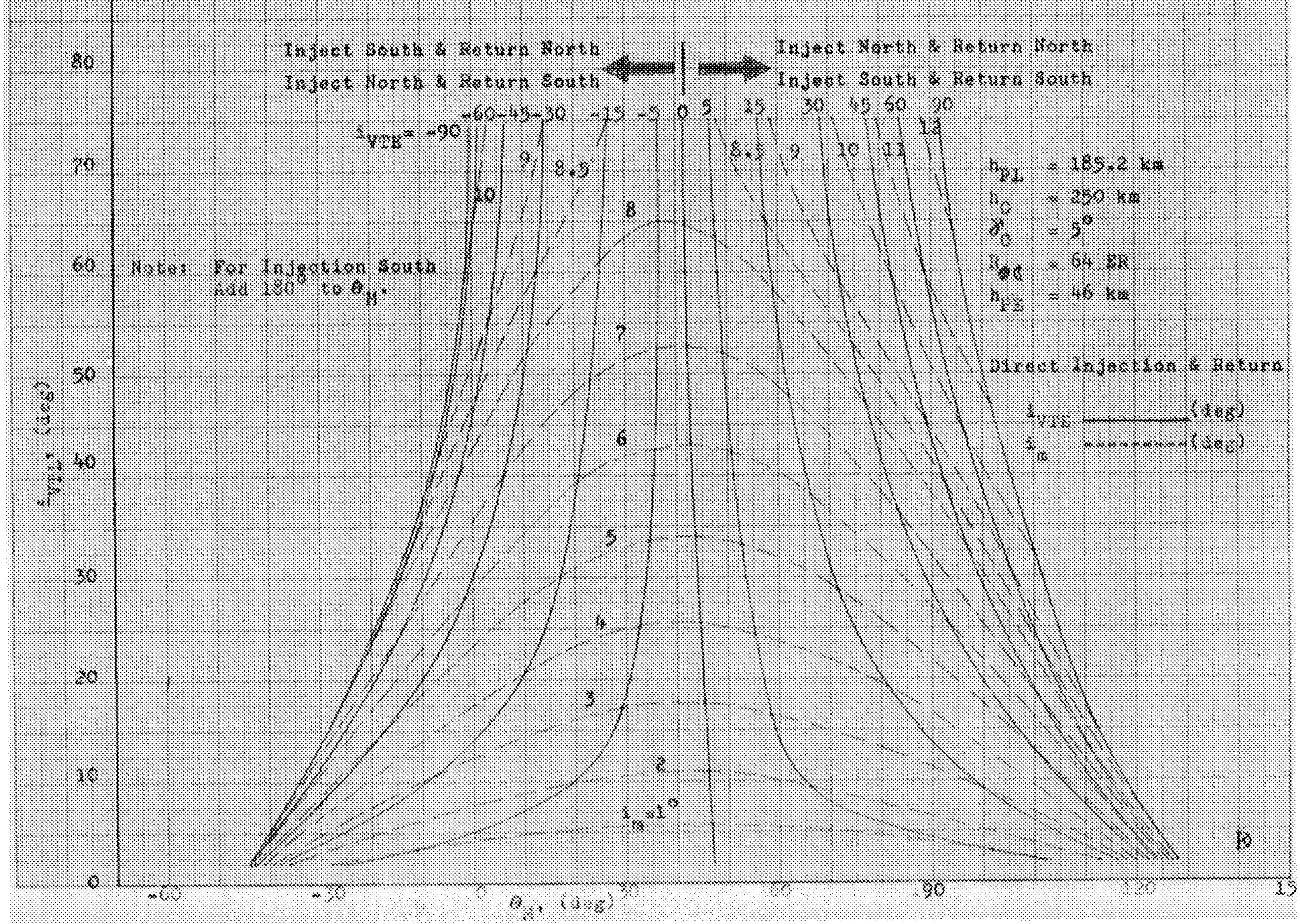


Fig. 29. Mission Planning Envelope (Circumlunar Trajectory Class)



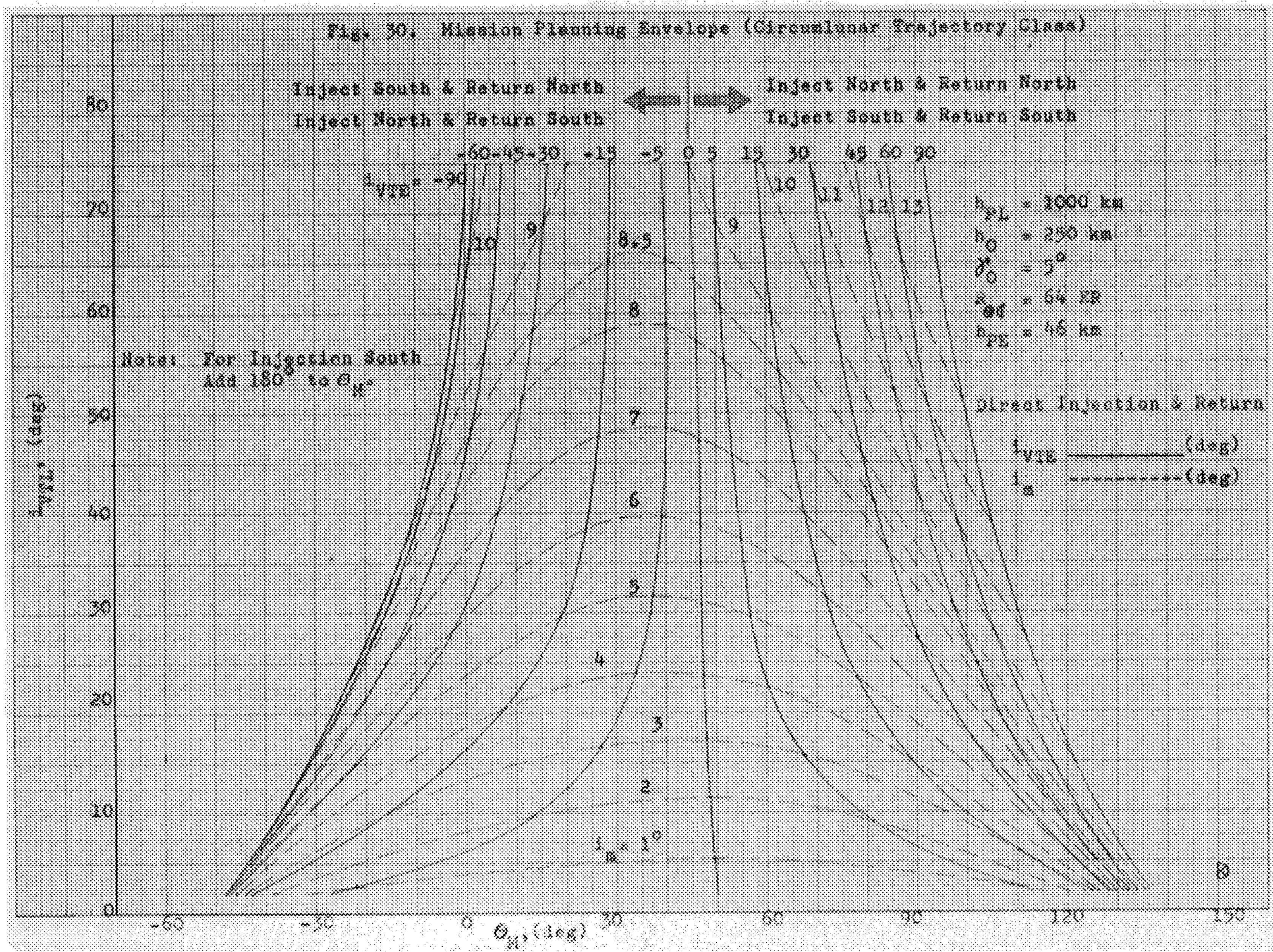


Fig. 30. Mission Planning Envelope (Circumlunar Trajectory Class)

Fig. 31. Mission Planning Envelope (Circumlunar Trajectory Class)

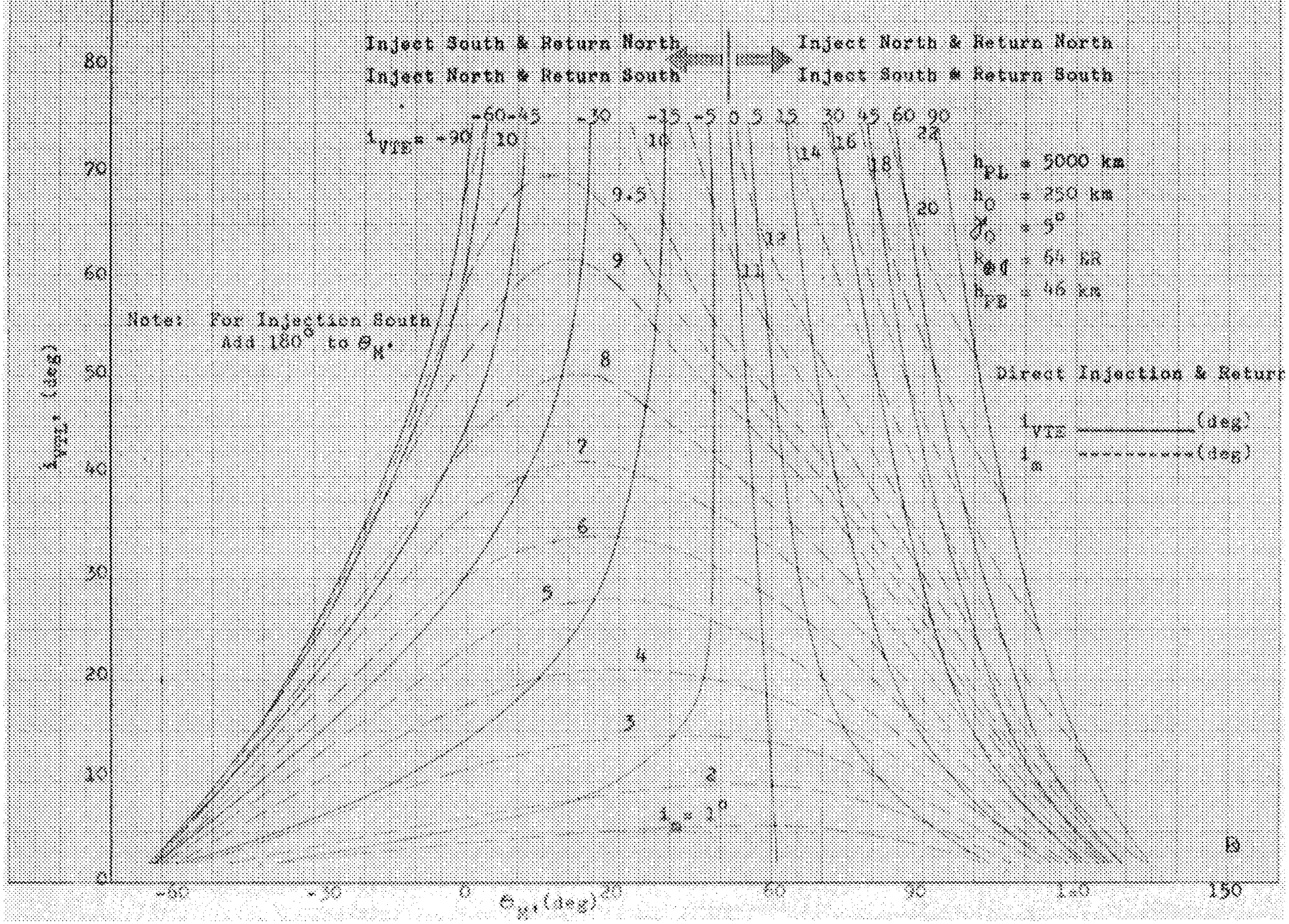


Fig. 32. Mission Planning Envelope (Approach Trajectory Class)

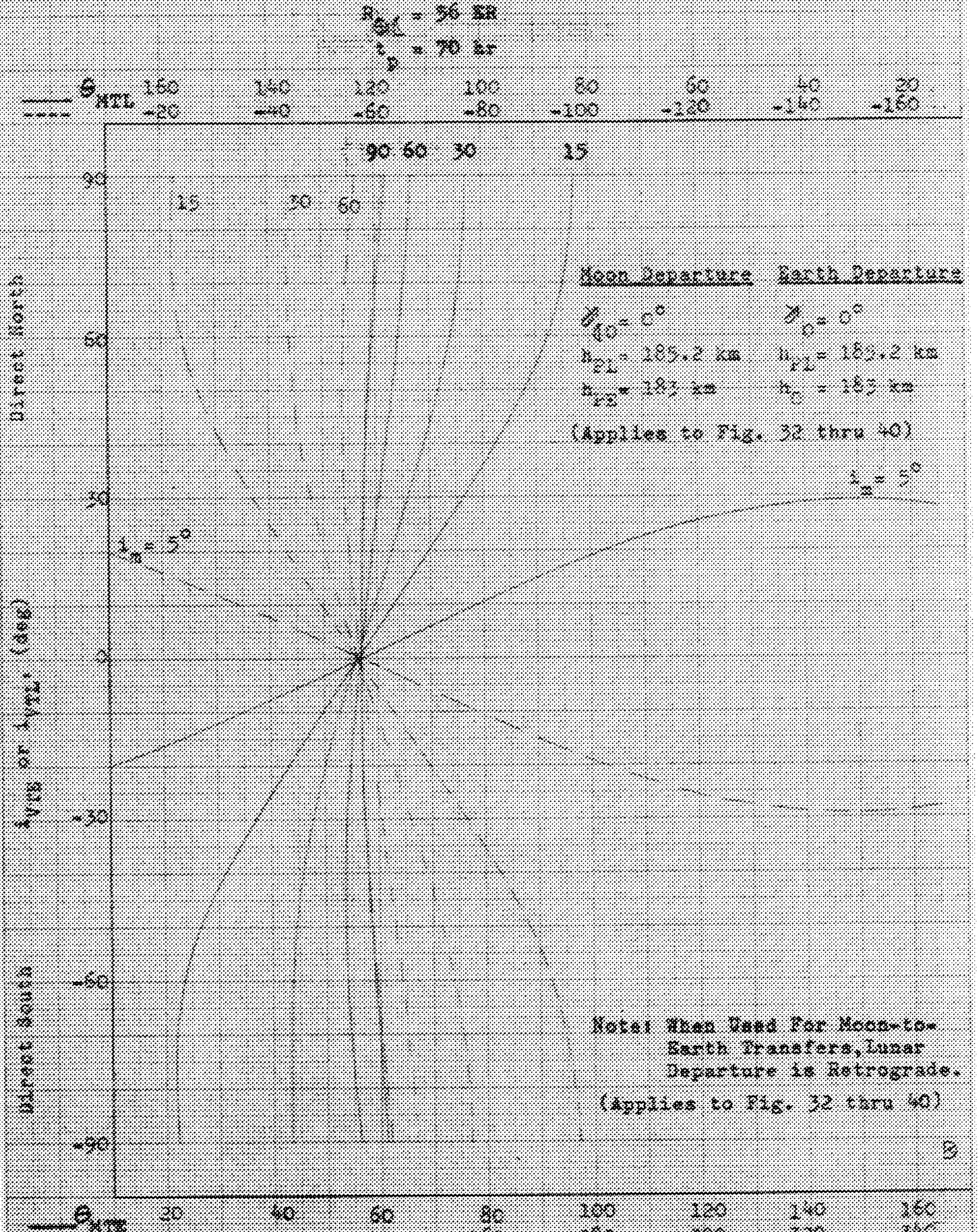


Fig. 33. Mission Planning Envelope (Approach Trajectory Class)

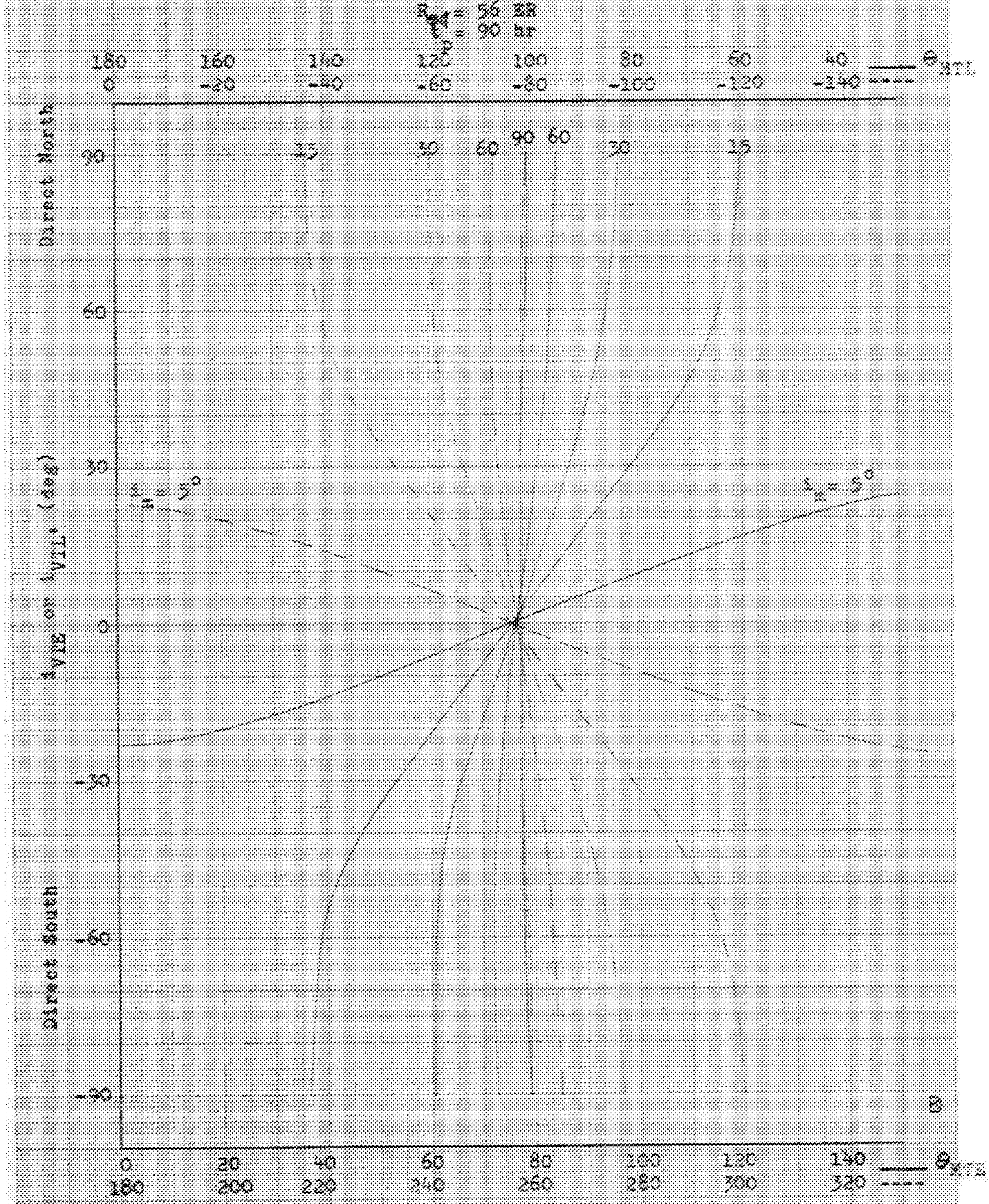


Fig. 34. Mission Planning Envelope (Approach Trajectory Class)

R_{PM} = 56 ER

t_p = 110 hr

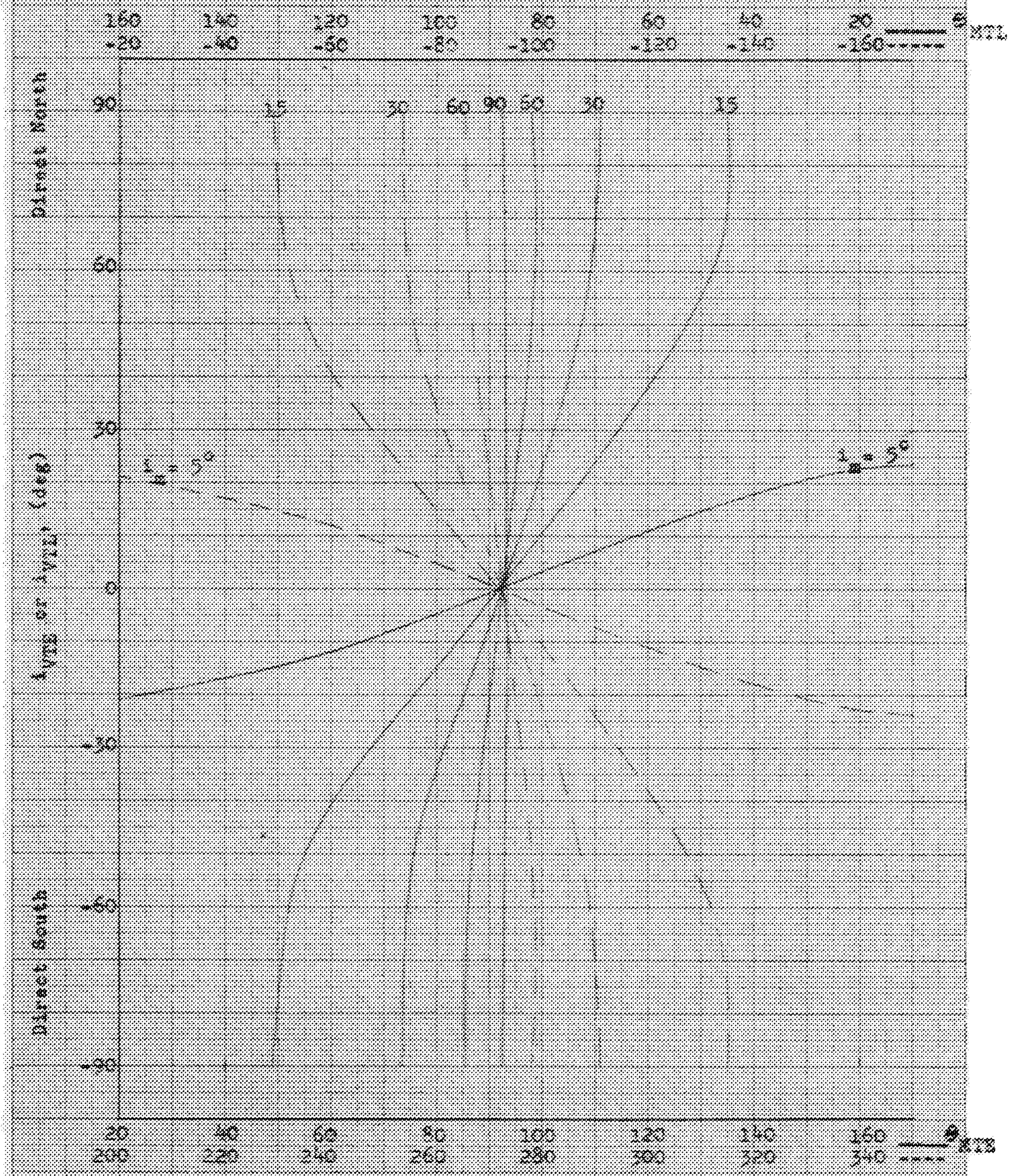


Fig. 35. Mission Planning Envelope (Approach Trajectory Class)

$R_{p_{☾}} = 60$ ER

$t_f = 70$ hr

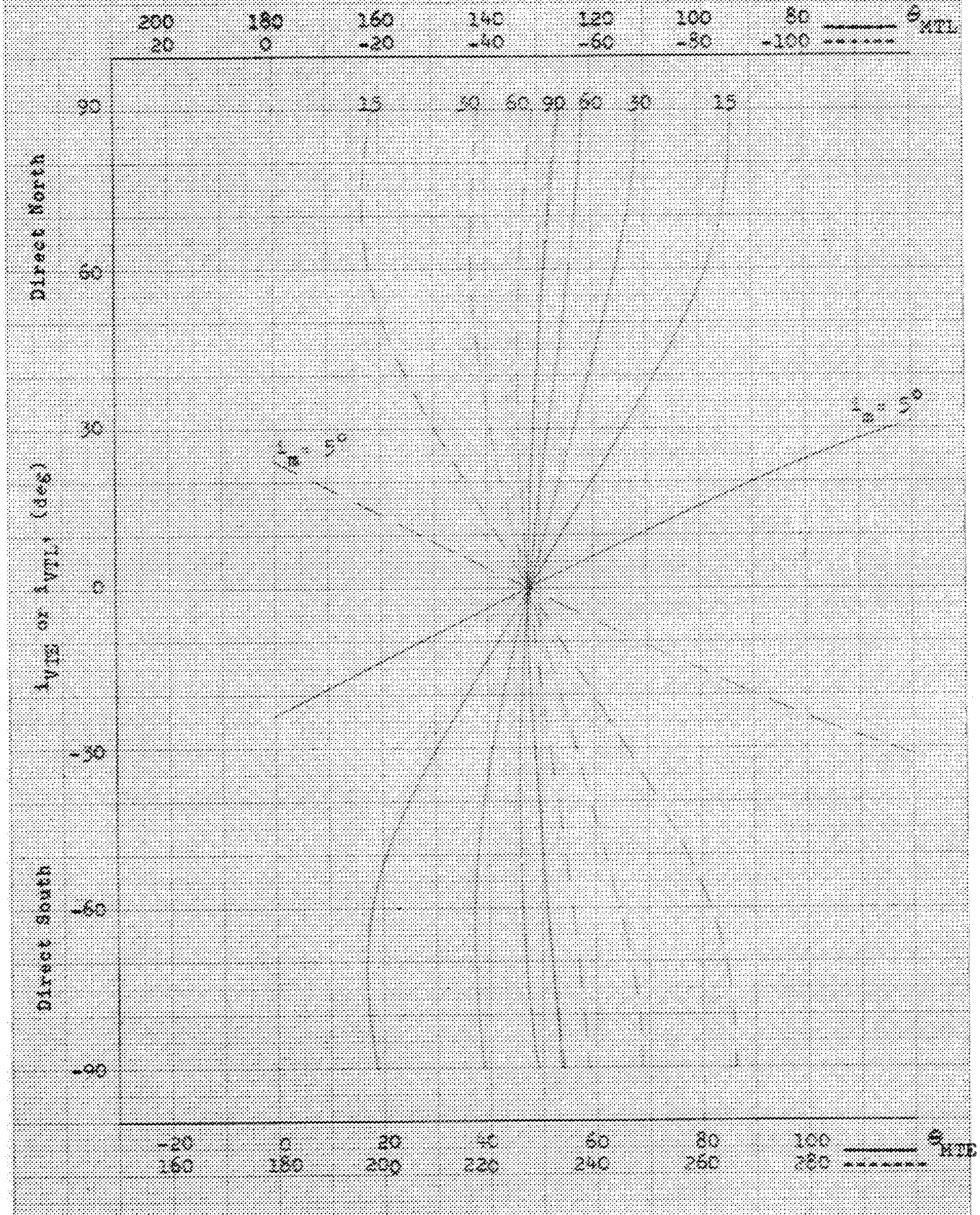


Fig. 36. Mission Planning Envelope (Approach Trajectory Class)

$R_{\oplus i} = 60$ ER

$t_F = 90$ hr

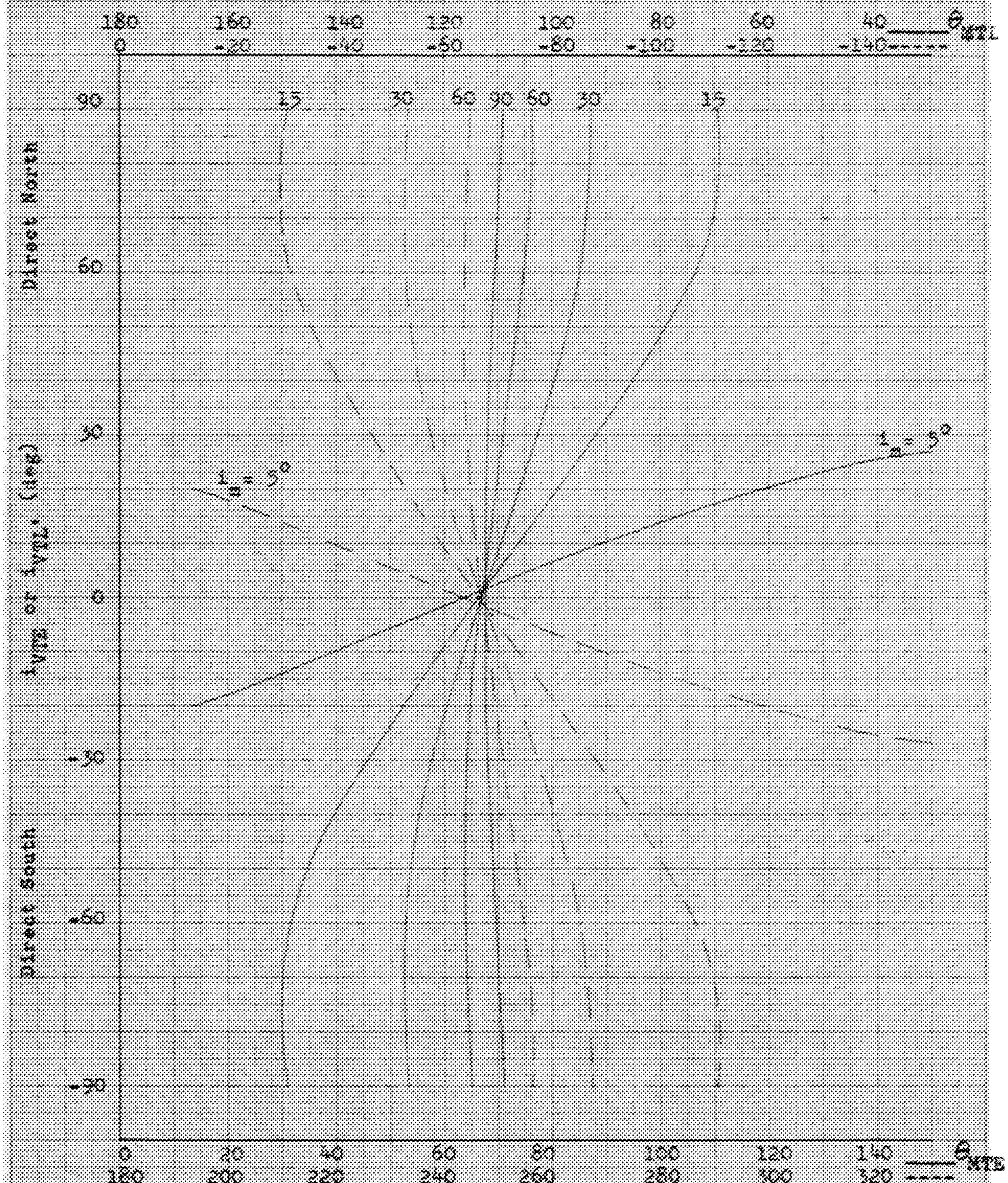


Fig. 37. Mission Planning Envelope (Approach Trajectory Class)

$R_{\oplus\mathbb{C}} = 60$ ER

$t_p = 110$ hr

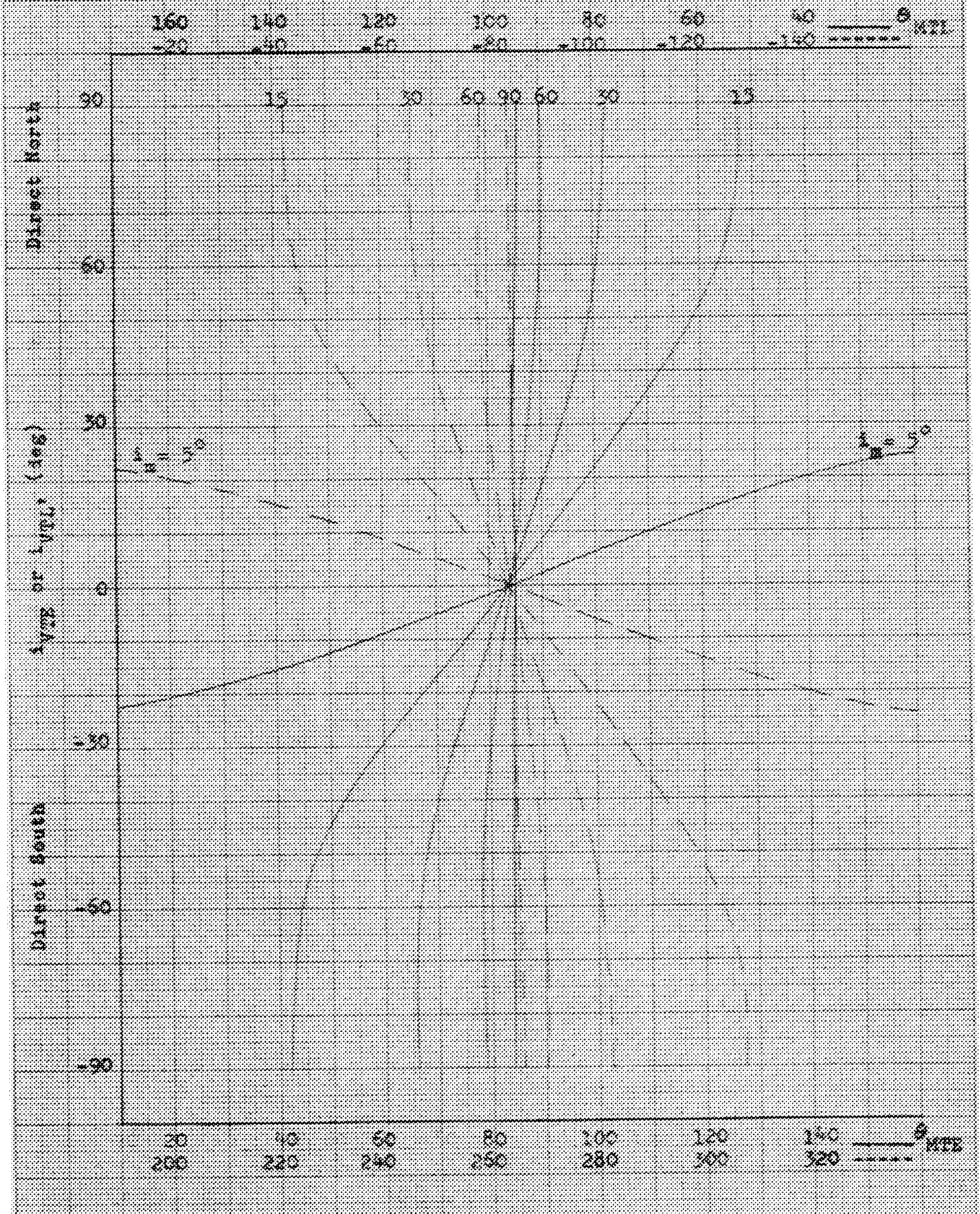


Fig. 38. Mission Planning Envelope (Approach Trajectory Class)

$R_{\oplus☾} = 64$ ER

$t_p = 70$ hr

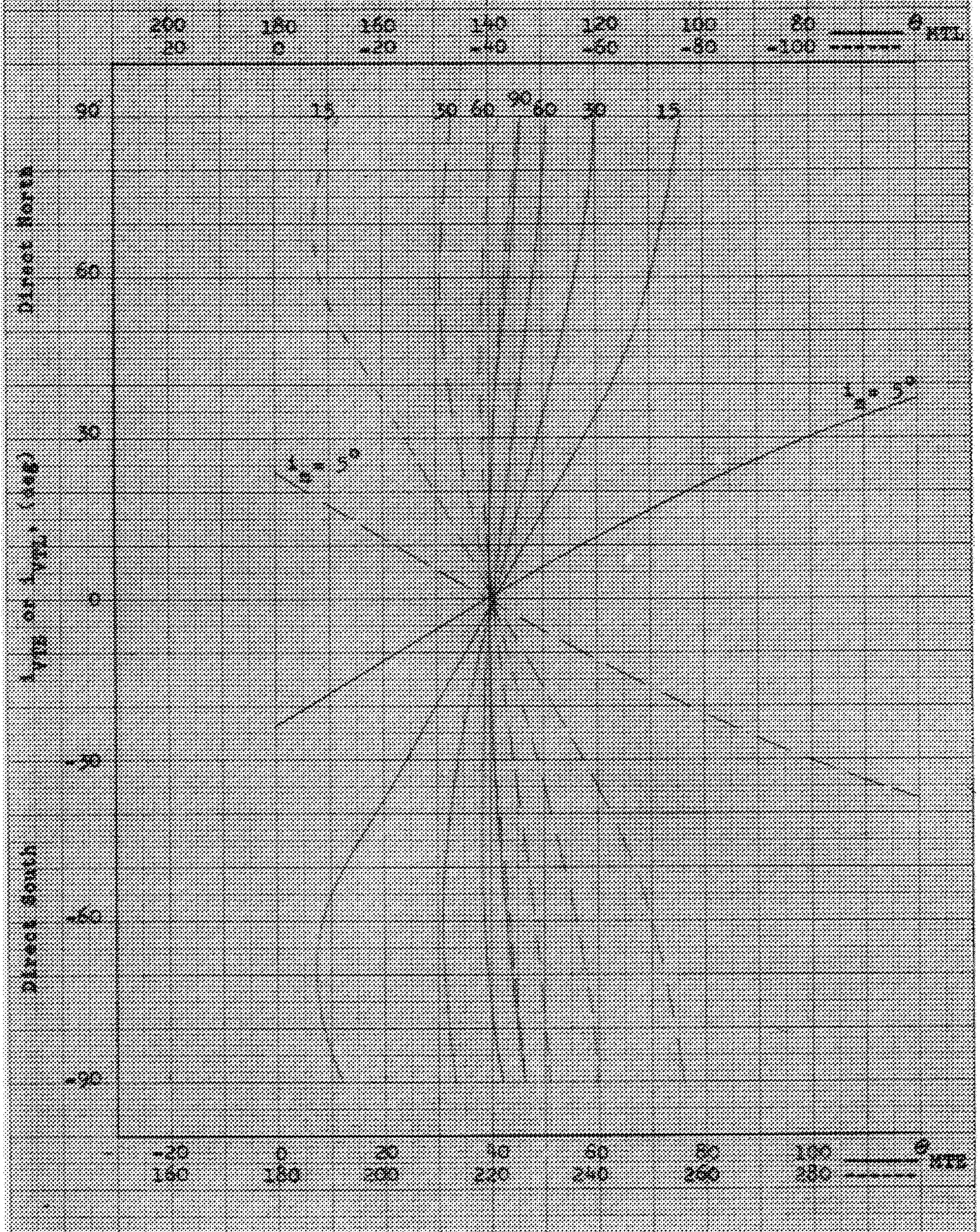


Fig. 39. Mission Planning Envelope (Approach Trajectory Class)

$R_{\oplus ☾} = 64$ ER

$t_p = 90$ hr

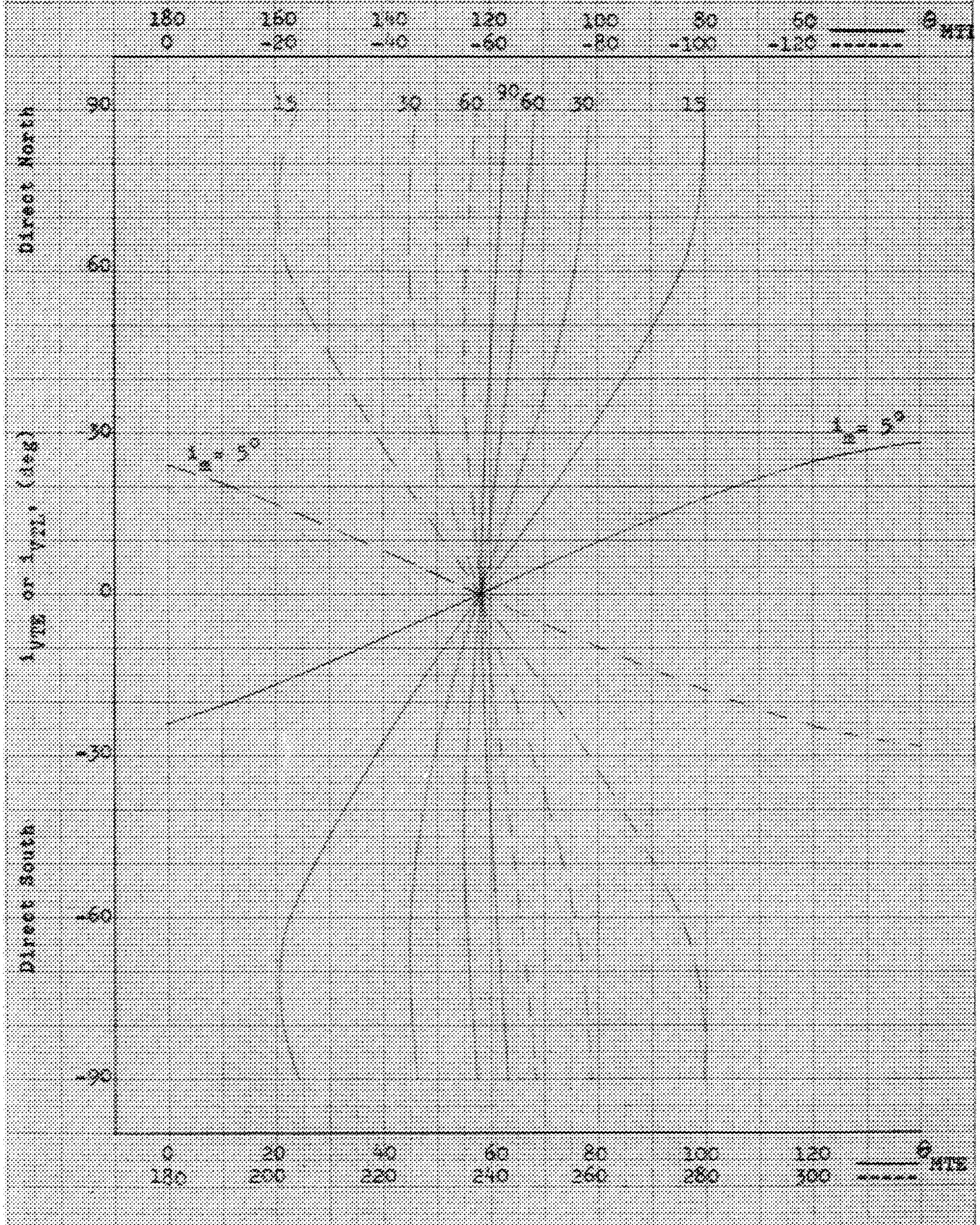


Fig. 40. Mission Planning Envelope (Approach Trajectory Class)

$R_{\oplus ☾} = 64$ ER

$t_p = 110$ hr

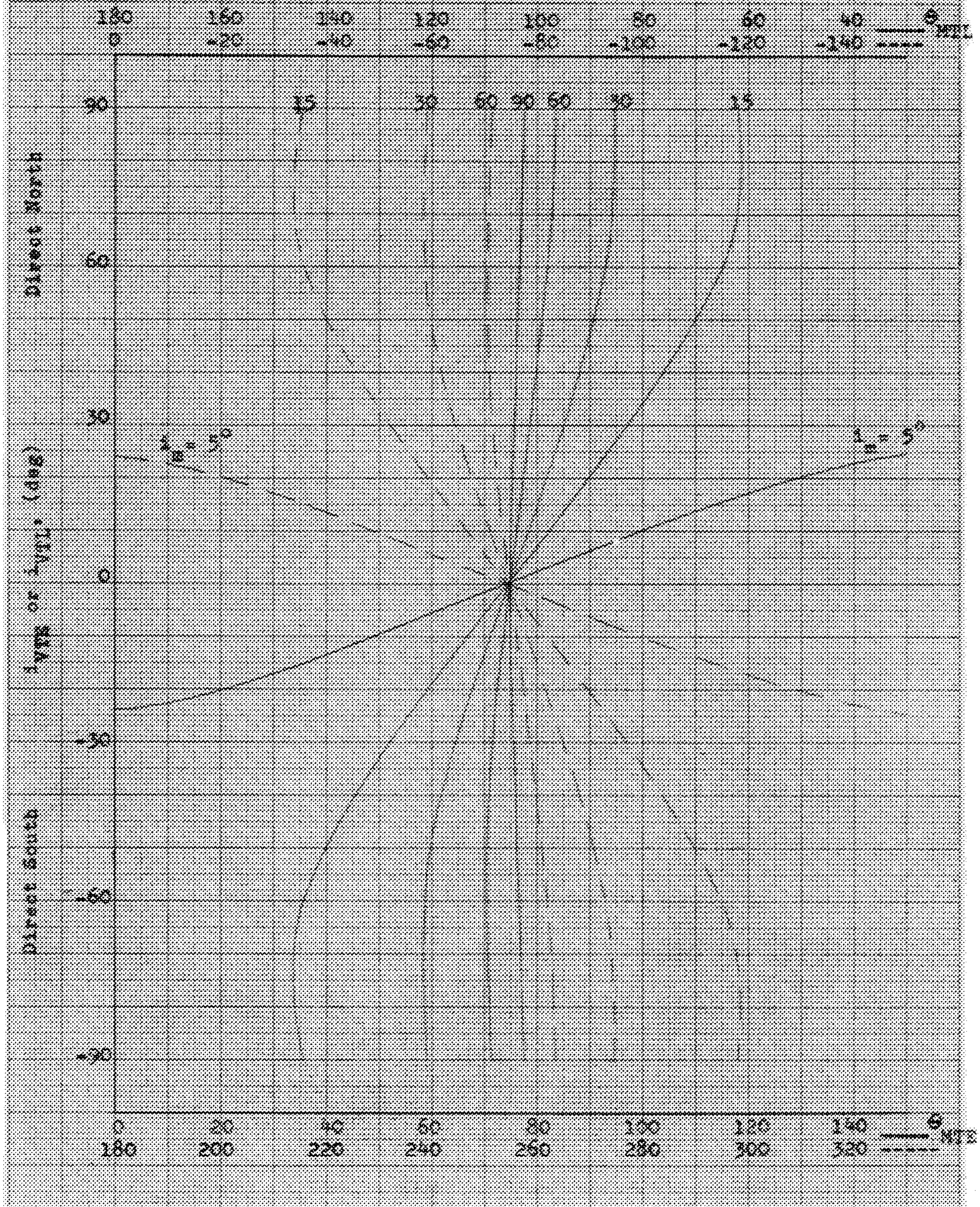


Fig. 41. Mission Planning Envelope (Circumlunar Trajectory Class)

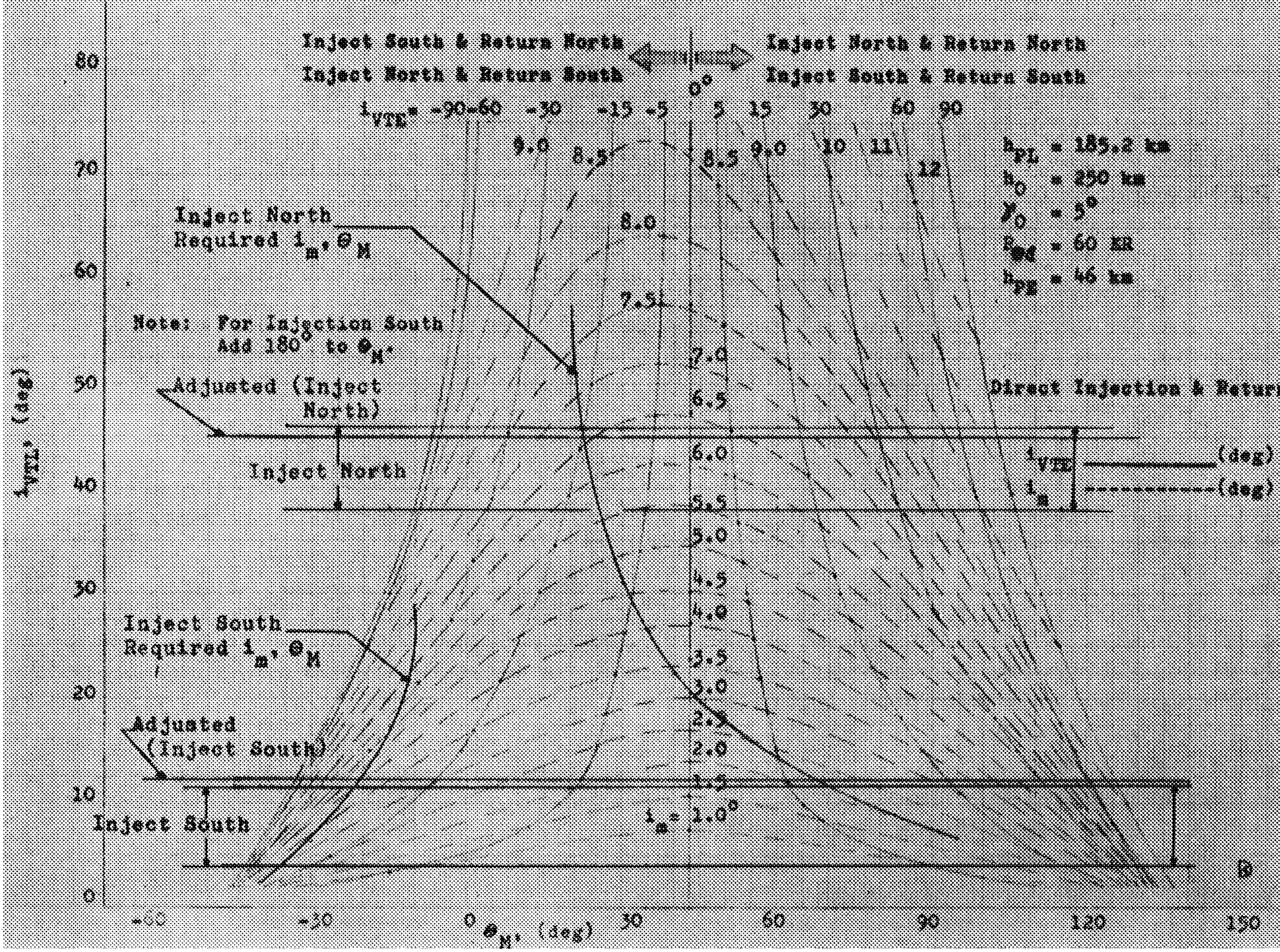
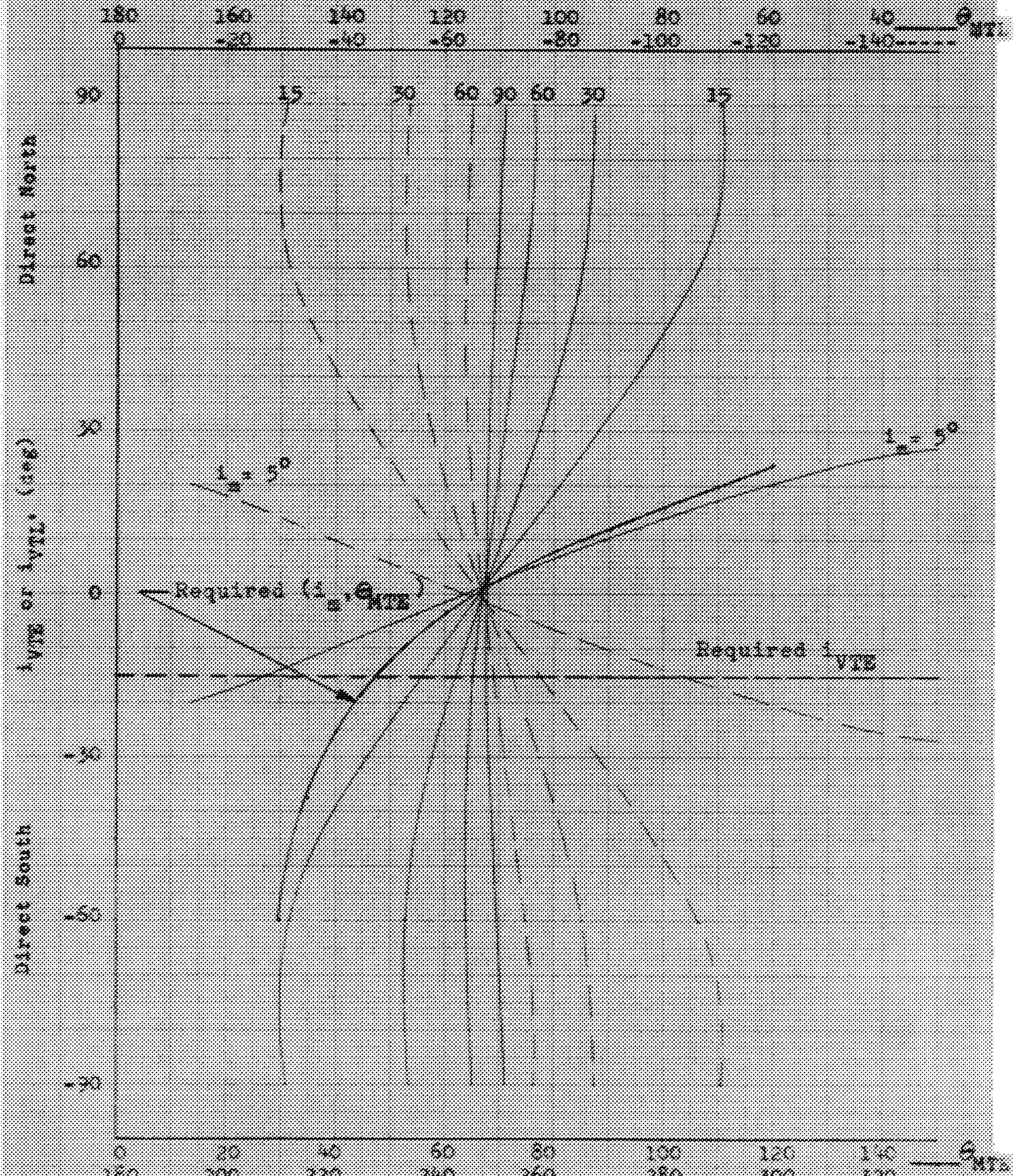


Fig. 42. Mission Planning Envelope (Approach Trajectory Class)
$R_{\oplus f} = 60$ ER
$t_p = 90$ hr



CHAPTER XII

BIBLIOGRAPHY

XII. BIBLIOGRAPHY

Any attempt to write a bibliography for lunar flight can meet with only partial success due to the vast amount of literature in this field. To be useful, the emphasis in this Bibliography has been on recent material and on material that can be obtained easily from either technical libraries or government agencies. In addition, the idea of subdividing the Bibliography by handbook chapter has been abandoned because too much material is common to several chapters.

To aid the user, this Bibliography has been subdivided into three sections. Section A lists sources that enable the engineer to keep current in the literature on lunar flight. The technical and semi-technical magazines which often contain information useful for lunar flight are given, and the ones containing proportionately the largest amount of material are emphasized by an asterisk. In addition, series of books and proceedings of meetings which are published periodically have been included.

Section B gives the bibliography of background material--essentially further references to the material of Chapters II, III and IV. Books on astronomy, space flight, and space environment, as well as reports and articles dealing with lunar flight in general and with environmental factors form the bulk of this portion of the Bibliography. For references on lunar ephemerides, the reader is referred to Subsection C-1e of Chapter III, and for references on current lunar maps to Subsection A-2g of Chapter III, since they are not repeated in the Bibliography.

Section C gives the bibliography for the various phases of lunar flight as discussed in Chapters V to X. It consists mostly of recent articles from technical journals, of NASA, ASTIA, and some company reports.

No bibliography is given for the mission analysis (Chapter XI of the Handbook), since similar material has been generated only recently, and is available in generally inaccessible internal company reports.

A. SOURCES OF FLIGHT MECHANICS MATERIAL

1. Technical Journals

Aero/Space Medicine, formerly Journal of Aviation Medicine

*American Institute of Aeronautics and Astronautics (AIAA) Journal

American Journal of Physics

*American Rocket Society (ARS) Journal, formerly Jet Propulsion

Archive for Rational Mechanics and Analysis

*Astronautica Acta

*Astronomical Journal

Astrophysical Journal

*British Interplanetary Society Journal

IRE Transactions

Journal of Applied Physics

*Journal of Astronautical Sciences

Journal of Atmospheric Sciences

Journal of Geophysical Research

*Journal of the Aero/Space Sciences, formerly Journal of the Aeronautical Sciences

Nature

Planetary and Space Sciences

Royal Aeronautical Society Journal

Soviet Astronomy

Zeitschrift für Flugwissenschaften

2. Semi-technical Journals

Aeronautics

*Aero/Space Engineering

Aero/Space Management

Air Force and Space Digest

American Geophysical Union Transactions

*Astronautical Sciences Review

*Astronautics

*Aviation Week

Canadian Aeronautics and Space Journal

Current Contents

Dissertation Abstracts

Electrical Engineering

*Missiles and Rockets

Missiles and Space

Scientific American

Space Flight

Space/Aeronautics, formerly Aviation Age

Sky and Telescope

U.S. Government Research Reports.

3. Series of Books and Proceedings of Meetings

Advances in Applied Mechanics (Academic Press)

Advances in Astronomy and Astrophysics (Academic Press)

Advances in Astronautical Sciences (Plenum Press, Mac Millan)

Advances in Geophysics (Academic Press)

Ballistic Missile and Space Technology (Academic Press)

Proceedings of the International Astronautical Congress (Springer Verlag, Vienna)

Progress in Astronautics and Rocketry(Academic Press)

Smithsonian Contributions to Astrophysics (Smithsonian Institute Astrophysics Laboratory)

Space Research (Interscience)

Vistas in Astronomy (Pergamon Press)

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APPENDIX A

APPENDIX A

GLOSSARY (REF. 1)

A

Aberration: apparent displacement of a body from its actual position due to the observer's motion, the object's motion and the finite speed of light.

Aberration, planetary: aberration including effects of the object's motion as well as the observer's motion during the time light travels from the object to the observer.

Aberration, stellar: aberration including only the effect of the earth's motion around the sun, mean value 29.9 km/sec.

Ablation: the gradual removal or erosion of an exposed surface of an object resulting from its high speed passage through a resistive medium.

Abort: the termination of a space mission after an emergency forces return to earth.

Albedo: fraction of total incident light reflected by a body.

Albedo, average geometric: ratio between the average brightness of the object to the brightness of a white screen of the same size normal to the incident light (lunar albedo 0.105).

Albedo, spherical: ratio of the light scattered in all directions by the object to the total incident light (lunar albedo = 0.073).

Almucantar: a parallel to the horizon.

Altitude (also elevation): a topocentric coordinate in the horizon system; the angular distance of an object above the horizon, measured on a vertical circle. Also synonymous with the height of an object above some surface.

Analytical integration: the specification of an explicit closed algebraic or series relation to represent the integral of a given function.

Angular momentum: the quantity $m\vec{r} \times \dot{\vec{r}}$ ($= r^2 \dot{\theta}$ in polar coordinates) constant for conic motion.

Anomaly: or angle; see true anomaly, mean anomaly, and eccentric anomaly.

Aphelion: the point on a heliocentric ellipse farthest from the sun.

Apocynthion (also aposelene or apolune): the point on a selenocentric elliptic orbit farthest from the moon's center.

Apofocus: the apsis on an elliptic orbit farthest from the principal focus or center of force.

Apogee: the point on a geocentric elliptical orbit farthest from the earth's center.

Apsis (plural, apsides): the point on a conic where the radius vector is a maximum or minimum. The line of apsides is the major axis extended indefinitely.

Argument of latitude: the angle in the orbit from the ascending node to the object in the direction of motion; the sum of the argument of perifocus and the true anomaly.

Argument of perifocus: the angular distance measured in the orbit plane in the direction of motion from the lines of nodes to line of apsides.

Aries: an astronomical constellation; a portion of the celestial sphere which contained the vernal equinox.

Aries, first line of: the direction of the vernal equinox (the name is a carryover from a time that the vernal equinox was in the constellation Aries).

Aspect: angular position of a body relative to its line of advance in orbit.

Astrodynamics: the engineering or practical application of celestial mechanics and other allied fields such as high altitude aerodynamics; geophysics; attitude dynamics; and electromagnetic, optimization, observation, navigation, and propulsion theory, to the contemporary problems of space vehicles. Astrodynamics is sometimes also meant to include the study of natural objects such as comets, meteorites and planets.

Astronomical unit (AU): the mean distance or semimajor axis of the orbit of a fictitious unperturbed planet having the mass (0.000,002,819 solar masses) and sidereal period (365.256,383,5 mean solar days) that Gauss adopted for the earth in his original determination of the gravitational constant K_s (= 0.017,202,089,95).

Approximately equal to 92,914,000 statute miles or 149,530,000 km.

Azimuth: a topocentric coordinate measured in the plane of the horizon from the north (or south) point on the horizon clockwise to the object.

B

Ballistic trajectory (also coast trajectory or free-flight trajectory): motion of the space vehicle without rocket burning or thrust forces.

Barker's equation: an equation that relates position to time for an object traveling in a parabolic orbit.

Barycenter: center of mass of a system of masses.

Base altitudes: reference altitudes or levels of the atmosphere between which the atmospheric temperature gradient is assumed to be a constant.

Boltzmann's constant: the ratio of the mean total energy of a molecule to its absolute temperature. Its value is 1.380×10^{-23} joule/°K.

Braking: the deceleration of a space vehicle by rocket thrust or by atmospheric drag.

Braking ellipses: a series of ellipses whose semi-major axes decrease due to the atmosphere of a planet when a vehicle attempts a landing on that planet.

Burnout: end of rocket burning for a particular rocket engine in a given stage of the rocket.

C

Call-down frequency: the frequency with which a vehicle can be recalled from orbit and landed at a specific site.

Cartesian coordinate system: a set of (usually three) mutually orthogonal straight coordinate axes which form a right-handed coordinate system.

Celestial equator: the great circle in which the plane of the terrestrial equator intersects the celestial sphere. The north celestial pole is the point of intersection of the earth's spin vector with the celestial sphere.

Celestial sphere: a hypothetical sphere of infinite dimensions, centered at the observer (or center of the earth or sun, etc.), on the inner surface of which the celestial bodies are projected and appear to move. This sphere is fixed in space, and thus, because of the earth's rotation, appears to rotate from east to west.

Centrifugal force: a fictitious position-dependent force that apparently arises when the motion of an object is observed with respect to a rotating coordinate system. The relationship yielding this "force" is $-m\vec{\omega} \times (\vec{\omega} \times \vec{r})$, where m is the mass of the object and $\vec{\omega}$ is the angular velocity vector of the rotating coordinate system.

Characteristic velocity: the sum of all absolute velocity changes required of a vehicle for a particular space flight (a measure of the total energy requirement for a flight).

Circle, galactic: fundamental plane of the galactic reference system (north pole at $12^h\ 44^m$ right ascension and +27° declination), inclined 62° to the celestial equator.

Circle, hour: secondary circles of the equatorial coordinate system, i.e., planes normal to the celestial equator.

Circle, secondary: great circles (or planes through the origin) which pass through the poles of a given coordinate system.

Circle, vertical: intersections of the celestial sphere by vertical planes in a horizontal coordinate system.

Circumlunar trajectory: a trajectory from the vicinity of the earth which passes behind the moon and returns ballistically to the vicinity of the earth.

Cislunar space: the region of space around the earth and moon, usually taken as being synonymous with the sphere of influence of the earth-moon system.

Collision parameter: the offset distance between the extension of a velocity vector of an object at a great distance from a center of attraction or repulsion and this center.

Colure, equinoctial: the plane, secondary to the equator, which passes through both the celestial poles and the equinoxes.

Colure, solstitial: the plane, secondary to the equator, which passes through both the celestial poles and the solstices.

Conjunction: a point in the orbit of a planet (or moon) where its celestial longitude equals that of the sun. If the alignment is sun-planet-earth, the planet is said to be in "inferior conjunction." This configuration is possible only with inferior planets; if it is planet-sun-earth, the planet is in "superior conjunction." Similarly, when the moon (or a superior planet) is between the earth and the sun, i.e., "new," it is said to be at conjunction.

Coordinate systems: one of a number of sets of celestial coordinate systems used in astronautics (Chapter XI).

(1) Ecliptic System uses the plane of the earth's orbit (ecliptic) as the reference. The axis of the poles of the ecliptic is at right angles to this plane. This system is most useful for intrasolar system work since all the planets move in or near the plane of the ecliptic.

(2) Equatorial System uses the celestial equator as the reference plane. The celestial equator and celestial poles coincide with extensions of the earth's equator and poles on the celestial sphere. This system is the one most commonly used in astronomy.

(3) Horizon System uses the observed horizon as the reference plane and is the common system of celestial navigation.

Coriolis force: a fictitious velocity dependent force that apparently arises when the motion of an object is reckoned with respect to a rotating coordinate system. The relationship yielding this "force" is $-2m\vec{\omega} \times \dot{\vec{r}}_r$, where m is the mass of the object, $\vec{\omega}$ is the angular velocity vector of the rotating coordinate system, and $\dot{\vec{r}}_r$ is the velocity of the object reckoned with respect to the rotating system.

Cosmic dust: fine dust particles (micrometeorites) that are concentrated in the solar system in the plane of the ecliptic (e.g., giving rise to the phenomenon of "zodiacal light") and also dispersed in a more rarefied manner in interstellar space, being more concentrated in the galactic spiral arms; also a component of comets.

Cosmic rays (direct): high-energy charged particles (e.g., with energies in excess of 100 Mev) such as protons, alpha particles and heavy nuclei which have apparently been ejected by stars and accelerated by vast magnetic fields in interstellar space.

Cosmoparticle: discrete material entities of submeteoritic mass, either in or from space. They may be "free" or individual molecules or atoms, or molecular or atomic constituents of any kind, e.g., ions, atomic nuclei, protons, neutrons, electrons, positrons, etc.

Cross product: or vector product (denoted by $\vec{A} \times \vec{B}$) of two typical vector quantities $\vec{A}$ and $\vec{B}$ can be defined either as a vector mutually perpendicular to both $\vec{A}$ and $\vec{B}$ with magnitude AB sin (A, B) or equivalently as

$$(A_yB_z - A_zB_y)\ I + (A_zB_x - A_xB_z)\ J + (A_xB_y - A_yB_x)\ K$$

where the subscripts denote the components of the vectors on the three orthogonal axes denoted by the unit vectors I, J, K.

Culmination: The time at which a heavenly body reaches the meridian of an observer. Upper culmination occurs near zenith, lower culmination near nadir.

D

Day, sidereal: the period of one rotation of the earth relative to inertial space (the stars), $23^h\ 56^m\ 04^s.090$ mean solar time.

Day, solar: the time between two successive upper (or lower) culminations of the sun, $24^h\ 03^m\ 56^s.556$ sidereal time.

Declination: the arc of an hour circle (great circles passing through the poles) intercepted between the celestial equator and the object; angular distance north or south of the celestial equator.

Definitive orbit: an orbit that is defined in a highly precise manner with due regard taken for accurate constants and observational data, and precision computational techniques including perturbations.

Differential correction: a method for finding from the observed minus computed (O - C) residuals small corrections which, when applied to the elements or constants, will reduce the deviations from the observed motion to a minimum.

Dip: the angular distance between the true horizontal and the observed horizon for an observer above ground level.

Direct motion: the term applied to eastward or counterclockwise motion of a planet or other object as seen from the North Pole (i.e., in the direction of increasing right ascension). Thus, it is motion on an orbit in which $i \leq 90$ degrees.

Diurnal: daily.

Diurnal motion: the apparent revolution of the heavenly bodies around the earth.

Dot product: or scalar product (denoted by $\vec{A} \cdot \vec{B}$) of two typical vector quantities $\vec{A}$ and $\vec{B}$ can be defined as AB cos (A, B) or equivalently as $A_xB_x + A_yB_y + A_zB_z$ where the subscripts denote the components of the vectors on three orthogonal axes.

Drag: the force occasioned by the passage of an object through a resistive medium acting in a direction opposite to that of the object's motion relative to the medium.

Drag coefficient: the total drag force acting on an object divided by one-half the local atmospheric density, the projected frontal area of the object, and the square of the magnitude of the velocity of the object relative to the resistive medium.

Drift, anomalistic: the variation or drift of a frequency source (e.g., a crystal oscillator) such that the frequency changes due to a variety of causes (e.g., temperature variation, component aging, etc.), none of which can be predicted in advance or completely controlled.

E

Eccentric anomaly: an angle at the center of an ellipse between the line of apsides and the radius of the auxiliary circle (which has radius equal to semimajor axis of ellipse and center at center of ellipse) through a point that has the same x-coordinate as a given point on the ellipse.

Eccentricity: the ratio of the radius vector through a point on a conic to the distance from the point to the directrix.

Eclipse: a name applied to cases where a nonluminous body passes into the shadow of another; eclipse of the sun means the interposition of the moon's disc between the observer and the sun.

Ecliptic: the great circle formed by the intersection of the orbital plane of the earth (the ecliptic plane) and the celestial sphere.

Ecliptic coordinate system: axes with the ecliptic as the fundamental plane and with spherical coordinates: celestial longitude and latitude.

Elements of orbit: any six independent constants defining the orbit, e.g., (1) orientation elements: Ω longitude of ascending node; i inclination of orbit plane; ω argument of perifocus; (2) dimensional elements: e eccentricity; a semimajor axis; (3) time element: T time of perifocus passage.

Elevation, angle of: the angle between the inertial velocity vector $\dot{\vec{r}}$ and the local horizontal, i.e., the plane normal to $\vec{r}$ passing through the vehicle.

Eliminant: a determinant that is formed when n - 1 linear unknowns are eliminated from a set of n equations. The elimination of x and y, for example, from

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

$$a_3x + b_3y = c_3$$

yields the eliminant:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Elongation, angle of: the angle between the direction to an object and to the center of the coordinate system reckoned at the observer.

Energy integral: one of the integrals of two-body motion expressing conservation of energy.

Entry angle: the angle between the velocity vector of a space vehicle relative to a resistive medium and the local horizontal.

Ephemeris (plural, ephemerdies): a table of calculated coordinates of an orbject with equidistant dates as arguments.

Ephemeris time (ET): uniform or Newtonian time, defined by mean frequency of rotation of the earth around the sun for the year 1900.

Epoch: arbitrary instant of time for which the elements of an orbit are valid (e.g., initial, injection, or correction time).

Equator, celestial: the great circle in which the plane of the terrestrial equator intersects the celestial sphere.

Equator, terrestrial: the circle in which the plane through the earth's center normal to its axis of rotation (the equatorial plane) intersects the earth's surface.

Equatorial bulge: the excess of the earth's equatorial diameter over the polar diameter (i.e., about 27 miles, 43 km); oblateness.

Equatorial satellite: a satellite whose orbit plane coincides with the earth's equatorial plane.

Equatorial system: rectangular axes referred to the equator as the fundamental plane and having spherical coordinates, right ascension and declination.

Equilateral triangle solutions: a particular solution of the three-body problem in which an object situated at one vertex of an equilateral triangle formed with the sun and a planet has a stable orbit. It was predicted by Lagrange (1772) and amply confirmed in the case of Jupiter. See Trojan asteroids.

Equinox, nutation of: arises from nutation of equator.

Equinox of date: position of equinox at epoch being used in discussion.

Equinox, precession of: arises from precession of equator.

Equinox, true: equals equinox or vernal equinox, q.v., "true" being used to emphasize distinction from mean equinox.

Equinoxes: intersections of the equator and ecliptic, the vernal equinox being the point where the sun crosses the equator going from south to north (descending node of earth's orbit).

Euler's equation: a relation in a parabolic orbit involving two radii vectors, their chord, and the time interval between them; discovered by Euler (1744).

Evection: a large perturbative term in the moon's longitude discovered by Hipparchus, amounting to 1° 15' at maximum.

F

Feasibility orbit: an orbit that can be rapidly and inexpensively computed on the basis of simplifying assumptions (e.g., two-body motion, circular orbit, three-body motion approximated by 2 two-body orbits, etc.) and yields an indication of the general feasibility of a system based upon the orbit without having to carry out a definitive orbit computation.

Free-molecule flow (or free-molecular flow): flow regime in aerodynamics in which molecules emitted from an object, as it passes through a resistive medium, do not affect the flow of oncoming molecules by scattering interactions, i.e., the mean free path of the emitted molecules is much longer than a characteristic linear dimension of an object.

G

Galactic system: a system based on the centerline of the milky way.

Gaussian gravitational constant, K_s: factor of proportionality in Kepler's third law; the numerical value depending on the units employed. See astronautical unit.

Geocentric: referred to the center of the earth as origin.

Geocentric parallax: see parallax.

Geocentric subvehicle point: the point where the radius vector from the geocenter to a space vehicle intersects the spheroid.

Geodetic subvehicle point: the point where a line through a space vehicle normal to the spheroid intersects the spheroid.

Geoid: the mean sea-level figure of the earth.

Geoidal surface: the mean sea-level surface of the earth; surface of gravitational equipotential.

Geometric meter: the standard meter.

Geopotential meter: a unit of length employed in reckoning geopotential altitude.

Gravitational potential: at a point, the work required to remove unit mass from that point to infinity.

Greenwich meridian: the zero meridian from which geographical longitude is measured (passes through the Greenwich Observatory, England).

Ground trace: a succession of subvehicle points on earth or on any other celestial body.

Ground swath: a region around the ground trace, the boundaries of which are specified by the lateral distance from the ground trace.

Guidance and control system: a system that actively counteracts or overcomes the effects of deviations (from nominal conditions) in order to accomplish the given mission with the desired degree of exactness. Navigational inputs allow the guidance and control system to sense these deviations.

Guidance law: the equations which are mechanized in the guidance and control system.

Guidance law, explicit: the guidance computer in the vehicle predicts and the vehicle is steered along a trajectory which brings it to the desired end conditions.

Guidance law, implicit: the vehicle follows a nominal trajectory while the guidance system is active.

H

Harmonics of the earth's gravitational field: a series representing the gravitational potential of the earth whose terms form a harmonic progression, i.e., include powers of the reciprocal of distance.

Heliocentric: referred to the center of the sun as origin.

Hohmann orbit: an elliptic heliocentric trajectory for interplanetary flight, having tangency to the earth at one apsis and to another planetary orbit (e.g., that of Venus or Mars) at the opposite apsis. More generally stands for any such doubly tangent transfer ellipse.

Horizon, apparent: the horizon formed by the horizontal plane through the position of the observer.

Horizon, rational: the horizon formed by the plane through the center of the earth parallel to the observer's horizon.

Horizon coordinate system: a system of topocentric coordinates either spherical (azimuth and altitude) or rectangular, having as reference plane the celestial horizon, which is perpendicular to the direction of gravity at the observer.

Horizon scanner: an optical device that senses the radiation discontinuity between a planet or lunar surface and the stellar background of space. It can be utilized to establish a "vertical" reference based upon a "visual" horizon (which differs from both the astronomical and geodetic horizon).

Horizontal plane: that plane perpendicular to the direction of gravity at any place.

Hour angle (LHA): angle between the observer's meridian and the hour circle passing through the object, a coordinate in the rotating equator system, positive toward west, 0 to 24 hr.

Hour circle: any one of the great circles that pass through the celestial poles and, therefore, are at right angles to the equator.

I

Inclination i: angle between orbit plane and reference plane (e.g., the equator is the reference plane for geocentric orbits and the ecliptic for heliocentric orbits).

Inertial axes: axes that are not in accelerated or rotational motion.

Injection: the addition of an "instantaneous" incremental velocity vector to the satellite velocity vector at a prescribed time and place to establish a new orbit.

Injection conditions: position and velocity of rocket at the instant when the thrusting of rocket motor ends and the ballistic portion of the trajectory begins.

Intermediate orbit: an orbit tangent to the actual (or disturbed) orbit, having the same coordinates but not velocity at point of tangency.

Inversion: in this context is meant to be synonymous with the numerical solution of a set of linear algebraic equations.

Ionosphere: the ionized portion of the atmosphere above about 60 km.

Isostatic equilibrium: a situation in which the pressure under the earth's surface is the same regardless of whether it is measured under a mountain, valley or ocean, i.e., lower density strata underlie mountains while higher density strata underlie oceans.

J

Jacobi's integral: an integral of the equations of motion in a rotating coordinate system which relates the square of the velocity and the coordinates of an infinitesimal body referred to the rotating coordinate system. The constant of integration associated with Jacobi's integral is known as Jacobi's constant.

Julian date: the number of mean solar days that have elapsed since midnight, January 1, 4713 BC; e.g., the Julian date of January 1, 1960 is 2,436,934, and of February 1, 1965 is 2,438,792, etc.

K

Kepler's planetary laws: (1) every planet moves in an ellipse about the sun with the sun at one focus; (2) every planet moves in such a way that its radius vector sweeps over equal areas in equal intervals of time; (3) the squares of the periods of revolution of two planets are to each other as the cubes of their mean distances from the sun.

K_e^{-1} min: the characteristic time for geocentric orbits, i.e., the time required by hypothetical satellites to move 1 radian in a circular orbit of radius a_e (equatorial earth's radius); equal to 13,447,052 min.

K_s^{-1} day: the characteristic time for heliocentric orbits, i.e., the time required for a planet at 1 astronomical unit to move 1 radian (or 1 a.u.) along its orbit; equal to 58.132,440,87 days.

L

Lagrangian solutions: particular solutions of the three-body problem in which an infinitesimal object moves under the attraction of two finite bodies (e.g., the sun and Jupiter) which revolve in circles around their center of mass and in which the distances from the infinitesimal object to the finite bodies remain constant. See also equilateral triangle solutions and synodic satellites, i.e., the so-called straight line solutions.

Lambert's equation: an equation of the 8th degree expressing the curvature of the apparent path of a body moving around the sun, as seen from the earth: discovered by Lambert (1771).

Latitude, astronomical: the angle between the direction of gravity through a point and the equatorial plane.

Latitude, celestial: the angular distance of an object north (+) or south (-) of the ecliptic plane; a coordinate in the ecliptic system.

Latitude, geocentric: the angle between the equatorial plane and a straight line from the observer to the center of the earth. It differs from astronomical and geodetic latitudes because of the oblateness of the earth, 0° to 90° north or south.

Latitude, geodetic (or geographic latitude): the angle between the plane of the equator and a normal to a reference spheroid. Geodetic and astronomical latitudes differ only because of local deviations in the direction of gravity, 0° to 90° north or south.

Least squares inversion: a solution of a set of overdetermined linear equations such that the sum of the squares of the residuals is a minimum.

Legendre polynomials: the coefficients $P_n(c)$ in the expansion $(1 - 2ch + h^2)^{-1/2} = \sum_{n=0}^{\infty} P_x(c)\, h^n$ where $P_0(c) = 1$, $P_1(c) = 1/2\,(3c^2 - 1)$, $P_3(c) = 1/2\,(5c^3 - 3c)$, or, in general, $(n + 1)\, P_n + 1(c) - (2n + 1)\, cP_n(c) + nP_{n-1}(c) = 0$.

Libration: (1) apparent or optical and physical tilting and side-to-side movements of the moon that render 18 percent of its surface alternately visible and invisible, (2) long-period orbital motions of the Trojan asteroids around the equilateral triangle points of the three-body Lagrangian solutions, (3) periodic perturbative oscillations in orbital elements.

Lift: the force arising from the passage of a vehicle through a resistive medium when the vehicle presents an asymmetrical form or orientation; which force acts in a direction normal to the object's motion relative to the medium.

Limb: the edge of the visible disk of the sun, moon, planet, etc.

Line of apsides: a line connecting the near to the far apsis, i.e., defines the major or transverse axis.

Line of nodes: the intersection of a reference plane and the orbit plane.

Line-of-sight: the apparent or observed direction of an object.

Longitude, celestial: the angular distance measured along the ecliptic from the vernal equinox eastward to the great circle passing through the object and normal to the ecliptic.

Longitude, ephemeris: analogous to ordinary geographic longitude, but referred to the ephemeris meridian, rather than to the Meridian of Greenwich.

Longitude, geocentric: the angular distance from the foot of the Greenwich meridian, measured along the equator, east or west, to the foot of the meridian through the place.

Longitude of ascending node: the angular distance from the vernal equinox measured eastward in the fundamental plane (ecliptic or equator) to the point of intersection with the orbit plane at the point that the object crosses from south to north.

Longitude of perifocus: sum of the angle in the fundamental plane between the vernal equinox and the line of nodes and the angle in the orbit plane between the line of nodes and the line of apsides, measured in the direction of motion.

Lunar equation: a factor required for reducing observations to the barycenter of the earth/moon system.

Lunar theory: the analytical theory of the motion of the moon. The lunar theories of Delavnay, Hansen, and Hill-Brown are used most frequently today.

Lunar unit (LU): the mean distance from the center of the earth to the center of a fictitious unperturbed moon having the mass and sidereal period of the moon. One lunar unit is approximately equal to 384,747 km or 239,122 statute miles.

Lunicentric: referred to the moon's center as origin; selenocentric.

M

Mach number: the ratio of the speed of a vehicle to the local speed of sound.

Macrometeorites: meteorites that are sufficiently massive to become fallen meteorites (and whose origin appears to be related to that of minor planets).

Magnetic storms: extensive disturbances in the earth's magnetic field.

Magnitude, stellar: a measure of the brightness of a star. A difference of five magnitudes represents a factor of 100 in brightness.

Mean anomaly: the angle through which an object would move at the uniform average angular speed n, measured from perifocus; $M = t\sqrt{\frac{\mu}{a^3}}$.

Mean center of moon (MCP): the point on the lunar surface intersected by the lunar radius that is directed toward the earth's center when the moon is at the mean ascending node and when the node coincides with the mean perigee or the mean apogee. The MCP is a specified distance from the crater Mösting A in the Sinus Medii.

Mean distance: the semimajor axis (it can be considered as an historical term).

Mean equinox of date: a fictitious equinox whose position is that of the vernal equinox at a particular date with the effect of nutation removed.

Mean free path: the path of a molecule when molecules are assumed to be smooth, rigid spheres with no external field of force acting on them; each molecule travels freely on a straight line between impacts with other molecules. The distance traversed between two successive impacts is called the free path and the average value of this distance the mean free path.

Mean solar day: the elapsed time between successive passage of the mean fictitious sun across the observer's meridian, 86,400 mean solar sec, the mean fictitious sun being a fictitious sun that moves along the celestial equator with the mean speed with which the true sun apparently moves along the ecliptic throughout the year.

Meridian: (1) Terrestrial meridians: great circles passing through North and South Poles, e.g., the observer's local meridian passes through his local zenith and the North and South Poles. (2) Celestial meridian: a great circle on the celestial sphere in the plane of the observer's terrestrial meridian.

Meridian, ephemeris: the geographical meridian which lies east of Greenwich by the amount 1.002738 times the difference (ET-UT).

Meridian passage: also called "transit" or "culmination" of a celestial object is marked by its crossing an observer's meridian.

Mesometeorites: intermediate meteorites having characteristic dimension of the order of a fraction of an inch that are stopped by the atmosphere, consumed, and are seen as common "meteors." The origin of these bodies appears to be related to that of comets

Meteor swarms: a large collection of mesometeorites (probably the remains of an "old" comet) that enters the earth's atmosphere and is seen as a swarm of meteors. The term is often applied to the actual collection of mesometeorites on heliocentric orbits in space.

Micrometeorites: very small meteorites (having a characteristic dimension of a few microns) that are stopped by the atmosphere without being consumed in flight or without producing luminous phenomena visible at the earth's surface.

Minor planets (or asteroids): small planets revolving about the sun, estimated to number more than 30,000, with diameters of more than 1 mile. The largest, Ceres, has a diameter of 488 miles.

Molecular scale temperature: the actual temperature of the atmosphere at any given height multiplied by the ratio of the mean molecular weight of the atmosphere at sea level to the mean molecular weight of the atmosphere at the given height.

Month, nodal: the time for one revolution of the moon with respect to either node.

Month, sidereal: the time between two successive arrivals of the moon at a given apparent place on the celestial sphere as indicated by the stars.

Month, synodic: the time for one revolution of the moon with respect to the apparent place of the sun, e.g., the time between conjunctions.

Moon's celestial equator: a great circle on the celestial sphere in the plane of the moon's equator, i.e., in a plane perpendicular to the moon's axis of rotation.

Moon's orbital plane (MOP): the instantaneous orbital plane of the moon around the earth, defined by the moon's geocentric radius and velocity vectors.

N

Nadir: the downward plumb-bob direction, or the point where the downward extension of the direction of a plumb-bob intersects the celestial sphere.

Navigation: the process of determining the position and velocity of a submarine, ship, airplane, or space vehicle by making observations from the vehicle of objects in the environment of the vehicle.

n-body problem: concerned with the gravitational interactions of masses m_i, m_j, $i, j = 1, 2, \ldots, n$ which are assumed homogeneous in spherical layers, under the Newtonian law. If $n = 2$, one has a two-body problem, while $n = 3$ is known as the three-body problem.

Newton's laws: Law of gravitation: Every particle of matter in the universe attracts every other particle with a force varying directly as the product of their masses and inversely as the square of the distance between them. Laws of motion: (1) Every particle continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by a force impressed upon it. (2) The rate of change of momentum is proportional to the force impressed, and takes place in the direction of the straight line in which the force acts. (3) To every action there is an equal and opposite reaction; or the mutual actions of two bodies are always equal and oppositely directed.

Nodal passage, time of: the time T_Ω when an object passes through the node from the southern hemisphere to the northern hemisphere.

Node: the points of intersection of the great circle on the celestial sphere cut by the orbit plane and a reference plane (e.g., the ecliptic or equator reference plane).

Node, ascending: the node in the reference plane through which the body passes from South to North.

Node, descending: the node in the reference plane through which the body passes from North to South.

Node, longitude of ascending: see longitude of ascending node.

Nominal orbit: the true or ideal orbit in which space vehicle is expected to travel.

Normal places: curve formed, when several observations are available very close together in time, by smoothing observed coordinates.

Numerical differentiation: a process that allows for the numerical evaluation of the derivative of quantity, given tabular values of the quantity.

Numerical integration: a process that allows for the numerical evaluation of a definite integral.

Nutation: short period terms in the precession arising from the obliquity, the eccentricity, and the inclination of the moon's orbit and the regression of its nodes (approximately a 19-year period).

O

Obliquity of the ecliptic: the inclination of the ecliptic to the celestial equator; the angle of approximately 23° 27' between the earth's orbital plane and its equator.

Occultation: the interruption of the light from one celestial body by the intervention of another.

Opposition: the position of an object when its celestial longitude is 180° from sun, i.e., opposite to sun. (Configuration possible only with moon and superior planets.)

Orientation angles: the classical orientation elements, i.e., the inclination, longitude of the ascending node, and longitude of perifocus.

Osculating orbit: an orbit tangent to the actual or disturbed trajectory, having the same coordinates and velocity at that instant.

P

Parallactic angle: the angle between the hour circle of and the vertical circle through a body.

Parallactic inequality: a secondary effect on the solar perturbations in the moon's longitude due to the ellipticity of the earth's orbit.

Parallax: (1) Geocentric parallax: the angle at the object subtended by the earth's equatorial radius; applied to objects in the solar system. (2) Heliocentric parallax: the angle at a star, etc., subtended by the radius of the earth's orbit; applied to objects outside the solar system.

Pericynthion: the point on a selenocentric orbit nearest the moon's center.

Perifocus: the point on an orbit nearest the central force.

Perigee: the point on a geocentric orbit nearest the earth's center.

Perihelion: the point on a heliocentric orbit nearest the sun.

Period: the time required for one complete circuit of the orbit.

Period, anomalistic: interval of time from one perifocus passage to the next.

Period, nodal (also draconic): interval of time from one nodal crossing to the next.

Period, sidereal: the time required for the projection of a planet or other body to make a complete circuit of the celestial sphere. This is the true period.

Period, synodic: the time between successive oppositions of a superior planet or successive inferior conjunctions of an inferior planet.

Perturbations: deviations from exact reference motion caused by the gravitational attractions of other bodies or other forces.

General perturbations: A method of calculating the perturbative effects by expanding and integrating in series.

Special perturbations: methods of deriving the disturbed orbit by numerically integrating the rectangular coordinates or the elements.

Piecewise continuous: a function that can be divided into a finite number of pieces such that the function is continuous on the interior of each piece and such that the function approaches a finite limit at the point of connection of one piece with another. In the context of the temperature profile discussion the term is used in a more restricted sense to imply a function that is divided into a finite number or series of connected linear pieces (straight line segments).

Planetocentric: referred to the center of a planet as dynamical center or origin of coordinates.

Planets: bodies in the solar system which move in essentially elliptical paths around the sun (see Kepler's laws).

Inferior planets:

Mercury
Venus

Superior planets:

Mars
Asteroids
Jupiter
Saturn
Uranus
Neptune
Pluto

Inner, or terrestrial, planets:

Mercury
Venus
Earth
Mars

Asteroids, or minor planets.

Outer, or major, planets:

Jupiter
Saturn
Uranus
Neptune
Pluto

Plasma: a collection of positive and negative ions that has no overall or gross charge.

Polar satellite: a satellite that passes over the north and south poles of the earth, i.e., that has an inclination of 90° with respect to the earth's equator.

Polar distance, ecliptic: complement of the celestial latitude.

Polar distance, north: complement of the declination.

Poles, celestial: the points in which the axis of rotation intersects the celestial sphere.

Poles, ecliptic: the points in which the normal to the ecliptic through the origin intersects the celestial sphere.

Poles, galactic: the points in which the fundamental galactic axis intersects the celestial sphere. The north galactic pole is at $12^h\ 44^m$ right ascension and +27° declination.

Position, apparent: coordinates of a celestial body as seen by an observer at the center of the earth referred to a coordinate system defined by the instantaneous equator, ecliptic, and equinox. The tabulated positions of the sun, moon, and planets in the American Ephemeris and Nautical Almanac are usually apparent positions.

Position, mean: coordinates of a celestial body referred to a coordinate system defined by the mean equator, ecliptic and equinox of date. This means that the periodic effects of nutation have been neglected.

Position, true: coordinates of a celestial body if corrections for planetary aberration are applied to the apparent position. A sequence of true positions as a function of time is known as a geometric ephemeris.

Potential function: see gravitational potential.

Poynting-Robertson effect: the gradual decrease in the orbital semimajor axis and eccentricity of a micrometeorite caused by the re-emission of radiant energy from the micrometeorite. The theory was first announced by Poynting and later improved and brought into conformity with the theory of relativity by Robertson.

Precession of the equinoxes: the slow, 26,000-year period westward motion of the equinoxes (and equator) along the ecliptic, arising from solar and lunar perturbations on the earth's equatorial bulge, which cause the earth's axis to precess.

Predicting a satellite's position: the six elements are the same in number as the three coordinates of position and the three components of velocity required to specify the launching conditions completely.

Primary: the body having the strongest gravitational field (most ponderous mass) in a system of bodies revolving about their common center of gravity. (Sun is the solar system's primary, earth is earth-moon system primary, etc.)

Prime meridian: the meridian defining 0° and 180° E or 180° W longitude. On earth the Greenwich meridian is the prime meridian.

R

Radiation pressure: the pressure acting on a surface exposed to incident electromagnetic radiation caused by the momentum transferred to the surface by the absorption and reflection of the radiation.

Ratios of the triangles: in the orbit determination methods of Gauss, Olbers, et al., the ratios of the triangles formed by the radii and the chords are assumed in a first approximation to be ratios of the sectors, which are the ratios of the corresponding time intervals by Kepler's second law.

Rectilinear orbit: a trajectory for which perifocus distance is zero and eccentricity is one.

Red shift, gravitational: an effect predicted by the General Theory of Relativity in which the frequency of light emitted by atoms in stellar atmospheres is decreased by a factor proportional to the (mass/radius) quotient of the star: confirmed observationally by the spectra of white dwarfs.

Re-entry: portion of a trajectory in the atmosphere of a planet; in the case of the earth it is usually taken as the portion below 400,000 ft or 122 km.

Re-entry corridor: all possible re-entry trajectories which do not produce excessive aerodynamic heating or deceleration.

Reduction to orbit: quantity added to celestial heliocentric longitude to give true longitude, q.v.

Reference ellipsoid (or spheroid): oblate spheroid closely approximating the geoid.

Reference orbit: an orbit, usually but not exclusively the best two-body orbit available, on the basis of which the perturbations are computed.

Refractive index (of a medium): the ratio of the speed of light in a vacuum to that in the medium, hence it is a measure of how greatly electromagnetic radiation rays are bent during their transit through a medium such as the earth's atmosphere.

Regression of the moon's nodes: the movement of the nodes of the moon's orbit westward along the ecliptic, due to solar perturbations, with period $\simeq$ 19 years.

Relativity effects: effects on a space vehicle trajectory and on time measurement arising by use of Einstein's special theory of relativity or of Einstein's general theory of relativity instead of the customary Newtonian mechanics for determining the trajectory. The fundamentals of these theories of relativity are discussed in Chapter IV of the Lunar Flight Handbook. Relativity effects are small in the weak gravitational field of the solar system if the space vehicle velocity is small compared to the speed of light. There are many such effects, the most prominent of which are: the time dilation predicted from the Lorentz transformation of special relativity; the time dilation, secular advance of perigee, and red-shift of spectral lines predicted by general relativity.

Rendezvous: the approach and contact of two vehicles in space.

Representation: the computation of the position of a space vehicle given the orbital elements and the time.

Residuals (O - C): differences between the observed and computed coordinates in the sense observed minus computed.

Residuals (O - 1): differences between the precomputed ideal observational data and the actual observed data on, for example, an interplanetary voyage.

Restricted n-body problem: the motion of n masses under their mutual gravitational attraction, but with one of the n masses having negligible mass and hence not influnencing the motion of the other (n - 1) masses. This term is usually applied to n = 3 (see also n-body problem).

Retrograde motion: westward or clockwise motion as seen from the North Pole, i.e., motion in an orbit in which $i > 90$ degrees (opposite earth's rotation).

Retrorocket: a rocket attached to a space vehicle whose thrust is directed in a general direction against the inertial velocity of the space vehicle.

Reynolds number: the ratio of inertial forces to viscous forces--it is proportional to the Mach number, vehicle diameter, and the density, or, in equivalent terms, proportional to the diameter of the space vehicle in mean free paths and the vehicle speed measured in terms of the average thermal speeds of gas molecules that constitute the oncoming flow.

Right ascension: angular position of an object (e.g., star) measured eastward along the celestial equator from the vernal equinox to the great circle passing through the north celestial pole and the star (hour circle). Right ascension is often expressed in hours, minutes, and seconds ($1^h = 15°$).

S

Scale height: the distance in which an isothermal atmosphere decreases in density from 1 to 1/e.

Secular terms: expressions for perturbations that are proportional to the time.

Selenocentric: referred to the center of the moon; lunicentric.

Selenocentric equatorial coordinates: a right-handed coordinate system centered at the moon with its three axes defined by the vernal equinox, north celestial pole (of the earth), and a direction perpendicular to these two, i.e., an equatorial coordinate system translated to the moon.

Selenographic coordinates: coordinates that are rigidly attached to the moon (as geographic coordinates are attached to the earth) defined by the moon's equator and prime meridian. See mean center of moon.

Semimajor axis: the distance from the center of an ellipse to an apsis; one-half the longest diameter; one of the orbital elements.

Semiminor axis: one-half the shortest diameter of an ellipse.

Semiparameter: semilatus rectum; the perpendicular distance from the conic to the semimajor axis through either focus (not to be confused with the generic term "parameters").

Setting circles: a graduated scale that can be read visually and indicates the direction (e.g., altitude and azimuth or right ascension and declination) in which a telescope is pointed. Ordinarily they are employed to set or point a conventional astronomical telescope in the proper direction to make a given observation.

Sidereal period of a planet: see period, sidereal.

Sidereal time: the hour angle of the vernal equinox. (See Chapter II for conversion of sidereal time to mean solar time).

Sidereal year: time required by the earth to complete one revolution of its orbit; equal to 365.25636 mean solar days.

Slip flow: a flow regime in aerodynamics in which there is some departure from continuum flow and the layer of compressible fluid immediately adjacent to the surface of an object is no longer at rest but has a finite tangential "slipping" velocity.

Solar flares: short-lived areas of brilliance (covering areas of 10 million square miles or so) on the sun's chromosphere that are associated with other solar activity. Often accompanied by bursts of emitted charged corpuscles and electromagnetic radiation. They reach several times normal brightness within one or two minutes and then subside slowly over 15 to 30 minutes.

Solar parallax: the ratio of the earth's equatorial radius to its mean distance from the sun.

Solar time, mean: hour angle of fictitious mean sun increased by 12 hours. (The fictitious mean sun is a fictitious sun moving on the celestial equator with a mean motion of the real sun.) See pages 474 to 476, American Ephemeris and Nautical Almanac for conversion of mean solar time to sidereal time.

Solar wind: those low energy particles, i.e., corpuscular radiation (electrons and protons) emanating from the sun. Typical flux rates are 10^8 to 10^{10} particles per cm^2 per second, and typical energies are 1000 to 100,000 electron volts for the protons and a few electron volts for the electrons.

Solstices: the two times a year when the sun's declination is greatest north or south (about June 22 and December 22).

Space range system: a system or network of observation stations, together with their associated communication links and computational facilities, that are utilized to observe and track space vehicles, e.g., the Pacific Missile Range, the National Space Surveillance System, etc.

Specular reflection: characterized by the relation that the angle of incidence equals the angle of reflection, in contrast to diffuse reflection.

Sphere, celestial: an imaginary reference sphere; generally considered to be of infinite radius, and having its visible representation in the sky.

Spheroid: an oblate ellipsoid which closely approximates the mean sea-level figure of the earth or geoid.

Stability of a point or orbit: a point or orbit is stable if the space vehicle will remain near the point or orbit if given a small displacement and velocity. The point or orbit is unstable if the space vehicle will depart from it rapidly.

Standard atmosphere: a table of atmospheric density as a function of altitude which is accepted as a standard and used as a model to portray a typical average atmospheric density variation.

Standard deviation: the square root of the arithmetic mean of the squares of the deviations from the mean; also called root mean square error and sigma deviation.

Stationary points: points in the apparent path of a planet, etc., against the star background where the object appears to stand still because relative to the observer it is moving only in the line of sight. Such a point occurs when a planet changes its apparent motion from direct to retrograde and vice-versa.

Station error: small, usually negligible, differences between the astronomical and geodetic latitudes, due to certain anomalies (such as a mountain) in the local gravitational field.

Stratosphere: a region in which the temperature remains constant from about 18 km up to a height of 30 to 35 km.

Surface-circular satellite: a hypothetical satellite on a circular orbit about the earth having a semimajor axis equal to the earth's equatorial radius. Hence, such a satellite would "skim the surface of the earth" as it revolved on its orbit.

Synodic satellite: a hypothetical satellite, situated 0.84 of the distance to the moon on a line joining the centers of the earth and moon and having the same period of revolution as the moon, according to the Lagrangian "straight line solution" of the three-body problem.

T

Terminator: the boundary between the illuminated and dark sides of a planet or satellite. Usually one distinguishes between a morning and an evening terminator.

Three-body problem: the problem of integrating the equations of motion of three bodies (e.g., sun-moon-earth) moving under their mutual gravitational attractions: directly soluble only in particular cases. See Lagrangian solutions.

Thrust: the force exerted on a vehicle, by the discharge of a gas or propellant, in accordance with the conservation of linear momentum.

Time, ephemeris: time reckoning based upon "constant" frequency rather than frequency of earth's rotation. The current difference between ephemeris and universal time is about 35 seconds.

Time dilation: the apparent slowing-down of moving clocks. This effect arises from the special and general theory of relativity.

Time of perifocal passage: the time when a space vehicle traveling upon an orbit passes by the nearer apsis or perifocal point.

Topocentric: referred to the position of the observer on the surface of the earth, as origin.

Topocentric parallax: the difference between the geocentric and topocentric positions of a satellite.

Topocentric equatorial coordinates: a right-handed coordinate system centered at the observer with its three axes defined by the vernal equinox, north celestial pole, and a direction perpendicular to these two, i.e., an equatorial coordinate system translated to the topos.

Tracking: the process of determining the position and velocity of a celestial body by making observations from earth by optical or electromagnetic means.

Trajectory sensitivities: the partial derivatives of dependent trajectory variables with respect to independent trajectory variables.

Transitional flow: a flow regime in aerodynamics between the free-molecule flow and slip-flow regimes in which the molecules emitted from the surface of an object affect the flow of oncoming molecules, i.e., in which the mean free path of the emitted molecules becomes comparable to a characteristic linear dimension of an object.

Transearth trajectory: trajectory from the vicinity of the moon to the vicinity of the earth.

Translunar trajectory: trajectory from the vicinity of the earth to the vicinity of the moon.

Transverse axis: the distance between the apsides --identical to the semimajor axis for elliptical orbits.

Triaxial ellipsoid: a solid aspherical figure which when cut or sectioned in three (orthogonal, normal or mutually perpendicular) directions exhibits three elliptical cross sections of differing semimajor axes and eccentricities.

Tropopause: the height (varying from about 9 km over the poles to 18 km over the equator) where the gradual decrease in temperature with elevation above sea level ceases.

True anomaly: the angle about the focus between the perifocus and the radius vector in the direction of the motion.

True equinox of date: the actual position of the equinox including both precession and nutation.

Twenty-four-hour satellite: a satellite whose orbital period is approximately 24 hr. If such a satellite is on circular equatorial orbit, then it will theoretically remain fixed or "stationary" relative to the rotating earth.

Two-body orbit: the motion of a body of negligible mass around a center of attraction.

U

Umbra: the dark central portion of the shadow of a large body such as the earth or moon (used in connection with eclipses). The outer, less dark shadow is known as the penumbra.

Unit vector: a vector whose magnitude or length is unity--utilized to define directions in space.

Universal time (UT): mean solar time referred to the meridian of Greenwich, slightly nonuniform owing to the irregular rotation of the earth.

V

Van Allen radiation belt: two toroidal-shaped zones or belts of charged particles roughly situated in the plane of earth's equator. The inner belt commences at about one-fifth on an earth's radius above the equator and extends out to a little less than one earth's radius. The outer belt is located at about two-and one-half earth radii from the earth at the equator and is about one-earth radius thick. Actually the outer belt has a cross section that is shaped somewhat like a banana and extends north and south of the equatorial plane two earth radii. The northern and southern extremes of the belt's cross section (at about 45 degrees latitude) approach the earth one-half of an earth radius closer than at the equator.

Variant orbits: computed orbits in which one of the initial conditions (or parameters) is varied slightly from those of the nominal trajectory--such orbits are utilized to compute numerical partial derivatives or to determine the effects of errors in launch conditions.

Variation of latitude: small periodic changes in the position of the earth's poles due to a "wobbling" of the axis of rotation about the geometrical axis (the shortest diameter) of the earth.

Vector component: the projection of a vector on a given axis in space, e.g., if it is the X-axis then the component of the vector A on this axis is denoted by A_x.

Vector equation: an equation, whose terms include vectors, that can be resolved into component equations; e.g., $\ddot{\underline{r}} = \frac{-\mu \vec{r}}{r^3}$ actually represents the three component equations:

$$\ddot{x} = -\mu x/r^3$$

$$\ddot{y} = -\mu y/r^3$$

$$\ddot{z} = -\mu z/r^3$$

where $\ddot{\underline{r}}$ has been replaced by its three components $\ddot{x}$, $\ddot{y}$, and $\ddot{z}$ and $\underline{r}$ by its three components x, y, and z.

Velocity, circular: the magnitude of the velocity required of a body at a given point in a gravitational field which will result in the body following a circular orbital path about the center of the field.

Velocity, escape (also parabolic velocity): the minimum magnitude of the velocity required of a body at a given point in a gravitational field which will permit the body to escape from the field.

Velocity, orbital: with respect to the planets, usually the mean magnitude of the velocity in orbit--computed as the total distance traveled in one circuit divided by the period.

Vernal equinox: that point of intersection of the ecliptic and celestial equator where the sun crosses the equator from south to north in its apparent annual motion along the ecliptic.

Vis viva integral: see energy integral.

Voice trajectory program (Volume of Influence Calculated Envelopes): a patched conic lunar mission trajectory program. It uses the analytical solutions of the two-body trajectories to construct a complete trajectory from the vicinity of the earth to the moon and back.

Y

Year: the orbital period of the earth. When unqualified, it refers to the equatorial or to the calendar year, depending on its use.

Year, anomalistic: the time interval between successive passes through perihelion = 365.259,641,34 + 0.000,003,04 T days (T denotes centuries since 1900).

Year, Besselian: a time reckoning in terms of actual rather than calendar years.

Year, calendar: a variable year containing either 365 or 366 days.

Year, equatorial (also tropical or ordinary year, not calendar year): interval between transits of the sun through the moon equator 365.242,198,79 - 0.000,006,14 T days.

Year, Julian: the year of the Julian calendar = 365.25 days.

Year, sidereal: the period of the earth relative to the stars = 365.256,360,42 + 0.000,000,11 T days.

Z

Zedir technique: the use of two cameras on a satellite whose optical axes are parallel, one of which photographs the sky (zenith) while the other simultaneously photographs the ground (nadir). Upon development and measurement, the photographs can be utilized to find the attitude of the camera's optical axis at the time of photograph.

Zenith: the point where the upward extension of the plumb-bob direction intersects the celestial sphere.

REFERENCE

1. "Flight Performance Handbook for Orbital Operations," Appendix A, Space Technology Laboratories, Inc. (Redondo Beach, California), September 1961.

APPENDIX B

COMPREHENSIVE LIST OF SYMBOLS

APPENDIX B

COMPREHENSIVE LIST OF SYMBOLS

The symbols are arranged alphabetically by chapters. Throughout the handbook a double subscript notation has been introduced to define the origin of and distances in a coordinate system. A geocentric equatorial coordinate system with origin at the center of the earth is denoted by the subscript "$\oplus$." However, if this coordinate system is translated to the center of the moon, for instance, then two subscripts separated by a comma are used: "$\oplus, \leftmoon$." A double subscript without a comma indicates the position of the object denoted by the second subscript relative to object denoted by the first subscript. Hence, $r_{\oplus\leftmoon}$ is the distance of the moon from the earth in any coordinate system. Sometimes it is desirable to emphasize the coordinate system in which the radius vector is measured. The radius vector of the vehicle from the center of the moon may be denoted by $\vec{r}_{\leftmoon\triangle}$, or, if the vector has components in a geocentric equatorial coordinate system, it may be denoted by $\vec{r}_{\oplus, \leftmoon \to \triangle}$.

In some chapters, it is possible to simplify this notation. Thus, in Chapter V, Earth Departure, most quantities refer to the earth, so the subscript "$\oplus$" has been deleted when the reference to the earth is obvious. A similar statement can be made for Chapter VIII, Descent to and Ascent from the Lunar Surface, where the majority of symbols refer to the moon, and where the reference to the moon is obvious in most cases.

Vectors are denoted by an arrow and their components in a matrix equation by curly brackets, while unit vectors are denoted by carats. Hence,

$$\vec{r}_{\oplus\leftmoon} \equiv \begin{Bmatrix} x_{\oplus\leftmoon} \\ y_{\oplus\leftmoon} \\ z_{\oplus\leftmoon} \end{Bmatrix}$$

is a vector from the earth to the moon and $\hat{r}_{\oplus\leftmoon}$ is a unit vector at the center of the earth in the direction of the moon. Matrices are denoted by square brackets and determinants by straight lines, i.e.,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

is the unit matrix, and

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \equiv 1$$

is a complex way of writing 1.

CHAPTER II

SYMBOLS

Latin Symbols

$A_{n,\ m}$	General spherical harmonic coefficient of order n and degree m
AU	Astronomical unit (unit of length, $1 AU = 149.53 \times 10^6$ km)
a, b, c†	Semiaxes of the triaxially ellipsoidal moon
$B_{n,\ m}$	General spherical harmonic coefficient of order n and degree m
$C_{n,\ m}$	General spherical harmonic coefficient of order n and degree m
$C_{2,\ 2}$	Coefficient expressing the ellipticity of the equator
f	Flattening or oblateness of a planet
f'	Flattening of the lunar equator
f*	Flattening of the lunar prime meridian
G	Universal gravitational constant
g	Gravitational acceleration
g_o	Gravitational acceleration at the surface of the equivalent sphere
h	Altitude
I_a, I_b, I_c	Moments of inertia of the moon about the a, b, c axes, respectively
J_n	N-th order zonal harmonic coefficient in the expansion of the gravitational potential in terms of zonal harmonics, $J_n = -C_{n,\ 0}$
K	Thermal inertia constant $K = (\lambda \rho c)^{-1/2}$, see page II-12
$K_{\odot}$	Gaussian value of the solar gravitational constant in units of $AU^{3/2}$/ mean solar day
L'	Lunar inequality (see page II-4 and Table 4)
LU	Lunar unit (unit of length, 1 LU = 384747.2 km)
M	Mass
$\overline{M}$	Molecular weight of air
MSD	Mean solar day
P_n ( )	Legendre polynomial
P_n^m ( )	Associated Legendre polynomial
p	Pressure
R	Radius of the sphere of equivalent volume as the planet
R*	Universal gas constant, $R^* = 8.31470 \times 10^3$ joules/kg° K
R_e	Equatorial radius
R_p	Polar radius
R_ϕ	Radius of celestial body at latitude ϕ
$R_{\phi\lambda}$	Radius of celestial body at latitude ϕ and longitude λ
$\vec{r}$	Radius vector
r*	Radius of action of the planet
$\overline{r}_{\oplus☾}$	Mean earth-moon distance for the restricted three-body force model ($\overline{r}_{\oplus☾} \equiv 1$ LU = 384,747.2 km)
$\overline{r}_{☾}$	Mean lunar distance for the n-body force model ($\overline{r}_{☾}$ = 384,402 km)
S	Solar constant (total radiated power of the sun crossing an area of 1 meter^2 at a distance of 1 AU
T	Absolute temperature
T_i	Freezing point of water on the absolute temperature scale
t	Time, nonabsolute temperature (i.e., temperature measured on the Celsius or Fahrenheit scales)
t_i	Freezing point of water on the nonabsolute temperature scales
U	Gravitational potential
UT	Universal time (Greenwich mean time)
V	Velocity
V_p	Parabolic velocity (two-body escape velocity)
W	Radiated power per unit area
$x_G\ y_G\ z_G$	Geographic coordinate system

† In this list of symbols, coordinates in a given coordinate system have been listed together.

$x_S\ y_S\ z_S$	Selenographic coordinate system
$x_☾\ y_☾\ z_☾$	Selenocentric lunar equatorial system

Greek Symbols

$\alpha_p,\ \epsilon_p,\ i_p$	Photometric coordinates of the point P (see page II-11)
$\beta_☾$	Angle between the y_S axis and r_S
Λ_S	Phase angle between the $x_☾$ and x_S axes at time t = 0
λ	Inhomogeneity factor of the moon
$\lambda,\ \phi$	Longitude, geodetic longitude
$\lambda,\ \phi'$	Longitude, geocentric latitude
$\lambda_☾,\ \phi_☾$	Selenographic longitude, latitude
μ	μ = GM: gravitational constant of the celestial body with mass M in laboratory units
ρ	Atmospheric density
ρ^*	Brightness factor used for albedo measurements
ρ_0^*	Normal albedo
σ	Standard deviation, Stefan-Boltzmann constant ($\sigma = 5.67 \times 10^{-8}$ watts/in.^{2} (°K)^4)
τ	Orbital period
$\omega_{\oplus☾}$	Mean angular velocity of the moon in its orbit around the earth
$\omega_\oplus$	Rotational rate of the earth around its axis
$\omega_☾$	Rotational rate of the moon around its axis

Subscripts and Superscipts

$\odot$ ††	Sun
$\oplus$ ††	Earth
☾ ††	Moon
Δ ††	Space vehicle
—	Mean value
d, h, m, s	Days, hours, minutes, seconds
°	Degrees (angular measure or temperature)

††These subscripts have been deleted if the reference to these bodies is obvious.

SYMBOLS

Latin symbols

A, ϵ *	Azimuth, elevation
a	Semimajor axis
C	Jacobian constant (constant of integration)
$D = \text{☾} - L$	Mean elongation of the moon from the sun
d	Number of ephemeris days from the epoch
EML	Earth-moon line
ET	Ephemeris time
e	Eccentricity
$F = \text{☾} - \Omega$	Argument of lunar perigee plus mean anomaly of the moon
$\tilde{F}$	Hamiltonian of the dynamical system
f	True anomaly (central angle in an alliptic orbit measured from perifocus)
G	Universal gravitational constant
$g_{\odot}, \ell_{\odot}, \ell'$**	Mean anomaly of the sun
$h_{\text{☾}}$	Altitude above mean lunar surface
I	Inclination of the mean lunar equator to the ecliptic
i	Inclination, $0° \leq i \leq 180°$
i_{em}	Inclination of the moon orbital plane (MOP) with respect to the earth's equatorial plane
$i_{\text{☾}}$	Inclination of the mean equator of the moon to the true equator or the earth
$i_{\epsilon m}$	Inclination of the moon's orbital plane with respect to the ecliptic
k	Variable introduced by Hansen's intermediate orbit
L, G, H, ℓ, g, h	Canonical orbital elements (Delaunay variables) defined in Subsection C-1
L	Mean longitude of the sun
ℓ	Mean anomaly (of the moon), which is simultaneously a Delaunay variable
ℓ, b	Selenographic coordinates of the earth: (total librations of the moon in longitude and latitude, respectively)
ℓ', b'	Optical librations of the moon in longitude and latitude
$\delta\ell, \delta b$	Physical librations of the moon in longitude and latitude
$\hat{L}$	Unit vector along the moon-earth line (MEL)
M	Mass
MCP	Mean center point of the moon
MEL	Moon-earth line
MOP	Lunar orbital plane
MST	Mean solar time
m	Parameter introduced by Hill in his lunar theory
n	Mean motion or mean angular velocity in the orbit
$P_n(\ )$	Legendre polynomial of order n in ( )
q	Perifocus distance
R	Disturbing function
R_e	Equatorial radius of the earth
$R_{\text{☾}}$	Mean radius of the moon
R_{ϕ}	Local radius of the earth
$r_{\oplus}$	Distance from the center of the earth
$r_{\text{☾}}$	Distance from the center of the moon
$\bar{r}_{\oplus\text{☾}}$	Mean distance between the center of the earth and center of the moon
r	Orbital radius (with respect to any reference body)
S	Angle between the earth-moon and earth-sun lines at earth
ST	Sidereal time
T	Number of Julian centuries (36,525 ephemeris days) from the epoch 1900 Jan 0.5 ET
T_{ω}	Time of perifocal passage
T_{Ω}	Time of ascending nodal crossing

*In this list of symbols coordinates in a given coordinate system have been listed together.
**All three symbols have been used to denote the mean anomaly of the sun in various parts of this chapter since all three are used frequently.

t	Time
t_0	Time of epoch, which is any time reference
U	Gravitational potential, work function, or negative of the gravitational potential energy. The symbol U is also used for gravitational field strength or potential per unit mass
UT	Universal time
V	Velocity (magnitude of)
W	Total energy
$\tilde{W}$	Function introduced by Hansen (Subsection C-1)
$x_b\ y_b\ z_b$	Body axis system
$x_e\ y_e\ z_e$	Inertial orbit plane coordinate system with origin at earth
$x_G\ y_G\ z_G$	Rectangular geographic coordinate system
$x_h\ y_h\ z_h$	Topocentric horizon system
$x_m\ y_m\ z_m$	Inertial orbit plane coordinate system with origin at moon
$x_0\ y_0\ z_0$	Inertial orbit plane coordinate system with origin at barycenter of the earth-moon system
$x'_0\ y'_0\ z'_0$	Inertial orbit plane coordinate system with origin at barycenter of the earth-moon-sun system
$x_P\ y_P\ z_P$	General fixed orbit plane system
$x_R\ y_R\ z_R$	Rotating orbit plane system--general rotating coordinate system
$x_{R☾}\ y_{R☾}\ z_{R☾}$	Rotating orbit plane system--coordinate system rotating with mean motion of the sun
$x_S,\ y_S,\ z_S$	Rectangular selenographic system
$x_\epsilon\ y_\epsilon\ z_\epsilon$	Geocentric ecliptic coordinates
$x_\Omega\ y_\Omega\ z_\Omega$	Fixed orbit plane system with x-axis directed to the ascending node
$x'_\oplus\ y'_\oplus\ z'_\oplus$	Geocentric equatorial system with $x'_\oplus$ directed toward true vernal equinox
$x_\omega\ y_\omega\ z_\omega$	Fixed orbit plane system with x-axis directed to perifocus
$x_\oplus\ y_\oplus\ z_\oplus$	Geocentric equatorial system with $x_\oplus$ directed toward mean vernal equinox of a specified epoch
$x_☾,\ y_☾,\ z_☾$	Selenocentric lunar equatorial system
$x_{☾,\Omega}\ y_{☾,\Omega}\ z_{☾,\Omega}$	Selenocentric lunar equatorial system with $x_{☾,\Omega}$ in the direction of the ascending node on ecliptic
y	Variable introduced in Hansen's intermediate orbit
z	Variable introduced in Hansen's intermediate orbit

<u>Greek symbols</u>

α	Constant of sine parallax of the moon
$\alpha,\ \delta$	Right ascension, declination
$\beta = \omega + f$	Angle between the mean ascending node and the instantaneous position of the vehicle
γ	Constant of inclination of the moon's orbit
Γ	Mean longitude of solar perigee in the orbit of the sun around the earth, i. e., angle in the ecliptic from the mean equinox of date to solar perigee
Γ'	Mean longitude of lunar perigee, i. e., angle in the ecliptic from the mean equinox of date to the mean ascending node of the lunar orbit and then to lunar perigee along the orbit
Δ	Angle along the mean equator of the moon from its ascending node on the true equator of the earth to its ascending node on the ecliptic of date
ϵ	Mean obliquity of the ecliptic
$\epsilon_0 = \tilde{\omega} + \ell_0$	Mean longitude at epoch t_0
θ	Orbital central angle of the moon after time t
$\theta' = \omega_☾ t$	Angle of rotation of the moon in in time t
Λ_G	Angle between the x_e and x_G axes at $t = 0$
Λ_M	Angle in the moon's equatorial plane from the ascending node to the lunar prime meridian
Λ_S	Angle between the $x_☾$ and x_S axes at $t = 0$
$\lambda,\ \phi'$	Longitude, geocentric latitude
$\lambda,\ \phi$	Longitude, geodetic latitude

$\lambda,\ \phi^*$	Longitude, astronomical latitude
$\lambda_\epsilon,\ \beta_\epsilon$	Celestial longitude, celestial latitude
$\lambda_☾,\ \phi_☾$	Selenographic longitude, latitude
$\lambda'_☾$	Selenographic colongitude
$\mu = GM_\oplus$	Gravitational constant of the earth
$\nu = \frac{M_☾}{M_\oplus + M_☾}$	Ratio of the mass of the moon to the total mass of the earth and moon
$\pi_☾$	Horizontal parallax of the moon
ρ	Change in I due to the physical librations of the moon
σ	Change in Ω due to the physical librations of the moon
Φ	Angle between the x_0 and x_R axes at time $t = 0$
ψ	Angle between the $x_☾,\ \Omega$ axis and $z_☾,\ \Omega x\ (\overrightarrow{MEL} \times z_☾,\ \Omega)$
Ω	Longitude of the mean ascending node (of the moon) measured from the mean equinox of date.
Ω'	Longitude of the mean ascending node (of the moon) measured from the true equinox
$\Omega_☾$	Right ascension of the ascending node of the mean lunar equator measured from true equinox of date along the true celestial (earth) equatorial plane
ω	Argument of perifocus, angle in the orbit plane between the ascending node and perigee
$\tilde{\omega} = \Omega + \omega$	Longitude of perifocus
ω_R	Angular velocity of the rotating orbit plane coordinate system
$\omega_\oplus$	Angular velocity of the earth about its axis
$\omega_☾$	Angular velocity of the moon about its axis
$\omega_{\oplus☾}$	Angular velocity of the earth-moon system about the barycenter
$☾ = \Omega' + \omega + \ell$	Mean orbital longitude of the moon (referred to true equinox)

Subscripts and Superscripts

$\odot$	Sun
$\oplus$	Earth
☾	Moon
Δ	Vehicle
0	Initial conditions (also barycentric inertial coordinates)
h, m, s	Hours, minutes of time, seconds of time
r, °, ', "	Revolutions, degrees, minutes of arc, seconds of arc
$\sim$	Hansen's intermediate orbit

SYMBOLS

Latin symbols

A_{ref}	Constant reference area characterizing the space vehicle
A_s	Area of the vehicle perpendicular to the earth-sun line
A	Area
A_a	Area of the space vehicle perpendicular to $\vec{V}_a$
A_e	Azimuth on earth
A_Δ	Vehicle surface area
$A_\odot$	Surface area of the sun
a, b, c	Principal axes of inertia of the moon
B	Ballistic coefficient $B = \frac{C_D A_a}{2M_\Delta}$
b_m	Lunar trajectory impact parameter, or the distance between the asymptote of the hyperbolic circumlunar orbit and the center of the moon
C_D	Drag coefficient
C_i	i-th value of the Jacobi constant (restricted three-body problem)
C_L	Lift coefficient
C_x, C_y, C_z	Aerodynamic coefficients in body axes x_b, y_b, z_b, respectively
c	Speed of light, c = 299,792.5 km/sec
$\vec{D}$	Atmospheric drag force
$\vec{D}_E$	Electromagnetic (drag) forces
$\vec{D}_M$	Meteoritic drag force
$\vec{D}_S$	Force due to solar radiation pressure
E	Energy per unit mass
E_P	Energy of a photon
$\overrightarrow{EML}$	Earth-moon line
$e_☾$	Eccentricity of the lunar orbit
$e_{m\Delta}$	Eccentricity of the space vehicle orbit around the moon
f	Dimensionless quantity defined by Eq (69)
$\vec{f}$	Acceleration (force per unit mass)
$\vec{f}_i$	Acceleration of the space vehicle due to oblateness of the i-th celestial body
G	Universal gravitational constant
$\vec{g}$	Gravitational acceleration
h	Planck's constant, $h = 6.625 \times 10^{-34}$ joule/sec
$\vec{h}$	Angular momentum per unit mass
h_e	Altitude above the earth
h_{PE}	Return vacuum perigee altitude
h_{PL}	Pericynthion altitude
$h_\oplus$	Altitude above earth's equator
I_a, I_b, I_c	Lunar moments of inertia about the three principal axes of inertia a, b, c
i_m	Inclination of the trajectory to the moon's orbital plane
i_{em}	Inclination of the lunar orbit to the earth's equatorial plane
i_{VE}	Inclination of the vehicle with respect to the earth's equatorial plane $0 \leq i_{VE} \leq 180°$
i_{VTE}	Inclination of the transearth trajectory to MOP
i_{VTL}	Inclination of the translunar plane to the MOP at the time of injection
i_{VTEQ}	Inclination of the transearth trajectory to the earth's equator
J_n	Expansion coefficients in the expansion of the earth's gravitational potential (empirical constants determined by measurements)
k	Boltzmann constant $k = 1.380 \times 10^{-23}$ joule/°K
k_v	Constant of proportionality relating the gravisphere to the volume of influence
L	Aerodynamic lift force
LOR	Lunar orbit rendezvous mission (a landing technique)
LU	Lunar unit ($\equiv \bar{r}_{\oplus ☾}$)
l, m, n*	Direction cosines
M	Mass
M_M	Total mass of meteorites striking the space vehicle from a given direction
MOP	Moon's orbital plane (x_E y_E plane)

*In this list of symbols the coordinates in a given coordinate system have been listed together.

m	Equivalent mass of a photon, also mass of molecule
N	Number of photon collisions per unit time per unit area
N_0	Number of molecules per unit volume
$\vec{n}_\Delta$	Nongravitational forces per unit mass acting on the space vehicle
$\vec{P}$	Momentum of a photon
$P_n(\ )$	Legendre polynomial of n-th order in ( )
p_s	Solar radiation pressure (force per unit area)
p_{s0}	Solar radiation pressure constant defined by Eq (81)
q	Dimensionless quantity defined by Eq (66)
q_e	Charge of an electron, $q_e = 1.602 \times 10^{-19}$ coulomb
q_s	Factor indicating the type of collision of the photon with the vehicle
R_e	Equatorial radius of the earth
R_{rel}	Relativistic corrections to the equations of motion
r, θ	Plane polar coordinates
$\vec{r}$	Radius vector
r_g	Radius of the lunar gravisphere
r_v	Radius of the lunar volume of influence
$\bar{r}_{\oplus☾}$	Lunar unit or earth-moon distance used in the restricted three-body problem, $\bar{r}_{\oplus☾} = 384747.2$ km
S, S'	Inertial coordinate systems, coordinate systems in general
T	Absolute temperature (°K)
$\vec{T}$	Rocket thrust force
$T(\ )$	Rotation matrix with angle of rotation ( )
t	Time, also total transit time to the moon
t_d	Time of departure from lunar orbit
t_e	Time of entry into lunar orbit
t_i	Impact time
t_p	Time of pericynthion (closest approach to moon)
U	Gravitational potential
U_i	Expansion terms in the earth's gravitational potential
$U_{☾i}$	Expansion terms in the lunar gravitational potential
$\vec{V}$	Velocity
$\vec{V}_a$	Vehicle velocity relative to the atmosphere
$\vec{V}_{\oplus a}$	Velocity of the atmosphere in the geocentric equatorial coordinate system
$V_{\oplus M}$	Average speed of the meteorites hitting the vehicle
$\bar{V}_{e\ell}$	Average thermal speed of an electron
$(V_{e\Delta})_{max}$	Maximum restricted three-body escape speed from earth
$(V_{e\Delta})_{min}$	Minimum restricted three-body escape speed from earth
V_{ep}	Two-body escape speed from earth or parabolic velocity of the earth
V_p	Space vehicle speed relative to the plasma
Voice	Volume of influence calculated envelopes trajectory computation technique
$W_\odot$	Total radiated power by the sun
W_Δ	Radiated power per unit area arriving at the space vehicle
$x_b\ y_b\ z_b$	Body axis coordinate system with origin at the vehicle center of gravity
$x_E\ y_E\ z_E$	Trajectory coordinates with origin at the center of the earth and the x_E-axis along the intersection of the translunar trajectory plane with the MOP and in the general direction of the moon (see Fig. 4 of Chapter IV)
$x_e\ y_e\ z_e$	Trajectory coordinates with origin at the center of the earth and axes parallel to the inertial barycentric coordinate system
$x_M\ y_M\ z_M$	Selenocentric trajectory coordinate system with the y_M-axis defined by the intersection of the $x_M\ y_M$ with the $x_E\ y_E$ planes
$x_m\ y_m\ z_m$	Trajectory coordinates with origin at the center of the moon and axes parallel to the inertial barycentric coordinates
$x_0\ y_0\ z_0$	Inertial coordinate system with origin at the earth-moon system barycenter
$x_R\ y_R\ z_R$	Rotating coordinates with origin at the barycenter and the x_R-axis along the earth-moon line

$x_S\ y_S\ z_S$	Selenographic coordinate system
$x_v\ y_v\ z_v$	Vehicle-centered coordinate system with the z_v-axis in the direction of the radius vector, x_v-axis in the general direction of motion
$x_w\ y_w\ z_w$	Wind axes with origin at the center of gravity (or, alternately, the center of pressure) of the vehicle, with the x_w-axis in the direction of $\vec{V}_a$
$x_\oplus\ y_\oplus\ z_\oplus$	Geocentric equatorial coordinate system with $x_\oplus$ axis directed toward the mean equinox
$x_{\oplus 0}\ y_{\oplus 0}\ z_{\oplus 0}$	Inertial equatorial coordinate system with origin at the barycenter of the physical model chosen and $x_{\oplus 0}$ axis directed toward the mean equinox
$x_{\oplus u}\ y_{\oplus u}\ z_{\oplus u}$	Geocentric equatorial coordinates of the space vehicle in a Keplerian (unperturbed) orbit

<u>Greek symbols</u>

α	Angle of attack
$\alpha,\ \delta$	Right ascension, declination
α_e	The angle between the x_e-axis and the x_R-axis
β	Relativistic correction factor (see table on p IV-37), $\beta = 1/\left(1 - \frac{V^2}{c^2}\right)^{1/2}$
β	Yaw angle
$\beta_☾$	Angle between the y axis and $\vec{r}_{☾\Delta}$ (see sketch on p IV-24)
β_{M0}	Orbital central angle of the ejection point from lunar orbit
γ_e	Inertial flight path angle with respect to earth, or angle between $\vec{r}_{e\Delta}$ and $\vec{V}_{e\Delta}$ minus 90°
γ_M	Inertial flight path angle with respect to the moon
Δi_{VTL}	Change in i_{VTL} due to lunar gravitational attraction or drift characteristic of the trajectory
Δr_g	Distance from the center of the moon to the center of the lunar gravisphere
Δr_v	Distance from the center of the moon to the center of the spherical lunar volume of influence
ΔV	Characteristic velocity of the thrust maneuver
$\Delta V_{e\Delta 0}$	Correction to the injection velocity
ΔV_g	Change in velocity due to the gravitational attraction of the moon (see sketch on p IV-12)
$\Delta\lambda_{0☾}$	Longitude difference between the injection point and the moon at impact or when $r_{e\Delta} = \bar{r}_{\oplus☾}$
ϵ	Emissivity of the sun
η_e	Angle between x_R and $\vec{r}_{e\Delta 0}$
η_m	Angle between the moon-earth line and $r_{☾\Delta}$
$\theta,\ \phi,\ \psi$	Rotation angles from body axes to vehicle-centered equatorial coordinate axes
$\theta_{0☾}$	Total in-plane angle from injection to the lunar orbit
θ_M	Angle between $(-\overrightarrow{EML})$ and the y_M axis
μ	GM, gravitational constant of body with mass M
ν	The reduced mass of the earth moon system; i.e., the ratio of the lunar mass to the sun of the lunar mass and earth mass $\nu = \frac{M_☾}{M_\oplus + M_☾}$
ν_p	Frequency of the radiation
ξ	Bank angle
$\xi_1,\ \xi_2$	Auxiliary angles defined on the sketch on page IV-18 and Eq (41)
$\xi,\ \eta,\ \zeta$	Differences between perturbed and unperturbed vehicle coordinates, i.e., $\xi_{\oplus\Delta} = x_{\oplus\Delta} - x_{\oplus u}$, etc.
ρ	Atmospheric density
ρ_M	Average density of meteoritic material in space
σ	Stefan-Boltzmann constant, $\sigma = 5.67 \times 10^{-8} \frac{\text{watts}}{m^2\ (°K)^4}$
$\tau_{\oplus☾}$	Period of rotation of the earth-moon system
Φ	The initial angle between the inertial and rotating coordinate systems, i.e., the x_0 and x_R coordinate axes, also planar angle from injection to the position of the moon at injection
Φ_Δ	Electrostatic potential of the vehicle

Φ^*	Lead angle of the moon (angle from the x_E-axis to the position of the moon at injection)
ϕ'	Geocentric latitude
$\phi_☾$	Selenocentric latitude (see sketch on p IV-24)
ψ_0	Angle measured from the intersection of the translunar plane with the moon's orbital plane (negative x_E axis) to the injection point
Ω	Right ascension of the ascending node
$\omega_{\oplus ☾}$	The rotational rate of the earth-moon line around the barycenter, $\omega_{\oplus ☾} = 2.661669484 \times 10^{-6}$ rad/sec
$\omega_\oplus$	Rotational rate of the earth about its axis $\omega_\oplus = 7.292115 \times 10^{-5}$ rad/sec

Subscripts and superscripts

$\odot$	Sun
$\oplus$	Earth
☾	Moon
$\triangle$	Vehicle
0	Injection conditions
u	Unperturbed (Keplerian) conditions
M	Meteoritic, pertaining to meteorites

CHAPTER V

SYMBOLS*

Latin Symbols

A	Inertial launch azimuth (great circle course on a nonrotating earth) measured clockwise from geographic north
h	Altitude
h_{BO}	Booster burnout altitude (launch phase)
I_{sp}	Specific impulse
i_{em}	Inclination of the MOP (moon's orbital plane) to the earth's equator
i_{VE}	Inclination of the translunar trajectory plane (or parking orbit plane, which is assumed the same) to the earth's equator
i_{VTE}	Inclination of the transearth trajectory plane to the MOP
i_{VTL}	Inclination of the translunar trajectory plane to the MOP
L/D	Lift-to-drag ratio
MOP	Lunar orbital plane
n	Number of satellite revolutions in parking orbit to achieve the correct phase relationship to rendezvous
r	Radius
T	Thrust (earth sea level force)
t	Time
t_a	Time of ascent to parking orbit
t_{aN}	Nominal ascent time
t_N	Nominal time of arrival at apogee
t_p	Time of pericynthion (closest approach to the moon)
t_{tr}	Transfer time from the intermediate orbit to the parking orbit
V	Velocity relative to an earth
V_{req}	Required velocity
W_0	Weight of the spacecraft at the start of rocket burning (earth sea level weight)
W_f	Weight of the propellant (earth sea level weight)
$\vec{x}_E$	Vector along the intersection between the translunar trajectory plane and the MOP in the general direction opposite to the lunar position

*Since the spacecraft and earth are the only celestial bodies considered and all parameters are referred to earth, the subscripts "$\oplus$" and "Δ" have been deleted for conciseness; i.e., $\vec{V}_{\oplus\Delta 0}$, the injection velocity of the vehicle relative to earth becomes V_0.

Greek Symbols

α	Inclination of the great circle course between the ascending node of the parking orbit and the launch site
β	Orbital central angle to be "gained" or "lost" by the spacecraft in the parking orbit
β_{req}	Required central angle of the "phantom" satellite to effect rendezvous
β_T	Orbital central angle of the rendezvous point
β_{TL}	Angle between the ascending node of the MOP to the intersection of the translunar trajectory with the MOP
γ	Local flight path angle measured from the local horizontal to the velocity vector
Δ	Increment (in general)
ΔA	Change in azimuth, which is the turn angle of the spacecraft's velocity vector
Δt_L	Launch time tolerance
ΔV	Characteristic velocity of the maneuver
ΔV_{cap}	Characteristic velocity of the total maneuvering capability of the spacecraft
$\Delta \beta'$	Central angle between the spacecraft and the phantom satellite
$\Delta \beta'_{req}$	Required central angle (or phase angle) for a Hohmann transfer to a parking orbit for rendezvous
$\Delta \lambda$	Longitude increment through which the launch site moves due to the earth's rotation
$\Delta \Phi$	Required shift in the x_E-direction due to a missed injection
$\Delta \Phi'$	Range extension increment
$\Delta \psi_0$	Increment in injection position angle (measured from the $-x_E$-axis) traversed during the injection phase

δ Central angle along the great circle course between the ascending node of the parking orbit and the launch site

ϵ Thrust angle, measured from the velocity vector to the thrust vector

ζ Mass ratio, $\zeta = \frac{W_f}{W_0}$

λ Longitude

λ_{NL} Inertial longitude of the launch site measured from the ascending node of the parking orbit

τ Orbital period

Φ Range angle (general)

Φ_a Ascent range angle (from launch to injection into the parking orbit)

Φ^* Lunar lead angle (measured from the x_E-axis to the lunar position at injection

ψ_0 Position angle at the completion of injection, measured from the $-x_E$-axis to the injection point, $\psi_0 = \psi_0' + \Delta\psi_0$

ψ_0' Position angle at the start of injection, measured from the $-x_E$-axis to the injection initiation point

ω Rotational rate of a satellite in circular orbit

Subscripts and Superscripts

A Abort conditions

a Ascent trajectory (from launch to injection into parking orbit)

i Intermediate or waiting orbit

j Conditions at a point during the injection burning phase

L Launch

N Nominal conditions

0 Injection conditions

P Parking orbit

R Re-entry conditions

CHAPTER VI

SYMBOLS

Latin Symbols

A	Angle at the space vehicle between the radius to earth to the radius to the moon
A_e	Inertial launch azimuth on earth
A^*	Angle between the edge of the lunar disc and a star
a	Semimajor axis
c	Velocity of light in vacuo, $c = 299,792.5$ km/sec
D	Apparent diameter of the moon
$\overrightarrow{EML}$	Earth-moon line
ER	Earth radius, unit of length with 1 ER = 6378.163 km
e	Eccentricity
h	Altitude
h_{PE}	Return vacuum perigee altitude
h_{PL}	Pericynthion altitude (altitude above the lunar surface at the point of closest approach to the moon)
i_m	Inclination of the vehicle orbit around the moon to the MOP
i_{VE}	Inclination of the translunar trajectory and earth parking orbit to the earth's equatorial plane
i_{VTE}	Inclination of the transearth trajectory plane to the MOP
i_{VTEQ}	Inclination of the transearth trajectory plane to the earth's equatorial plane
i_{VTL}	Inclination of the translunar trajectory plane to the moon's orbital plane
$\overrightarrow{MEL}$	Moon-earth line
MOP	Moon's orbital plane
$\hat{m}$	Unit vector from the space vehicle toward the earth
n	Number of the abort way-station, which is equivalent to the number of sidereal days or complete earth rotations
$\hat{n}$	Unit vector from the space vehicle toward a star
$\hat{n}'$	Unit vector from the space vehicle toward the moon
R	Radius in units of ER
$R_{\oplus}$	Radius of the equivalent spherical earth, $R_{\oplus} = 6371.02$ km
$R_{☾}$	Radius of the equivalent spherical moon, $R_{☾} = 1738.16$ km
$R_{\oplus ☾}$	Earth-moon distance in units of ER
$\vec{r}$	Radius vector (from the center of the earth)
$\hat{s}_1, \hat{s}_2$	Unit vectors from the space vehicle in the direction of stars 1 and 2, respectively
T	Total flight time in a circumlunar trajectory, measured from injection to return vacuum perigee
t	Time
t_p	Flight time to pericynthion
t_R	Time from the decision to abort the mission to re-entry
V	Velocity
V_p	Parabolic velocity (two-body earth escape speed)
$x_E\ y_E\ z_E$ †	Geocentric nonrotating Voice coordinate system
$x_P\ y_P\ z_P$	Orbit plane coordinate system illustrated in the sketch on p VI-12
$x_R\ y_R\ z_R$	Rotating rectangular coordinate system of the restricted three-body problem discussed in Subsection B-2 of Chapter III
$x_v\ y_v\ z_v$	Vehicle-centered coordinate system, with the z_v-axis in the direction of the local vertical, the x_v-axis along the local horizontal in the flight direction; the y_v-axis completes the right-handed Catesian system by definining a lateral direction.

Greek Symbols

β_M	Orbital central angle of the pericynthion point, measured positively toward the north from the descending node of the orbit relative to the MOP

†In this list of symbols the coordinates of a particular coordinate system have been listed together.

γ	Apparent angular diameter of the moon
γ_0	Flight path angle at translunar injection (angle between the local horizontal and the velocity vector)
ΔA	Change in azimuth during the abort maneuver
ΔP	Angular error in the line of position, which is the radius vector from the prime body to the space vehicle
Δt	Time increments for timing abort defined on p VI-26
ΔV	Equivalent velocity impulse to enter a circular orbit around the moon from the circumlunar trajectory
ΔV_A	Equivalent velocity impulse of the abort maneuver
ΔV_p	Midcourse guidance correction required to correct the position of the pericynthion point
ΔV_{TOT}	Total midcourse guidance correction
ΔV_v	Midcourse guidance correction required to reorient the velocity vector at pericynthion
$\Delta \Phi_R$	Range angle of the required longitudinal maneuverability from reentry
ϵ	Angle between the vehicle velocity vector before and after the abort maneuver
θ_M	Angle between the $\overrightarrow{EML}$ and the descending node of the lunar orbit with respect to the MOP
$\lambda_☾, \phi_☾$	Selenographic longitude, latitude
$\hat{\rho}$	Unit vector perpendicular to $\hat{n}$
σ_1, σ_2	Angle between $\vec{r}_{\oplus\Delta}$ and stars 1 and 2, respectively
σ_s	Angle between $\hat{s}_1$ and $\hat{s}_2$
Φ^*	Moon lead angle (angle from the lunar position to the x_E-axis measured in the MOP at the time of translunar injection)
Φ_A, Φ_R, Φ_0	Range angles for timing abort, defined on p VI-27
ψ_0	Angle between the $-x_E$-axis and the injection point measured in the translunar trajectory plane

Subscripts and Superscripts

A	Abort trajectory and maneuver conditions
0	Injection conditions
T	Translunar trajectory conditions prior to the abort maneuver
TE	Transearth trajectory conditions
TL	Translunar trajectory conditions
'	Denotes desired conditions (i.e., other than catalogued conditions)
$\oplus$ [††]	Earth
☾ [††]	Moon
Δ [††]	Space vehicle

[††] These subscripts have been omitted when reference to the respective body is obvious

SYMBOLS*

Latin Symbols

a	Semimajor axis
$C_{2,2}$	Second-order, second degree harmonic coefficient in the expansion of the gravitation potential U in terms of spherical harmonics
C_2	A particular value of the Jacobi constant (see also Chapter III)
$\vec{D}$	Atmospheric drag force
ER	Earth radii (unit of length, 1 ER = 6378.163 km)
e	Eccentricity
$\vec{F}$	Gravitational force
f	True anomaly
G	Universal gravitational constant
$g_{\oplus 0}$	Average acceleration due to gravity at earth sea level (conversion factor from absolute to gravitational units)
$g_{☾}$	Acceleration due to moon's gravity
h	Altitude
h_{PL}	Pericynthion altitude
I_{sp}	Specific impulse
i**	Orbital inclination (general)
i_m	Inclination of lunar orbit with respect to the MOP
i_{VTE}	Inclination of the transearth trajectory to the MOP
i_{VTL}	Inclination of the translunar trajectory to the MOP
J_2	Second-order zonal harmonic coefficient in the expansion of the gravitational potential U in terms of spherical harmonics
LR	Lunar radii (unit of length, 1 LR = 1738.16 km)
ℓ	Mean anomaly
M	Mass
M_0	Initial space vehicle mass
M_e	Final space vehicle mass
MOP	Moon's orbital plane
m	$m = \frac{2\pi/\tau_0}{\omega_{☾}}$, ratio of the angular velocity of the lunar satellite in its orbit to the rotational rate of the moon about its axis
p	Semi-latus rectum
$R_{\oplus ☾}$	Earth-moon distance in units of ER
$R_{☾}$	Radius or the equivalent spherical moon, $R_{☾}$ = 1738.16 km
R	Radius of moon in direction of earth
$\vec{r}$	Radius vector
r_0	Average orbital radius for unperturbed orbits, equivalent to the semi-latus rectum of the unperturbed orbit
$\bar{r}_0$	Average orbital radius if oblateness effects are taken into account
r_{PL}	Pericynthion radius (the pericynthion is the point of closest approach to the moon)
r_S	Distance from the origin in the selenographic coordinate system
$r_{☾}$	Distance from the origin in the selenocentric coordinate system
T	Thrust
t	Time
t_b	Total rocket burning time
U	Gravitational potential
$V_{☾c}$	Circular orbit velocity for an orbit around the moon
W_0	Initial space vehicle weight
$\dot{W}_f$	Propellant weight flow rate
x_S, y_S, z_S***	Coordinates in selenographic coordinate system
$x_{☾}, y_{☾}, z_{☾}$***	Coordinates in selenocentric coordinate system

*The moon symbol "☾" has been deleted in many cases where the reference to the moon of that particular quantity is obvious.

**In this chapter i refers primarily to the orbital inclination with respect to the lunar equator.

***In this list of symbols the coordinates in a given coordinate system have been listed together.

Greek Symbols

β — Orbital central angle measured along the orbit from the ascending node, $0° \leq \beta < 360°$

γ — Flight path angle with respect to the lunar local horizontal

$\Delta r_{☾}$ — $\Delta r_{☾} = r_{☾} - r_{☾0}$, difference in radius between a perturbed and a Keplerian orbit

ΔV — Equivalent velocity increment of the orbit entry or departure maneuver

$\Delta\lambda_{N\Delta}$ — Longitude of the subsatellite point relative to the ascending node

$\Delta\lambda_{☾}$ — Longitudinal shift between two successive satellite ground tracks

$\Delta\tau_{\beta}$ — $\Delta\tau_{\beta} = \tau_{\beta} - \tau_0$, difference between the nodal and Keplerian periods

δ_S — Declination of the subsatellite point

θ_{MTL} — Angle between the earth-moon line and the line toward the descending node of the lunar orbit

Λ_S — Angle between the $x_{☾}$ and x_S axes at time $t = 0$

λ_{AN} — Selenographic longitude of the ascending node

$\lambda_{☾}$, ϕ — Selenographic longitude, latitude

μ — $\mu = GM$, gravitational constant of the celestial body with mass M

$\rho_{☾SL}$ — Atmospheric density at the surface of the moon

$\rho_{\oplus SL}$ — Earth sea level atmospheric density

σ_c — Predicted instrument reading

σ_R — Instrument reading

τ — Orbital period

τ_0 — Keplerian orbital period

τ_r — Anomalistic period for a perturbed orbit

τ_{β} — Nodal period for a perturbed orbit

$\Phi_{☾}$ — $\Phi_{☾} = U_{☾} - \frac{\mu_{☾}}{r_{☾}}$, potential of the perturbating gravitational force; perturbation potential; perturbing function

Ω — Longitude of the mean ascending node

Ω' — Longitude of the instantaneous ascending node

ω — Argument of perifocus (angle in the orbital plane from the ascending node to the perifocus)

$\omega_{☾}$ — Rotational rate of the moon about its axis $\omega_{☾} = 2.661699484 \times 10^{-6}$ rad/sec

Subscripts and Superscripts

0 — Denotes value for unperturbed (Keplerian) orbit

$\odot$ — Sun

$\oplus$ — Earth

☾ — Moon

Δ — Space vehicle

$-$ — Average value of a quantity if oblateness effects are taken into account

CHAPTER VIII

SYMBOLS*

Latin Symbols

a	Acceleration
f_0	Thrust acceleration or thrust-to-mass ratio $f_0 = \frac{T}{M_0}$
F_τ, F_σ	Tangential forces, normal forces in a two-dimensional trajectory
$g_{\oplus 0}$	Acceleration due to earth gravity at sea level
$g_{☾}$	Acceleration due to lunar gravity
$\bar{g}_{☾}$	Average value of $g_{☾}$
$g_{☾ 0}$	Acceleration due to lunar gravity at the surface of the moon
H	Hamiltonian function
h	Altitude above the lunar surface
h_A	Abort altitude
h_{AL}	Apocynthion altitude (farthest departure from the moon)
h_{PL}	Pericynthion altitude (closest approach to the moon)
h_{BO}	Burnout altitude (on a lunar ascent trajectory)
I_{sp}	Specific impulse (earth sea level value)
LM	Landing module (entire translunar space vehicle lands on the lunar surface, sometimes also applied to the shuttle)
M	Instantaneous total mass of the vehicle
M_f	Fuel mass
M_0	Initial total mass of the vehicle
M_p	Total propulsion system mass (less fuel mass)
$\dot{M}$	Mass flow rate, or rate of fuel expenditure by rocket engine
MOM	Lunar orbital module
MOP	Moon's orbital plane
p	Maximum resistive pressure at impact
P	Maximum penetration depth of a hard impact spike
$\vec{r}$	Radius vector
$R_{☾}$	Lunar radius
t	Time
t_b	Burning time
t_t	Tilt time
$\vec{T}$	Thrust vector (earth sea level force)
$\vec{u}$	Control vector with control variables as components
V	Velocity in a nonrotating coordinate system
V_{ex}	Rocket exhaust velocity
V_i	Impact velocity
W	Vehicle weight (earth sea level weight)
W_0	Vehicle weight at initiation of burning (earth sea level weight)
W_f	Weight of propellant expended (earth sea level weight)
x_ℓ , y_ℓ **	Trajectory coordinate system with origin at the subsatellite point on the lunar surface
x, z	Trajectory coordinate system defined in Subsection B-2e
$x_t y_t$	Inertial two-dimensional trajectory coordinate system with origin at the center of the moon with x_t in the lunar equatorial plane and y_t toward the northernmost point of the descent plane
$x_{\ell t}$	Total horizontal distance of translation
x_t, y_t, z_t	Inertial three-dimensional trajectory coordinate system with unspecified rotation
$\vec{X}$	State vector describing the behavior of the system

*In most cases the subscript "☾" has been deleted since the symbols are referred to the moon, i.e., $\gamma_{☾}$, the flight path angle with respect to the local horizontal at the moon is denoted by γ, $h_{☾ BO}$, the burnout altitude on the moon is denoted by h_{BO}, etc.

***In this list of symbols, coordinates in a given coordinate system have been listed together.

Greek Symbols

α	Orbital central angle describing a safe abort window
β	Orbital central angle
γ	Flight path angle with respect to local horizontal
γ_t	Flight path angle with respect to inertial axes
ΔV	Characteristic velocity of the maneuver $\Delta V = -V_{ex} \ell n\,(1-\zeta)$
ΔV_A	Characteristic velocity of the abort maneuver
ΔV_c	Characteristic velocity of the circularizing maneuver in the abort trajectory
ζ	Mass ratio $\zeta = \frac{W_f}{W_o}$
Θ	Orbit departure angle (angle from deorbit to pericynthion of the descent trajectory)
θ	Thrust vector angle with respect to local horizontal (also referred to as pitch angle)
$\bar{\theta}$	Thrust vector angle with respect to the inertial (- x_t)-axis, $\bar{\theta} = 180° - \theta_t$
θ_t	Thrust vector angle with respect to inertial x_t - axis (also referred to as inertial pitch angle)
$\mu_☾$	$\mu_☾ = GM_☾$ the gravitational constant of the moon
ξ	Separation angle (orbital central angle between the LM and the MOM)
τ	Orbital period (Keplerian orbit), also used to denote final time
Φ	Range angle in the trajectory plane
Φ_b	Range angle from the end of ascent burning to orbit injection
Φ_0	Range angle from lunar lift-off to lunar orbit injection
Φ_d	Range angle from lunar deorbit to initiation of the braking maneuver
ϕ_t	Roll angle
ψ_t	Yaw angle
ω	Trajectory constant $\omega = \left[\mu_☾ / R_☾^3\right]^{1/2}$
ω_{vc}	Command pitch rate
ω_{wc}	Command yaw rate

Subscripts and Superscripts

A	Abort
c	Command
f	End conditions (sometimes used for denoting propellant)
j	j^{-th} rocket burning phase during descent
nom	Nominal conditions
t	Total, or referring to the inertial $x_t y_t z_t$ coordinate system
0	Initial conditions
☾	Moon (when the reference to the moon is obvious this symbol will be deleted)
$\triangle$	Vehicle (when the reference to the vehicle is obvious this symbol will be deleted)

CHAPTER IX

SYMBOLS

Latin Symbols

$\overrightarrow{EML}$	Earth-moon line
ER	Earth radii (unit of length, 1 ER = 6378.163 km)
h_0*	Translunar injection altitude (relative to earth)
h_{PE}	Vacuum perigee altitude of the transearth trajectory
h_{PL}	Pericynthion altitude, also transearth injection altitude
i_m	Inclination of the vehicle orbit to the MOP
i_{VTE}	Inclination of the transearth trajectory plane to the MOP, $-180° \leq i_{VTE} \leq 180°$
i_{VTL}	Inclination of the translunar trajectory plane to the MOP $-180° \leq i_{VTL} \leq 180°$
$\overrightarrow{MEL}$	Moon-earth line
MOP	Lunar orbital plane
$R_{☾}$	Radius of the moon (the radius of the equivalent spherical moon)
$R_{\oplus ☾}$	Earth-moon distance measured in units of earth radii (ER)
$r_{☾\Delta}$	Orbital radius of the circular lunar orbit
t	Time
t_p	Flight time from transearth injection near the moon to vacuum perigee
V_0	Translunar injection velocity (relative to a nonrotating geocentric coordinate system)
$V_{☾0}$	Injection velocity into the transearth trajectory
$V_{☾c}$	Velocity of the vehicle in the circular lunar orbit
$V_{☾\Delta}$	Vehicle selenocentric velocity (nonrotating coordinate system)
$V_{\oplus\Delta}$	Vehicle geocentric velocity
$V_{\oplus ☾}$	Geocentric velocity of the moon
x_v, y_v, z_v**	Vehicle-centered coordinate system defined in Subsection B-2
$x_{☾}, y_{☾}, z_{☾}$	Selenocentric (nonrotating) coordinate system

Greek Symbols

β_M	Orbital central angle of the vehicle in lunar orbit $-180° \leq \beta_M \leq 180°$, measured positively toward north from the descending node
$\gamma_{☾}$	Flight path angle with respect to the lunar local horizontal
γ_0	Translunar injection flight path angle with respect to the local horizontal on earth
ΔV	Equivalent velocity impulse of the transearth injection maneuver
ΔV_p	Midcourse guidance velocity correction for future position changes
ΔV_{TOT}	Total midcourse guidance velocity correction
ΔV_v	Midcourse guidance velocity correction
θ_M	Angle in the MOP between the $\overrightarrow{EML}$ and the line toward the descending node $-180° \leq \theta_M \leq +180°$ measured positively toward the east. This angle is similar to the orbital element "longitude of the ascending node"
ψ_0	Translunar injection position measured in the translunar trajectory plane from its intersection with the MOP to the injection position

Subscripts and Superscripts

0	Injection conditions
TE	Transearth
TL	Translunar

*The earth symbol has been omitted from the translunar injection conditions.

**In this list of symbols coordinates in a given coordinate system have been listed together.

CHAPTER X

SYMBOLS

Latin Symbols

A	Aerodynamic reference area of the vehicle
A_H	Aerodynamic heating reference area
B	Ballistic coefficient, $B = \frac{C_D A}{2M}$
B_{eff}	Effective ballistic coefficient (i.e., if the oblateness and dispersions of the atmosphere is absorbed in the ballistic coefficient
C_D	Drag coefficient
C_L	Lift coefficient
C_R	Resultant aerodynamic force coefficient
D	Drag
F_{PE}	Perigee parameter defined by Eq (4b)
g	Local acceleration due to gravity
g_0	Acceleration due to gravity at sea level (conversion factor from mass to weight in a gravitational system)
h	Altitude
h_{max}	Apogee altitude of the skip trajectory
h_{PE}	Vacuum perigee altitude
h_{PL}	Pericynthion altitude (closest approach to the moon)
I_{sp}	Specific impulse of fuel
i_m	Inclination of the vehicle orbit to the MOP
i_{VE}	Inclination of the vehicle orbit to the earth's equator
i_{VTE}	Inclination of the transearth trajectory plane to the MOP
i_{VTL}	Inclination of the translunar trajectory plane to the MOP
J_2	Second-order zonal harmonic coefficient in the expansion of the earth's gravitational potential in terms of zonal harmonics
k	Number of satellite revolutions during which satellite period dispersions occur
L	Lift
L/D	Lift-to-drag ratio
M	Mass
$\overline{M}$	Molecular weight of the atmosphere
MOP	Moon's orbital plane
m	Number of days
N	Number of satellite revolutions with N = 1.0 at the first ascending node
n	Number of satellite revolutions
p	Integral number of satellite revolutions
Q_H	Total heat transfer to the vehicle
q	Dynamic pressure, $q = \frac{1}{2} \rho_\infty \ V^2$
q_H	Heating rate of the vehicle
$\vec{R}$	Resultant aerodynamic force $\vec{R} = \vec{L} + \vec{D}$
R^*	Gas constant, $R^* = 8.31439 \times 10^3 \ \frac{\text{joules}}{°K - kg}$
$R_\oplus$	Radius of the equivalent spherical earth
R_e	Equatorial radius of the earth
R_{lat}	Lateral range (measured perpendicular to the re-entry trajectory)
R_{long}	Longitudinal range
r	Radius
T	Total flight time for a circumlunar trajectory (time from translunar injection to earth-return vacuum perigee); also, temperature measured in degrees Kelvin
t	Time
t_d	Descent time (time from deorbit to landing)
t_R	Total time to re-enter (time from re-entry to landing)
u, v	Horizontal and vertical velocity components (see sketch on page X-3)
$\overline{u}$	Independent variable introduced by Chapman which has the physical meaning of a dimensionless horizontal velocity parameter which gives horizontal velocity as a fraction of circular satellite velocity, $\overline{u} = \frac{u}{\sqrt{gr}} = \frac{u}{V_c}$

V	Velocity
V_c	Circular velocity
V_p	Parabolic velocity
$(\overline{V}_R)_i$	Re-entry velocity parameter $(\overline{V}_R)_i = \left(\frac{V_R}{\sqrt{gr}}\right)_i = \left(\frac{V_R}{V_c}\right)_i$
W	Weight
Z	Dependent variable introduced by Chapman and defined by Eq (3)

Greek Symbols

β	Logarithmic density gradient of the atmosphere $\beta = -\frac{d}{dr}(\ell n \rho_\infty)$
β_e	Orbital central angle during ballistic flight
γ	Flight path angle (angle between the local horizontal and the velocity vector)
ΔA	Change in azimuth
ΔV	Equivalent velocity impulse of the maneuver
ΔV_d	Equivalent velocity impulse of the deorbit maneuver
$\Delta \tau_d$	Dispersions in the orbital period
ζ	Mass ratio $\zeta = \frac{M_f}{M_0}$
θ_M	Angle between the earth-moon line and the descending node of the lunar orbit on the MOP
λ, ϕ'	Longitude, geocentric latitude
μ	Gravitational constant of the earth
ξ	Bank angle
ρ, ρ_∞	Atmospheric density; also referred to as free-stream density
τ_β	Nodal period (time from one ascending nodal crossing to the next)
Φ_c	Central angle from re-entry to landing
$\dot{\Omega}$	Regression rate of the ascending node
$\omega_\oplus$	Rotational rate of the earth around its axis

Subscripts and Superscripts

b	Ballistic flight conditions
e	Atmospheric exit conditions for a skip re-entry
f	Landing site conditions
PE	Vacuum perigee conditions
R	Re-entry conditions
0	Initial conditions in the parking orbit on earth return
$\oplus$ [†]	Earth

[†] The earth symbol has been deleted in the cases when the reference to earth is obvious.

CHAPTER XI

SYMBOLS

Latin Symbols

A — Inertial azimuth on earth, $0° \leq A < 360°$, measured from geographic north

C_0, D_0, E_0 — Constants appearing in empirical Eqs (21) and (22)

$\overrightarrow{EML}$ — Earth-moon line

EOP — Earth's orbital plane

ER — Earth radii (a unit of length, 1 ER = 6378.163 km)

$\overrightarrow{ESL}$ — Earth-sun line

$\overrightarrow{GW}$ — Vector from the center of the earth to the intersection of the Greenwich meridian with the earth's equator (or the MOP or the ecliptic)

h — Altitude

h_{PE} — Vacuum perigee altitude

h_{PL} — Pericynthion altitude (pericynthion = point of closest approach to the moon)

i — Phase angle (see Section B)

i_{em} — Inclination of the MOP relative to the earth's equatorial plane

i_{MOP} — Inclination of the true lunar orbital plane (MOP) with respect to the true lunar equator

i_m — Inclination of the vehicle in lunar orbit to the MOP

$(i_T)_{MOP}$ — Inclination of the morning terminator relative to the MOP

i_T — Inclination of the morning terminator relative to the lunar equator

i_{VE} — Inclination of the vehicle relative to the earth's equator

i_{VM} — Inclination of the vehicle in lunar orbit relative to the true lunar equator

i_{VTE} — Inclination of the transearth trajectory to the MOP

i_{VTEQ} — Inclination of the transearth trajectory to the earth's equator

i_{VTL} — Inclination of the translunar trajectory to the MOP

$\hat{K}(K_1, K_2, K_3)$ — Unit vector from the center of the moon to a specific lunar feature, with components K_1, K_2, K_3 in the x_{MOP}, y_{MOP}, z_{MOP} directions, respectively

$\hat{L}$ — Unit vector in the MOP defining the required value of θ_M

ℓ, b — Lunar librations in longitude and latitude (selenographic coordinates of the earth)

$\ell_\odot$, $b_\odot$ — Selenographic coordinates of the sun

$\overrightarrow{MCP}$ — Mean center point (vector from the center of the moon in the direction of the mean center point)

$\overrightarrow{MEL}$ — Moon-earth line (vector from the center of the moon to the center of the earth)

MOP — Moon's orbital plane

$\overrightarrow{MSL}$ — Moon-sun line (vector from the center of the moon to the center of the sun)

R — Radius measured in units of ER

$R_M \equiv R_{\oplus☾}$ — Earth-moon distance in units of ER

r — Radius

T — Julian date; also, total flight time for a circumlunar trajectory (time from translunar injection to return vacuum perigee)

t — Time

t_E — Time of transearth injection

t_p — Flight time to pericynthion from translunar injection

t_R — Time at re-entry

UT — Universal time

V — Velocity

V_p — Parabolic or two-body earth escape velocity

x_M, y_M, z_M* — Selenocentric trajectory coordinate system (see Chapter IV, Section C)

x_{MOP}, y_{MOP}, z_{MOP} — Selenocentric nonrotating coordinate system with the x_{MOP}-axis defining the intersection of the true MOP with the true lunar equator

* In this list of symbols, coordinates in a given coordinate system have been listed together.

x_S, y_S, z_S	Selenographic coordinate system
$x_{\oplus}$, $y_{\oplus}$, $z_{\oplus}$	Geocentric equatorial (nonrotating) coordinate system
$x_{\oplus☾}$, $y_{\oplus☾}$, $z_{\oplus☾}$	Geocentric equatorial position of the moon
$x_{\oplus\odot}$, $y_{\oplus\odot}$, $z_{\oplus\odot}$	Geocentric equatorial position of the sun

Greek Symbols

β	Angle measured from the x_{MOP}-axis to the $\overrightarrow{MEL}$
β_L	Orbital central angle of the launch site measured positively northward from the ascending node
β_{M0}	Transearth injection position from lunar orbit, or orbital central angle of the vehicle in the lunar orbit, measured positively toward north from the ascending node
β_0'	Orbital central angle of the translunar injection point, measured positively northward from the ascending node
$\beta_{☾}$	Orbital central angle of the moon, measured from the ascending node of the lunar orbit around earth, $0° \leq \beta_{☾} < 360°$
γ	Flight path angle relative to the local horizontal on earth
ΔV	Equivalent velocity impulse to decelerate into or accelerate from lunar orbit
ΔV_i	Equivalent velocity impulse to change i_{VTE} by Δi_{VTE}
ΔV_{TE}	Equivalent velocity impulse to change the time of arrival at vacuum perigee by Δt
$\Delta\lambda$	Longitude increment, $\Delta\lambda$ with various subscripts and their interpretations given in the text and sketches of Section G
$\Delta\Phi$	Required shift in the x_E- direction (which defines the intersection of the translunar or transearth trajectory plane with the MOP)
δ	Angle measured from the $\overrightarrow{MCP}$ to the x_{MOP}- axis
$\delta_M \equiv \delta_{☾}$	Declination of the moon
ϵ_T	Selenographic longitude of the morning terminator, measured positive to the west
$(\epsilon_T)_{MOP}$	Longitude of the morning terminator, measured in the MOP from the $\overrightarrow{MEL}$
ζ	Hour angle, measured in the earth's equatorial plane westward from the midnight meridian to the ascending node of the lunar orbit around the earth
θ	Angle from the sun-earth line to the instantaneous position of the moon, projected into the ecliptic plane
θ_M	Angle in the MOP between the $\overrightarrow{EML}$ and the line toward the descending node, measured positively to the east
λ, ϕ'	Longitude, geocentric latitude
λ_{MOP}, ϕ_{MOP}	Lunar longitude, latitude defined with respect to the $\overrightarrow{MEL}$ and the MOP
$\lambda_{☾}$, $\phi_{☾}$	Selenographic longitude, latitude
ξ	Auxiliary angle defined in Section G
Φ_{LO}	Range angle from launch to injection in the earth parking orbit
ψ_0	Angle measured from the intersection of the translunar trajectory plane with the MOP to the injection point
Ω_{MOP}	Longitude of the ascending node relative to the MOP, or angle in the true lunar equator, measured from the intersection with the MOP to the ascending node of the orbit
$\omega_{☾}$	Rotational rate of the moon about its axis
$\omega_{\oplus☾}$	Angular velocity of the moon in its orbit around earth

Subscripts and Superscripts

adj	adjusted
d, h, m, s	days, hours, minutes of time, seconds of time
L	Launch
0	Injection
PE	Return vacuum perigee
TE	Transearth
TL	Translunar
$\odot$	Sun

⊕	Earth
☾	Moon
△	Space vehicle

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